# Coupling of Mechanical and Optical Methods for Simulations and Measurements

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*Summary* In order to investigate and analyse the dynamical-optical behavior of high precision optical systems like lithographic objectives and astronomical telescopes with low computational cost, integrated modeling methods and strategies are proposed. During an astronomical observation, even small mechanical vibrations can be sufficient to produce aberrated images. The mechanical behavior of such optical systems can be described by a combination of rigid body motion and small deformations. This leads to the utilization of elastic multibody approaches and model order reduction methods in combination with techniques from optical simulations. As a result, the exposed image influenced by mechanical excitations and structural deformations can be simulated and assessed.

## INTRODUCTION

High precision optical systems are very sensitive with respect to mechanical influences. In order to investigate and analyse the dynamical-optical behavior of high precision optics, integrated modeling strategies and methods are proposed, see also [1], [2] and [3]. For deriving a simplified mechanical model for time simulations with low computational cost, the method of elastic multibody systems (EMBS) in combination with model order reduction methods is used. Mechanical and optical simulation models are derived and implemented according to the workflow shown in Figure 1. In order to clarify these methods, mirror and lens applications are chosen as academic examples. Ground-based telescopes are highly resoluting optical systems consisting of precise mirrors. They are accurately mounted and they are very sensitive with respect to vibrations. During the observation time, small mechanical vibrations can be sufficient to produce inacceptably aberrated images. Even the adaptive optical unit or other motion systems can unintentionally excite the whole construction. Similar effects can happen at wafer scanning systems which use lithography objectives to project structures in a reticle onto wafers. Thereby, lens deformations and related stresses also influence the optical behavior during the exposure time [4].

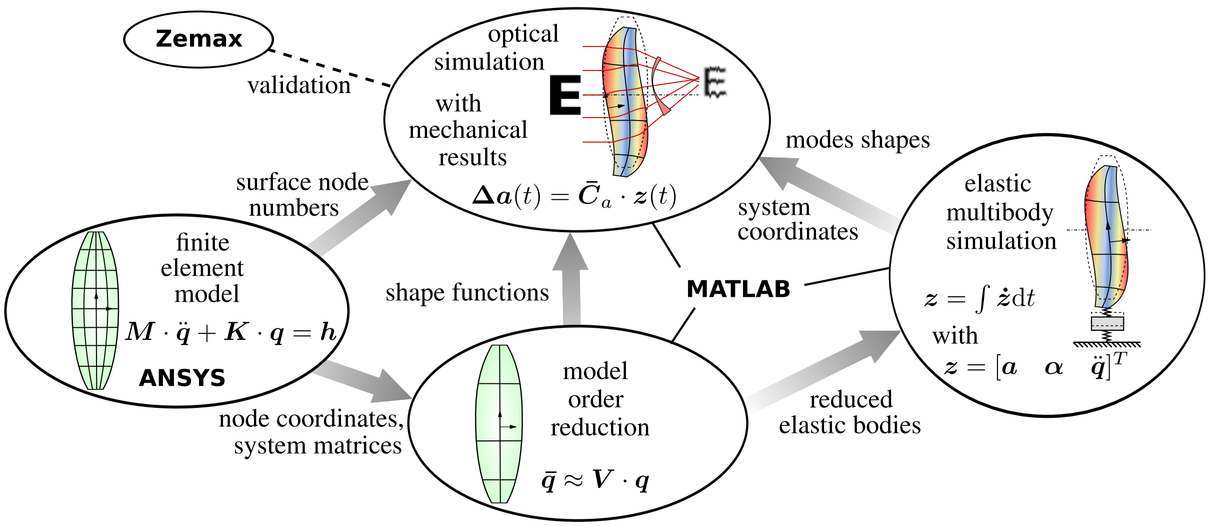


Figure 1: Basic procedure for describing dynamical-optical simulations.

## MECHANICAL MODEL

The mechanical model is based on EMBS with the floating frame of reference. In contrast to a detailed and large global model based on the finite element method, a reduced and modular EMBS leads to low computational effort for simulations in the time domain. Furthermore, the system can be analysed and assembled step by step and it is able to include only the relevant dynamical behaviour due to appropriate model order reduction methods. For a single elastic body in an EMBS, the structure of the equations of motion read

$$\left[\begin{array}{cc|c} m\boldsymbol{I} & m\tilde{\boldsymbol{c}}^T(\boldsymbol{q}) & \boldsymbol{C}_t^T \\ m\tilde{\boldsymbol{c}}(\boldsymbol{q}) & \boldsymbol{J}(\boldsymbol{q}) & \boldsymbol{C}_r^T \\ \hline \boldsymbol{C}_t & \boldsymbol{C}_r & \bar{\boldsymbol{M}}_e \end{array}\right] \cdot \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{\alpha} \\ \ddot{\boldsymbol{q}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \bar{\boldsymbol{D}}_e \cdot \dot{\boldsymbol{q}} + \bar{\boldsymbol{K}}_e \cdot \boldsymbol{q} \end{bmatrix} + \boldsymbol{h}_\omega = \boldsymbol{f}_a + \boldsymbol{f}_r, \tag{1}$$

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whereby the reduced equations of motion describing the deformation of the elastic body are embedded and the rigid body dynamics are included according to [5]. These equations can describe the mechanical system behaviour and the consideration of the optical behaviour is discussed in the following.

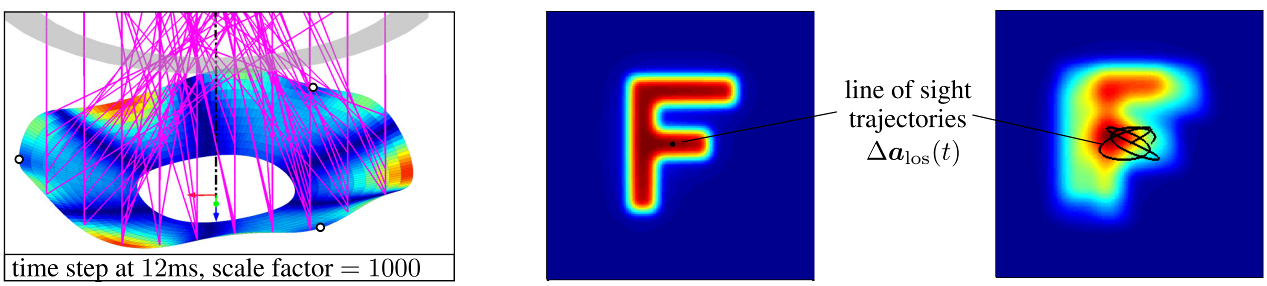
## OPTICAL MODEL

For sequential ray tracing through several lenses and mirrors during the optical simulation, a continuous description of the mechanically deformed surfaces is required. Thereby, the line of sight (LOS) describes the displacement of the center of intensity and the wavefront aberrations fitted by Zernike polynomials characterise the blurring and image quality [6]. In order to take also wave optical effects during an image simulation into account, Fourier optical methods are utilized. Furthermore, the relationship between a kinematic degree of freedom given by the generalized coordinates $\boldsymbol{q}$ and the relative aberrations $\Delta\boldsymbol{a}$ containing the LOS and the Zernike coefficients $c_j$ can be expressed by the kinematic-optical sensitivities. They can be collected in the matrix $\boldsymbol{C}_a$. This corresponds to the relations

$$\boldsymbol{\Delta a} = [\underbrace{\Delta x_{\mathrm{los}} \quad \Delta y_{\mathrm{los}}}_{\Delta\boldsymbol{a}_{\mathrm{los}}} \quad \underbrace{c_1 \quad c_2 \quad \dots \quad c_j \quad \dots \quad c_{\bar{k}}}_{\Delta\boldsymbol{a}_{\mathrm{wfa}}}]^T = \boldsymbol{C}_a \cdot \boldsymbol{q} \approx \underbrace{\boldsymbol{C}_a \cdot \boldsymbol{V}}_{\bar{\boldsymbol{C}}_a} \cdot \bar{\boldsymbol{q}}. \tag{2}$$

The so-called matrix of optical mode shapes $\bar{\boldsymbol{C}}_a$ is derived in combination with the projection matrix $\boldsymbol{V}$ of the model order reduction and can be obtained by means of ray tracing analyses during a static mode shape deformation.

Figure 2 illustrates results of an exemplary dynamical-optical simulation. On the left, a snapshot of a deformed mirror structure is visualized in an exaggerated manner. On the right, the time-accumulated irradiance is depicted, which delivers the exposed image of a regarded object.




(a) ray tracing simulation (b) neglecting mechanical vibrations (c) considering mechanical vibrations

Figure 2: Results of the dynamical-optical simulation.

## INVESTIGATIONS AND CONCLUSIONS

The influence of different model order reduction methods will be presented and analysed for an exemplary integrated model. It is based on an annular lightweight mirror with a parabolic shape made of aluminum. Here, the influence of the model order reduction methods is analysed and the mechanical-optical transfer behaviors of different wavefront aberrations are investigated. It is shown, that the mechanical-optical sensitivities depend on the frequency of excitation. Moreover, it is investigated, whether the sensitivities can be used during the model order reduction in order to describe the overall behavior with an optimized model. Finally, the performance of different simulation strategies is assessed and the application limits are identified.

### References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