The impact of the labour share on growth in the XXth century

Slim Bridji[a], Matthieu Charpe[a]

[a]International Labour Organization*

May 18, 2015

Abstract

This paper discusses the impact of the labour share on growth using historical national accounts for three countries: the United Kingdom from 1856-2010, France from 1896-2010 and the United States from 1898-2010. The value added of this paper is the use of data over a longer timespan than the usual system of national account series and the performance of single country estimations in contrast with existing panel data analysis. Another contribution of this paper is to perform a time-frequency analysis and a time-varying analysis of the relation between (functional) income distribution and growth. We find evidence of common information between growth and income distribution at low frequency, with the labour share leading growth. We also show that the sign of the coefficient associated with the labour share is negative at high frequencies and turns positive at low frequencies. Lastly, the coefficient associated with the labour share increases over time. ———————

Keywords: Distribution, growth, income distribution, Goodwin
JEL CLASSIFICATION SYSTEM: E24, **D31**, **N10**.

*We would like to thank Marco Gallegati, Philippe Askenazy, Michael Brei, Christian Proaño, Martin Ravallion, Willi Semmler and Engelbert Stockhammer for helpful suggestions.

1 Introduction

This paper assesses the impact of the distribution of income between labour and capital on economic growth for three countries: the United Kingdom from 1856-2010, France from 1896-2010 and the United States from 1898-2010 using the database of Piketty and Zucman (2014) as well as that of Groth and Madsen (2013). The link between the labour share and growth is studied using a historical time series covering at least the XXth centuries (and more), which enables a more long-term perspective on this topic. An additional feature of our work is the performance of a time-frequency analysis to test whether the sign of the relation between growth and the labour share changes across frequencies and over time.

The main motivation for this work is the renewed interest in studying functional income distribution. Although a major topic of study for classical economists, this issue has rarely been taken up in economics in recent times with the exception of a small part of the profession such as the Post-Keynesian for instance. This renewed interest is linked to the increasingly documented erosion in the labour share, which has called into question the previously well accepted stylized fact that the labour share is constant. The stability of factors shares has been long considered a stylized fact of macroeconomics. It is sometimes called the Bowley's law and has been alleged true by many economists such as Kaldor (1955) for instance.[1] This assertion has been somewhat unquestioned with the exception of Solow (1958) who, for example, argued that large changes in the labour share take place at industry level. More recently, the issue of the decline in labour share in the past two to three decades has been discussed in economic literature as for example by Guscina (2006), Young (2010) or Elsby et al. (2013). In particular, Piketty (2013) argues that profit share can be characterized by a U shape function of time: 35-40% in the XIX^{th} century, 20-25% in middle of the XX^{th} century and 25-30% in the early XXI^{st} century. It follows that the labour share cab be seen as an inverted U shape function of time. Beyond the debate about the long run stability of the labour share, Blanchard and Giavazzi (2003) argue that functional income shares experience significant fluctuations in the short to medium run. It would therefore be natural to enquire about the impact of changes in functional income distribution on growth.

The transmission channel from the labour share to growth can be understood only indirectly through modern macroeconomics, as single representative agent models rule out any impact of changes in income distribution by definition. A first transmission channel exists via the labour-demand effect associated with changes in labour cost, as for example, in search and matching models.[2] Higher wages reduce the surplus from an additional match in the labour market and lead firms to reduce vacancy posting. This is translated into

[1] Bowley's law has been labelled as such by Samuelson. For more details about the origin of the labour share stability in economists see Kramer (2011).

[2] See Gertler and Trigari (2009) or Gali et al. (2011) for instance.

lower employment and lower production. Another transmission channel relates to credit constrained households. Rule-of-thumb households create a link between labour income and consumption decisions[3]. In a model with endogenous credit constraints, Kumhof et al. (2015) show that lower wages can lead to over-indebtedness and a Minsky moment.[4] Previously, Goodwin (1967) highlighted the profit squeeze effect of a higher labour share.

This paper also attempts to determine whether the link between income distribution and growth differs across different time horizons. The main motivation for applying frequency analysis to the question of distribution and growth is that both theoretical models as well as empirical studies point to different sign of the relation that can be related to different time scale. Theoretical DSGE models[5] with a search and matching module of the labour market point to the negative impact of higher wage on vacancy posting at the business cycle frequency. Models with endogenous credit constraints address economic dynamics that take place over a longer time frame than business cycle DSGE models. In Kumhof et al. (2015), the model is calibrated on an annual frequency and generates 50-year impulse responses following highly auto-regressive shocks $\rho = 0.96$. Similarly, there has been much debate with respect to the frequency relevance of the Goodwin model. Goodwin (1967) argues that his paper is a "model of cycles in growth rates", which offers "better prospects than the more usual treatment of growth theory or of cycle theory", which leaves open the question of the time scale of his contribution.[6] Atkinson (1969), for instance, discusses the time scale of the Goodwin model, concluding that the model is better suited to the 16-22 year Kuznets cycle than the trade cycle.

Empirical studies of functional income distribution also point to different properties of the labour share at different time scales. The labour share shows strong counter-cyclical behaviour at the business cycle frequency. This can be explained by labour hoarding and/or a lag in the adjustment of wages to economic fluctuations. On the other hand, Bentolila and Saint-Paul (2003) show that the labour share is determined by technology as well as by the bargaining power of workers and firms in the long run (proxied by trade and financial globalization).

The time-frequency analysis is performed using wavelet analysis. The wavelet analysis allows the identification of local and global dynamic properties of a signal process (e.g. a stochastic process) at different time scales (time horizons). In other words, the wavelet analysis makes use of a transform that decomposes a signal process into different time

[3] An example of DSGE models with rule-of-thumb households is Galí et al. (2007).

[4] A shock on the bargaining power of workers is one way to modify income distribution in DSGE model with a search and matching on the labour market. The impact on functional income distribution may be limited as a change in the real wage is offset to some extent by an opposing change in labour demand. In order to produce large distribution effects Kumhof et al. (2015) model a bargaining shock while the employment level is kept constant.

[5] Dynamic Stochastic General Equilibrium model.

[6] A recent version of the Goodwin model can be found in Chiarella and Flaschel (2000)

horizons. The wavelet transform consists of two basic functions: the mother wavelet function and the father wavelet function. The former is used to capture the properties of the signal process at high frequencies (detailed parts of a signal) while the latter is used to capture the properties of the signal process at low frequency (smooth part of a signal). In order to obtain the different detailed parts of a signal process, the mother function is dilated and shifted. Wavelets have been increasingly applied in economics, especially in finance as for example, by Gallegati and Ramsey (2013), Gallegati et al. (2011) or Aguiar-Conraria and Soares (2012). The choice of wavelet is motivated as HP filters have been criticized for overdifferencing, which may cause spurious autocorrelation (see Canova (1998) for a discussion of the pros and cons of different filtering techniques). Other frequency methods such as Fourier transforms lose the time dimension of the data, while wavelets combine both the frequency and the time dimensions. Additionally, Fourier transforms are not well suited to non-stationary data as they are global methods. A single disturbance in the data under a Fourier transform will be translated into the entire series. On the contrary, wavelets are local as they are constructed over a finite interval of time. The local features of wavelets make them well suited to study series that include breaks, such as those produced by wars for instance.

The drawback of using historical data is the estimation of a stable coefficient over a long period of time, despite the major transformations experienced by the three countries considered over the XX^{th} century. In order to avoid this shortcoming, frequency analysis is combined with time analysis made possible by wavelet-specific tools such as power spectrum, coherency analysis and cross-wavelet power. We also look at the stability of the relation between distribution and growth over time by performing rolling correlation and rolling regression using different size of the window.

This paper adds to the literature estimating the consumption, investment and competitiveness effects associated with a change in functional income distribution as in Stockhammer et al. (2009) for instance. Existing studies usually only go back to the 1960s, as national account series are not uniformly available before this date. This limits empirical investigation to either performing single country estimation on a restricted number of data points or to performing panel data regression. The former is limited by the degrees of freedom while the latter pools together heterogenous countries and estimates an average effect across countries. Closely related to this is the literature testing the Goodwin model. Mohun and Veneziani (2006) conclude, based on US data since 1950, that the Goodwin model applies to the business frequency, while Kauermann et al. (2012), using quarterly data since 1950 and applying penalized spline regression, argue in favour of a long run Kondratiev cycle between income distribution and growth.

This paper also shares similarities with the inequality-growth debate. There is vast empirical literature assessing the growth effect of changes in personal income distribution using GINI indexes as a proxy for personal income distribution. The literature has evolved from using cross-sectional data as in Alesina and Rodrik (1994) towards using panel data as the database on GINI indexes has improved. This literature tests for a large range of transmission channels such as the level of economic development, non-linear effects (Banerjee and Duflo, 2003) and duration of growth spells (Berg and Ostry, 2011) for instance. Interestingly, Halter et al. (2014) show that inequality affects growth positively in the short run and negatively in the long run. A difference with our work is that they look at inequality, while we look at factor shares. Another difference is that they do not use a time-frequency specific methodology to test for the time dimension whereas we do.[7] Lastly, few attempts have made use of the top income share as a proxy for income distribution. Andrews et al. (2011) find a small positive effect of an increase in top income on growth using a panel data for 12 countries over the period 1960-2000.

The paper is organized as follows: Section 2 discusses the data source and the definition of the labour share with a focus on the measure of self-employment income. Section 3 is methodological and presents the main properties of wavelets as well as wavelet-specific instruments such as power spectrum, coherency analysis and cross-wavelet power. Section 4 discusses the distribution of information over time and across frequencies as well as the covariance between growth and income distribution over time and across frequencies using continuous wavelet analysis. Section 5 performs scale by scale correlation and regressions to test whether the sign of the relation changes across frequencies. This section makes use of discrete wavelet analysis. Section 6 discusses the stability of the relation over time and section 6 concludes.

2 The labour share of income in France, the UK and the USA

2.1 Definitions and trends

This section describes the data source and the various definitions of the labour share of income. The data source for France and the UK are taken from Piketty and Zucman (2014) (PZ from now on), while the data source for the United-States is taken from Groth and Madsen (2013). For consistency, the analysis presented in the following sections is reproduced in the appendix using the PZ data for the United-States.

The labour share of income measures the share of income that goes to labour as opposed to the capital share of income, which measures the share of income that goes to capital. Labour income is based on national account and sums up different components of the compensation of employees. The definition of the labour share ls based on the PZ data in France and the UK is described in eq 1. Total labour income is the sum of the compensation of employees paid by corporation ce_c and paid by the government ce_g.

[7] Dominicisn et al. (2008) provide a meta-analysis and calls for the use of single country estimation.

$$ls = \frac{(ce_c + ce_g + ce_c * ndp_{hh}/ndp_c)}{ndp - pt} \tag{1}$$

Total labour income is also augmented by the imputation of the labour income of the self-employed defined as the labour share in the corporate sector $\left(\frac{ce_c}{ndp_c}\right)$ multiplied by the net domestic product of the non-corporate business sector ndp_{hh}. ndp_c is the net domestic product of the corporate sector. This imputation assumes that the distribution of income between labour and capital in the non-corporate business sector is identical to the distribution of income between labour and capital in the corporate sector. The denominator is a measure of national income: net domestic product ndp minus production taxes. Subtracting production taxes from the denominator corresponds to a measure of the labour share at factor costs.

$$cs = \frac{(gcp_c - cd_c + ndp_h + (gcp_c - cd_c) * ndp_{hh}/ndp_c)}{ndp - pt} \tag{2}$$

The capital share cs is defined in eq 2 as the sum of gross corporate profits net of capital depreciation in the corporate business sector $(gcp_c - cd_c)$, the net domestic product in the housing sector ndp_h and the imputed capital share in the non-corporate business sector divided by a measure of national income $(ndp - pt)$. The imputed capital share in the non-corporate business sector is the capital share in the corporate sector $\left(\frac{gcp_c - cd_c}{ndp_c}\right)$ multiplied by the net domestic product of the non-corporate business sector ndp_{hh}. Under this definition, the labour share of income and the capital share of income sums to one and can be used interchangeably. This imply a small difference with the definition used in PZ, which incorporate foreign labour income in the numerator and net foreign factor income in the denominator. However, the differences are small as shown in Figure 1.[8]

In the United-States, primary income distribution data goes back to 1929 in the Piketty-Zucman database.[9] The definition follows the definition of the labour share in France as the self-employment labour income is imputed from the labour share in the corporate sector. The time series are therefore shorter than in the other two countries. The series also start with the Great Depression and shows large fluctuations at the beginning of the series.

An alternative is the database by Groth and Madsen (2013), which provide labour share data based on historical source before 1960 and OECD data after 1960. For the

[8] The definition used in PZ refers to Table FR.11b and Table UK.11a downloaded from http://piketty.pse.ens.fr/fr/capitalisback. The labour share is defined as $ls_{pz} = \frac{ce_c + ce_g + ce_f + ce_c * ndp_{hh}/ndp_c}{ndp + nffi - pt}$, while the capital share is defined as $cs_{pz} = \frac{ci_c + ci_h + ci_g + ci_f + ci_c * ndp_{hh}/ndp_c}{ndp + nffi - pt}$.

[9] These definition refers to Tables US.11 and US.10.

United-States, the historical data are taken from Liesner (1989). The labour share is defined as total labour costs over value-added in the corporate non-agricultural private sector (this definition is labelled ls_{gm} in the rest of the paper). This definition is slightly more restrictive than ls or ls_{pz} as it excludes some sectors from the definition. However, it is close to the definition of the labour share in the corporate sector $\left(\frac{ce_c}{ndp-pt}\right)$ and has the advantage of starting in 1898. The original time series in Groth and Madsen (2013) ended in 2001 and have been updated using OECD data.

[Figure 1 about here.]

Figure 1 displays the labour share in the three countries considered. Each figure displays the labour share ls as well as the definition ls_{pz}. In the three countries considered, the evolution of the labour share over the period 1950-2010 is in line with existing studies. The labour share increases until the 1970s before to decline up to the Great Recession in the late 2000s. A difference between France and the other two countries is that both the increase and the decrease are much more gradual. The labour share is constant in the 1960s and increases abruptly in the 1970s. The correction is as abrupt as the increase and took place almost entirely in the 1980s with the labour share stabilizing at a lower level than the pre-1970s level. There is little differences between the ls definition and the ls_{pz} definition in France and the UK. In the US, the rise in the labour share in the 1970s is much less apparent under the ls_{gm} definition than under the two alternative definitions. Additionally, the decline in the labour share since the 1980s is much more pronounced under the ls_{gm} definition.

The labour share shows a similar pattern in France and in the UK between 1900 and 1950. The labour share declines up to the Great Depression and recovers afterwards. The labour share then cumulates with the Second World War. In both countries, the two World Wars have had an important impact on the labour share. Both wars are characterized by a large increase in the labour share as these events are associated with large capital destruction, poorly functioning market economies, and a large increase in the size of public sector and public employment at least via military enrollment. While in France, the labour share quickly returns to its pre-crisis level after World War I, in the UK, the war had a threshold effect the labour share being higher than its pre-war level. The threshold effect is associated with an increase in the labour share in the corporate sector that remains high until the World War II. In the United-States, the labour share displays a slightly different evolution. It first increases up to the World War I and then declines up to the Great Depression before to recover during World War II. The labour share series date back to 1855 in the UK. Over the second half of the XIXth century the labour share first declines up to 1875 and then recovers up at the end of the century.

Piketty (2013) describes the evolution of the capital share in France and the UK as a U shape function of time over the period going from the beginning of the XIX century and the beginning of the XXI century. The capital share is around 35%-40% in the XIX century, it then drops to 20%-25% in the middle of the XXth century and then recovers to 25%-30% at the beginning of the XXIth century. It would follow that the evolution of the labour share is an inverted U shape function of time. Figure 1 is restricted to the XXth century in France and starts only in 1856 in the United-Kingdom. The average labour share is 0.73 in France between 1896 and 1939. It then increases to 0.78 over the period 1950-1974 (excluding World War II) and then drops to 0.76 between the period 1993-2010 (excluding the 70s and 80s boom and bust). The amplitude of the fluctuations are much more restricted than that described by Piketty as he describes fluctuation by 5 percentage point to 10 percentage point. Similarly, in the UK, the labour share is on average respectively at 0.73, 0.77 and 0.73 over the same subperiod (the first period starting in 1856 rather than 1896). In the United-States, the labour is 0.64, 0.65 and 0.61 over the same subperiods. In the United-States, the labour share is quite constant until the marked decline from the end of the 1970s. As underlined previously, the steady increase in the labour share in the post World War II is less marked using the Groth and Madsen series as the sectoral coverage is restricted to the corporate non-agricultural private sector. Regardless of whether the labour share is U shaped, the labour share displays large fluctuations. The question arises whether this fluctuations have an impact on economic growth.

Explaining changes in the labour share is not straightforward as it results from both technological changes and modification in the bargaining power of workers. Bentolila and Saint-Paul (2003) shows for instance that the wage share depends on the capital output ratio and the type of production function considered. Contrastingly, institutional variables usually include proxy for trade and financial globalization, as well as labour market regulation. Harrison (2002) or Jaumotte and Tytell (2007) find evidence that trade openness affects negatively the labour share of income in both high and low income countries. Jayadev (2007) underlines the negative impact of capital account openness and financial globalization on the the labour share of income. Lastly, labour market regulation has ambivalent effect on functional income distribution, as it may have opposite price effect and quantity effect (see Checchi and Garcia-Penalosa (2010)). Recently, Elsby et al. (2013) pointed to the importance of offshoring.

2.2 Imputing the Self-employment labour income

One important issue is to account for the labour income of the self-employed. As self-employment labour income cannot be directly observed, the imputation method chosen may affect the trend in the labour share (Gollin, 2002). There are usually three alternative adjustments: attributing all self-employment income to labour income (ls_{se1}), measuring national income net of the non-corporate business sector (ls_{se2}) and imputing the distribution of labour and capital of self-employment income based on the corporate sector either using production data as in Piketty and Zucman (2014) or employment data as in Bentolila and Saint-Paul (2003).

Figure 2(d) illustrates the three possibilities in the case of France. ls_{se1} is defined as $ls_{se1} = \frac{ce_c + ce_g + gmi_{hh} + ce_{hh} - cd_{hh}}{ndp - pt}$ with $(gmi_{hh} + ce_{hh} - cd_{hh})$ being equivalent to the net domestic product of the non-corporate business sector.[10] The main difference with eq 1 is that the decline in the size of the non-corporate business sector after world war II generates a decline in the labour share of income under this definition. The definition ls_{se1} clearly over-estimates the decline in the labour share of income in recent years. The second alternative for self-employment income is $ls_{se2} = \frac{ce_c + ce_g}{ndp - pt - (gmi_{hh} + ce_{hh} - cd_{hh})}$. This definition subtracts from national income the size of the non-corporate business sector. The assumption is that the capital labour distribution in the non-corporate business sector is similar to the distribution in the rest of the economy. The main difference with the eq 1 is that the labour share shows an upward trend over the period 1945-1970. These two alternative ways to account for self-employment income shows that the imputation of the self-employment labour income in eq 1 is comprised between these two alternative definitions over most of the sample.

The labour share computed in Piketty and Zucman for the U.K. differs from the labour share computed in France with respect to the measurement of self-employment labour income. In the case of France, self-employment income is imputed based on the share of labour income in the corporate sector. In the case of the U.K., self-employment income is imputed using employment data. Self-employment income is the sum of three components: non-corporate wages, labour share of agricultural self-employment net income and labor share of non-agricultural self-employment net income.[11]

$$ls_{pz}^{uk} = \frac{ce_c + ce_g + ce_f + ce_{hh} + ce_{sea} + ce_{sena}}{ndp + nfi - pt - pt_f} \tag{3}$$

with ce_c wages and social contributions paid by corporations, ce_g wages and social contributions paid by govt, ce_f net foreign labor income, ce_{hh} wages and social contributions paid by non-corporate business (and households), ce_{sea} labor share of agricultural self-employment net income and ce_{sena} labor share of non-agricultural self-employment net income. pt is product taxes (total) and pt_f is net foreign product taxes and subsidies.[12]

[10] gmi_{hh} is gross mixed income of self-employment, ce_{hh} is wages and social contributions paid by non-corporate business and cd_{hh} is capital depreciation of non-corporate business.

[11] This definition refers to Table UK.11a.

[12] According to Piketty and Zucman (2014): "Net foreign product taxes and subsidies were computed as the difference between current account total and components (excluding this term); for years 1997-2010, where detailed foreign product taxes and subsidies series are available, the sum and total coincide almost perfectly."

The non-corporate wage is based on the authors' assumption regarding the evolution of wages over the period 1855 to 1876 and is then kept fixed up to 1986. The labour share of agricultural self-employment net income is based on the compensation of employees in the farm sector adjusted for the self-employment ratio in the farm sector. The labour share of the non-agricultural self-employment net income is based on average wage of salaried workers multiplied by a scaling factor (largely ad hoc) times non-agricultural self-employment. The details of this definitions and imputations is discussed further in the appendix 9. Figure 2(b) shows the two definitions ls and ls_{pz}^{uk}. The difference between the two lines reflects the different imputation methods. The differences are larger than in the case of France but the trends is similar under both definitions. For the sake of comparability, the imputation of the self-employment labour income in the UK follows eq 1. However, the results of the regression analysis carried out in section 5 is reproduced in appendix 10 using the ls_{pz}^{uk} definition.

3 Wavelet analysis

It is well known that macroeconomic time series may show different relationships at different time horizons. Indeed, economic agents pursue different objectives at different time horizons (frequencies or scales). This has led economists to use spectral tools, such as the Fourier analysis, to explore relationships between macroeconomic time series across frequencies. However, the Fourier analysis presents some important limitations. First, if the spectral analysis is able to identify the main cyclical co-movements in the data it fails to capture their transient relations and when changes in the cyclical co-movements occur. This is attributable to the fact that in spectral analysis, data are, by definition, explored only through the frequency domain; time information of the data are then fully lost. Second, the spectral analysis is suitable only with stationary time series which is quite restrictive as most of the macroeconomic time series exhibits non-stationary patterns (Aguiar-Conraria and Soares, 2011; Gençay et al., 2002).

To overcome the limitations of the spectral analysis, some economists proposed to use instead wavelet tools (for instance Aguiar-Conraria and Soares, 2012; Gallegati and Ramsey, 2013; Gallegati et al., 2011; Ramsey et al., 2010). Indeed, wavelet analysis is able to map all the information of a time series into specific frequencies and time (Aguiar-Conraria and Soares, 2011; Gençay et al., 2002). The wavelet approach considers two kinds of wavelet transforms, mapping original time series into functions of time and frequency, that is the continuous and discrete wavelet transforms. The continuous wavelet transform shows highly redundant information on the data. Thereby, the results obtained with the continuous wavelet transform offer a clear picture on the co-movements of time series across time and frequencies. In turn, the discrete wavelet transform does not show redundant information on the data. Indeed, contrary to the continuous wavelet transform, it is computed for only a selection of points on the time-frequency space. Nonetheless, the discrete wavelet transform can be convenient when one is interested in applying the multivariate time-domain econometric tools on the transformed time series (Aguiar-Conraria and Soares, 2011). To gain redundancy from the discrete wavelet transform, one can instead make use of the maximum overlap discrete wavelet transform. The maximum overlap discrete wavelet transform is a redundant transform (but less than the continuous wavelet transform) because while it is computed for a selection of frequency points, it considers every points in time (Aguiar-Conraria and Soares, 2011).

In our empirical work we consider both kinds of wavelet transforms. While we use the continuous wavelet tools to get a precise description of how labour share relates to growth across time and frequencies in France, the UK and the US, we use the discrete wavelet tools to make then our work comparable with the existing literature on inequality and growth which has usually relied on standard time series econometric methods. Hence, in the subsequent subsections we introduce in a more formal way the continuous and discrete wavelet analyses.

3.1 Continuous wavelet analysis

The content of this subsection relies highly on Aguiar-Conraria and Soares (2011), Aguiar-Conraria and Soares (2012), Grinsted et al. (2004) and Lilly and Olhede (2010).

3.1.1 The mother wavelet function

A mother wavelet is a function of time that spans on the real space, $\psi(t) \in L^2(\mathbb{R})$, that satisfies the following admissibility condition

$$0 < C_\psi := \int_{-\infty}^{\infty} \frac{|\Psi(\omega)|}{|\omega|} d\omega < \infty \tag{4}$$

where C_ψ is the admissibility condition and $\Psi(\omega)$ denotes the Fourier transform, a function of angular frequency ω. Assuming that $\psi(t)$ is a function with sufficient decay, the previous admissibility condition (4) can be restated as follows

$$\Psi(0) = \int_{-\infty}^{\infty} \psi(t)\, dt = 0. \tag{5}$$

The assumed decaying property of the mother wavelet function enables the localization in both time and frequency.

3.1.2 The continuous wavelet transform

From a mother wavelet function $\psi(t)$, one can get a set $\psi_{\tau,s}(t)$ of wavelet daughters by scaling and translating $\psi(t)$

$$\psi_{\tau,s}(t) := \frac{1}{|s|}\psi\left(\frac{t-\tau}{s}\right), \quad s,\tau \in \mathbb{R},\ s \neq 0$$

where s is a parameter that controls the width of the wavelet and τ is a parameter that controls the location of the wavelet in the time domain. The parameter s, named scale or dilation parameter, governs the position of the mother wavelet function in the frequency domain. Taking large scale values, $|s| > 1$, means that the mother wavelet function is dilated to capture low frequency features of the data. Instead, taking small scale values, $|s| < 1$, means that the mother wavelet function is compressed to capture high frequency features of the data.

The continuous wavelet transform (CWT) of a time series $x(t)$ is a projection of $x(t)$ onto a specific mother wavelet function $\psi(t)$

$$W_x(\tau, s) = \int_{-\infty}^{\infty} x(t)\frac{1}{|s|}\psi^*\left(\frac{t-\tau}{s}\right)dt \tag{6}$$

where * denotes the complex conjugate. Under the admissibility condition (5), the CWT does not alter the energy (variance) of $x(t)$[13].

It is interesting to note that the wavelet transform actually defines a time-scale representation rather than a time-frequency representation. Thus, to get a time-frequency representation one needs to convert scales into angular frequencies ω or into Fourier frequencies f (number of cycles per unit of time) according to the following correspondence

$$\begin{aligned} \omega(s) &= \frac{\omega_\psi}{s} \\ f(s) &= \frac{\omega_\psi}{2\pi s} \end{aligned} \tag{7}$$

where ω_ψ denotes a specific frequency to be properly chosen. Researchers have proposed different rules for calculating ω_ψ. This means that the (inverse) relation between scales and frequencies is a matter of interpretation.

3.1.3 The continuous wavelet tools

The (local) wavelet power spectrum of a time series $x(t)$ is defined as follows

$$WPS_x(\tau, s) = |W_x(\tau, s)|^2 \tag{8}$$

[13]Henceforth, $x(t)$ can be recovered from its CWT

It shows how the local variance of $x(t)$ changes across time and scales/frequencies.

The continuous wavelet approach offers as well some interesting tools to analyse how two time series relate to each other across time and frequencies: the cross wavelet power, the wavelet coherency and the wavelet phase-difference. Before introducing these cross-wavelet tools, a definition for a cross-wavelet transform of two time series $x(t)$ and $y(t)$ is needed

$$W_{x,y}(\tau,s) = W_x(\tau,s)\,W_y^*(\tau,s) \tag{9}$$

The cross-wavelet power of two time series $x(t)$ and $y(t)$ which is defined as follows

$$XWP_{x,y}(\tau,s) = |W_{x,y}(\tau,s)|$$

shows how the local covariance between these two time series varies across time and scales/frequencies. Hence, the cross-wavelet power is a useful tool to identify regions in the time-scale/frequency space where $x(t)$ and $y(t)$ have high common power.

The wavelet coherency of two time series $x(t)$ and $y(t)$ which is defined as follows

$$0 \leq R_{x,y}(\tau,s) = \frac{|S(W_{x,y}(\tau,s))|}{\left[S\left(|W_x(\tau,s)|^2\right)S\left(|W_y(\tau,s)|^2\right)\right]^{1/2}} \leq 1 \tag{10}$$

shows how the local correlation between these two time series changes across time and scales/frequencies. S is a smoothing operator in both time and scale.

The phase-difference of two time series $x(t)$ and $y(t)$ which is defined as follows

$$\phi_{x,y}(\tau,s) = \arctan\left(\frac{\mathcal{I}(W_{x,y}(\tau,s))}{\mathcal{R}(W_{x,y}(\tau,s))}\right) \tag{11}$$

shows how the causal relationship between these two time series evolves across time and scales/frequencies[14]. $\mathcal{R}(X)$ and $\mathcal{I}(X)$ denote respectively the real and the imaginary part of X. $\phi_{x,y}(\tau,s) = 0$ means that $x(t)$ and $y(t)$ move together at the specified time-frequency (τ,s); when $\phi_{x,y}(\tau,s) \in \left(0,\frac{\pi}{2}\right)$ $x(t)$ and $y(t)$ are positively correlated (in phase) with $x(t)$ in the lead; when $\phi_{x,y}(\tau,s) \in \left(-\frac{\pi}{2},0\right)$ $x(t)$ and $y(t)$ are again positively correlated (in phase) but with $y(t)$ in the lead; $\phi_{x,y}(\tau,s) = \pi$ or $\phi_{x,y}(\tau,s) = -\pi$ mean that $x(t)$ and $y(t)$ are negatively correlated (in anti-phase); when $\phi_{x,y}(\tau,s) \in \left(\frac{\pi}{2},\pi\right)$ $x(t)$ and $y(t)$ are negatively correlated with $y(t)$ in the lead; and when $\phi_{x,y}(\tau,s) \in \left(-\pi,-\frac{\pi}{2}\right)$ $x(t)$ and $y(t)$ are again negatively correlated but with $x(t)$ in the lead.

[14] Sometimes the phase-difference is alternatively defined as follows

$$\phi_{x,y}(\tau,s) = \arctan\left(\frac{\mathcal{I}(S(W_{x,y}(\tau,s)))}{\mathcal{R}(S(W_{x,y}(\tau,s)))}\right)$$

where the phase-angle is computed from the smoothed cross-wavelet transform instead of the cross-wavelet transform. The definition used in the main text is more convenient because it is consistent with the individual phases of the time series.

3.1.4 The choice of the mother wavelet function

The wavelet literature proposes many kinds of mother wavelet functions with different characteristics. Thus, it is important to make a choice of the mother wavelet function according to the aim of the empirical study undertaken. If, as in our case, the research interest is on the cycles synchronism of several time series one should use a complex-valued mother wavelet function. Indeed, a complex-valued mother wavelet function is able to separate the amplitude and phase information of the time series[15]. Among the complex-value mother wavelet functions, the analytic mother wavelet functions present an appealing characteristic[16]. The associated analytic wavelet transform is able to estimate the instantaneous amplitude and the instantaneous phase of the time series in the neighbourhood of each point in the time-scale space.

In our work we use one particular analytic mother wavelet function, the Morlet wavelet function, which is defined as follows

$$\psi_{\omega_0}(t) = \frac{1}{\sqrt{\pi}} e^{i\omega_0 t} e^{-\frac{t^2}{2}} \tag{12}$$

with $\omega_0 > 5$[17]. The Morlet wavelet function is appealing because it has four interesting properties besides being analytic. First, the different rules for calculating ω_ψ provide the same result: $f(s) = \frac{\omega_0}{2\pi s}$. Second, taking $\omega_0 = 6$, the correspondence between Fourier frequencies and scales is highly simplified: $f = \frac{6}{2\pi s} \approx \frac{1}{s}$. Third, the joint time-frequency concentration of the Morlet wavelet function is optimal. Fourth, the Morlet wavelet function offers the best compromise between time and frequency concentration.

3.2 Discrete wavelet analysis

The content of this subsection relies highly on Gençay et al. (2002), Percival and Walden (2006), Ogden (2012), Ott (2012), Percival (2014).

3.2.1 The multiresolution wavelet analysis

The basic idea of the multiresolution wavelet analysis (MRA) is to decompose a time series x_t into several components with different cycle periodicities:

$$x_t = s_{J,t} + \sum_{j=1}^{J} d_{j,t},\ t = 0, \ldots, N-1 \tag{13}$$

[15]Information on both amplitude and phase of the time series can be extracted only if the wavelet transform is actually complex-valued.

[16]A wavelet function is called analytic if the associated Fourier transform is supported only in $\mathbb{R}^+$: $\Psi(\omega) = 0$ for $\omega < 0$.

[17]This condition guarantees that the Fourier transform of the Morlet wavelet function is supported only in $\mathbb{R}^+$

where J denotes the number of scales or multiresolution components to consider. The component $d_{j,t}$, named the jth level wavelet detail, represents the change in the time series x_t on a scale of length $\lambda_j = 2^{j-1}$. The component $s_{J,t}$, named the Jth level wavelet smooth, represents the cumulative sum of the changes in the time series x_t, that is the long term changes in x_t. The scale levels j can also be interpreted in the time domain. In particular, $d_{j,t}$ captures the oscillations of x_t within a window of $\left[2^j, 2^{j+1}\right]$ periods. As well, $s_{J,t}$ captures the long term oscillations of x_t over more than 2^{j+1} periods. Those periods can be measured for instance in days, month, quarters, or years. The MRA is performed using the discrete wavelet transform or its alternative form, that is the maximal overlap discrete wavelet transform, which we describe below.

3.2.2 The maximal overlap discrete wavelet transform

The maximal overlap discrete wavelet transform (MODWT) of a time series x_t is represented by the following matrix equation

$$\boldsymbol{w} = \mathcal{W}\boldsymbol{x} \tag{14}$$

where $\boldsymbol{x} = (x_0, x_1, \cdots, x_t, \cdots, x_{N-1})'$ is a $(N \times 1)$ vector of the observations of x_t, $\boldsymbol{w} = (\boldsymbol{w}_1, \boldsymbol{w}_2, \cdots, \boldsymbol{w}_j, \cdot, \boldsymbol{w}_J, \boldsymbol{v}_J)'$ is the $((J+1)\,N \times 1)$ vector of MODWT coefficients, and $\mathcal{W}$ is a $((J+1)\,N \times N)$ matrix defining the MODWT. The elements $\boldsymbol{w}_j$ in the vector $\boldsymbol{w}$ are $(N \times 1)$ vectors of wavelet coefficients. The wavelet coefficients in $\boldsymbol{w}_j$ characterize the behavior of $\boldsymbol{x}$ on a scale of length $\lambda_j = 2^{j-1}$. The element $\boldsymbol{v}_J$ in the vector $\boldsymbol{w}$ is the $(N \times 1)$ vector of scaling coefficients. The scaling coefficients in $\boldsymbol{v}_J$ characterize the long term behavior of $\boldsymbol{x}$, that is on a scale of length $2\lambda_J = 2^J$. The matrix $\mathcal{W}$ has submatrices such as

$$\mathcal{W} = \begin{bmatrix} \mathcal{W}_1 \\ \mathcal{W}_2 \\ \vdots \\ \mathcal{W}_J \\ \mathcal{V}_J \end{bmatrix}$$

The $(N \times N)$ submatrices $\mathcal{W}_1, \mathcal{W}_2, \cdots, \mathcal{W}_J$ are made up of rescaled wavelet filters $\boldsymbol{h}_j/2^j$, $j = 1, \ldots, J$, arranged as follows

$$\mathcal{W}_j = \left[\boldsymbol{h}_j^{(1)}/2^j, \boldsymbol{h}_j^{(2)}/2^j, \cdots, \boldsymbol{h}_j^{(N-2)}/2^j, \boldsymbol{h}_j^{(N-1)}/2^j, \boldsymbol{h}_j/2^j\right]'$$

where vector $\boldsymbol{h}_j^{(k)}$ is the circularly shifted vector $\boldsymbol{h}_j$ by factor k

$$\begin{aligned} \boldsymbol{h}_j &= [h_{j,N-1}, h_{j,N-2}, \cdots, h_{j,1}, h_{j,0}]' \\ \boldsymbol{h}_j^{(1)} &= [h_{j,0}, h_{j,N-1}, h_{j,N-2}, \cdots, h_{j,2}, h_{j,1}]' \\ \boldsymbol{h}_j^{(2)} &= [h_{j,1}, h_{j,0}, h_{j,N-1}, h_{j,N-2}, \cdots, h_{j,3}, h_{j,2}]' \end{aligned}$$

and so on. That is

$$\mathcal{W}_j = 2^{-j}\begin{bmatrix} h_{j,0} & h_{j,N-1} & h_{j,N-2} & h_{j,N-3} & \cdots & h_{j,3} & h_{j,2} & h_{j,1} \\ h_{j,1} & h_{j,0} & h_{j,N-1} & h_{j,N-2} & \cdots & h_{j,4} & h_{j,3} & h_{j,2} \\ h_{j,2} & h_{j,1} & h_{j,0} & h_{j,N-1} & \cdots & h_{j,5} & h_{j,4} & h_{j,3} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ h_{j,N-2} & h_{j,N-3} & h_{j,N-4} & h_{j,N-5} & \cdots & h_{j,1} & h_{j,0} & h_{j,N-1} \\ h_{j,N-1} & h_{j,N-2} & h_{j,N-3} & h_{j,N-4} & \cdots & h_{j,2} & h_{j,1} & h_{j,0} \end{bmatrix} \tag{15}$$

The $(N \times N)$ submatrix $\mathcal{V}$ is made up of the rescaled scaling filter $\boldsymbol{g}_J/2^J$, arranged as follows

$$\mathcal{V}_J = \left[\boldsymbol{g}_J^{(1)}/2^J, \boldsymbol{g}_J^{(2)}/2^J, \cdots, \boldsymbol{g}_J^{(N-2)}/2^J, \boldsymbol{g}_J^{(N-1)}/2^J, \boldsymbol{g}_J/2^J\right]'$$

where vector $\boldsymbol{g}_J^{(k)}$ is the circularly shifted vector $\boldsymbol{g}_J$ by factor k

$$\begin{aligned} \boldsymbol{g}_J &= [g_{J,N-1}, g_{J,N-2}, \cdots, g_{J,1}, g_{J,0}]' \\ \boldsymbol{g}_J^{(1)} &= [g_{J,0}, g_{J,N-1}, g_{J,N-2}, \cdots, g_{J,2}, g_{J,1}]' \\ \boldsymbol{g}_J^{(2)} &= [g_{J,1}, g_{J,0}, g_{J,N-1}, g_{J,N-2}, \cdots, g_{J,3}, g_{J,2}]' \end{aligned}$$

and so on. That is

$$\mathcal{V}_J = 2^{-J}\begin{bmatrix} g_{J,0} & g_{J,N-1} & g_{J,N-2} & g_{J,N-3} & \cdots & g_{J,3} & g_{J,2} & g_{J,1} \\ g_{J,1} & g_{J,0} & g_{J,N-1} & g_{J,N-2} & \cdots & g_{J,4} & g_{J,3} & g_{J,2} \\ g_{J,2} & g_{J,1} & g_{J,0} & g_{J,N-1} & \cdots & g_{J,5} & g_{J,4} & g_{J,3} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ g_{J,N-2} & g_{J,N-3} & g_{J,N-4} & g_{J,N-5} & \cdots & g_{J,1} & g_{J,0} & g_{J,N-1} \\ g_{J,N-1} & g_{J,N-2} & g_{J,N-3} & g_{J,N-4} & \cdots & g_{J,2} & g_{J,1} & g_{J,0} \end{bmatrix} \tag{16}$$

Thus, the $J+1$ MODWT coefficients – wavelet and scaling coefficients – can be obtained as follows

$$\begin{aligned} \boldsymbol{w}_j &= \mathcal{W}_j \boldsymbol{x},\ j = 1, \ldots, J \\ \boldsymbol{v}_J &= \mathcal{V}_J \boldsymbol{x} \end{aligned} \tag{17}$$

The wavelet detail and smooth components of the MRA, $d_{j,t}$ and $s_{J,t}$, can be computed from the MODWT as follows

$$\begin{aligned} \boldsymbol{d}_j &= \mathcal{W}_j' \boldsymbol{w}_j,\ j = 1, \ldots, J \\ \boldsymbol{s}_J &= \mathcal{V}_J' \boldsymbol{v}_J \end{aligned} \tag{18}$$

with $\boldsymbol{d}_j = (d_{j,0}, d_{j,1}, \ldots, d_{j,t}, \ldots, d_{j,N-1})'$ and $\boldsymbol{s}_J = (s_{J,0}, s_{J,1}, \ldots, s_{J,t}, \ldots, s_{J,N-1})'$.

In practice and for efficiency interest, the MODWT is computed using a *pyramid algorithm* implemented in J iterations. In particular the pyramid algorithm does not construct the matrix $\mathcal{W}$. Instead, it compute the MODWT coefficients using filtering operations in a *cascading way*. On each iteration j of the algorithm, the input is filtered to compute the jth-level wavelet and scaling coefficients, $\boldsymbol{w}_1$ and $\boldsymbol{v}_1$ respectively. In the first iteration, the input is the vector of data $\boldsymbol{x}$. In the subsequent iterations $j = 2, \ldots, J$, the input is instead the vector of scaling coefficients constructed in the previous iteration $\boldsymbol{v}_{j-1}$.

3.2.3 Wavelet and scaling filters

Here, we define the wavelet and scaling filters and present their fundamental properties. Let the finite real-valued sequence $\{h_l : l = 0, \ldots, L-1\}$ be a wavelet filter of width $L \in 2\mathbb{N}$. A wavelet filter must have the following properties

$$\begin{aligned} &\sum_{l=0}^{L-1} h_l = 0 \\ &\sum_{l=0}^{L-1} h_l^2 = 1 \\ &\sum_{l=0}^{L-1} h_l h_{l+2n} = 0,\ \forall n \in \mathbb{Z}^* \end{aligned} \tag{19}$$

with $h_l = 0\, \forall l \in \mathbb{Z} \setminus [0, L-1]$, $h_0 \neq 0$ and $h_{L-1} \neq 0$. The first property states that a filter integrates to zero; the second property states that a filter has unit energy; and the third property states that a filter is orthogonal to its even shifts. Note that L must be even so that the orthogonality condition can be held. The scaling filter of width L $\{g_l : l = 0, \ldots, L-1\}$, also a finite real-valued sequence, is associated to the wavelet filter as follows

$$g_l = (-1)^{l+1} h_{L-1-l},\ l = 0, \ldots, L-1 \tag{20}$$

with $g_l = 0\, \forall l \in \mathbb{Z} \setminus [0, L-1]$. The scaling filter has the same properties as the wavelet filter.

The wavelet literature propose several kinds of wavelet filters, among which the most popular are the Haar wavelet filter and the Daubechies wavelet filters. The Haar wavelet filter is a filter of width $L = 2$ and can be defined by its wavelet filter coefficients $h_0 = 1/\sqrt{2}$ and $h_1 = -1/\sqrt{2}$ or equivalently by its scaling filter coefficients $g_0 = g_1 = 1/\sqrt{2}$. The family of Daubechies wavelet filters of width $L = 2, 4, 8, 16, \ldots$ can be characterized by the squared gain function for its scaling filters, denoted $\mathcal{G}(f)$ with f the Fourier frequency[18].

[18] The squared gain function of a filter is a tool to capture the frequency domain properties of that filter.

However, several sequences of scaling filter coefficients of the form $\{g_l : l = 0, \ldots, L-1\}$, can share the same squared gain function $\mathcal{G}(f)$[19]. All possible sequences $\{g_l\}$ with the same $\mathcal{G}(f)$ are defined by the transfer function $G(f) = [\mathcal{G}(f)]^{1/2} \exp(i\theta(f))$, where $\theta(f)$ is the phase function. The scaling filters can be derived with a procedure called *spectral factorization*[20]. In general the literature focuses on two kinds of spectral factorization. The first one is called the extremal phase factorization and it defines the extremal phase filters of width L named $D(L)$[21]. The energy (variance) of an extremal phase filter is mainly concentrated in the start of its impulse response. The extremal phase wavelet filter of width $L = 2$, $D(2)$, is identical to the Haar wavelet filter as they both define the same sequence $\{h_l\}$. The $D(4)$ is in turn defined by the following wavelet filter coefficients: $h_0 = \frac{1-\sqrt{3}}{4\sqrt{2}}$, $h_1 = \frac{-3+\sqrt{3}}{4\sqrt{2}}$, $h_2 = \frac{3+\sqrt{3}}{4\sqrt{2}}$ and $h_4 = \frac{-1-\sqrt{3}}{4\sqrt{2}}$. However, the extremal phase filters of width $L > 4$ do not have closed-form expressions in the time domain for their wavelet and scaling coefficients. The second kind of spectral factorization is named *least asymmetric* factorization and it defines the least asymmetric filters of width L denoted by $LA(L)$. This factorization leads to select the *most* symmetric filter. This is obtained using a linear phase function $\theta(f)$. Note that filters $LA(L)$ and $D(L)$ are identical for $L = 2$ $L = 4$ and $L = 6$ and differ for $L = 8, 10, 12, \ldots$

Another concept of the filtering theory to introduce is the notion of cascade of filters, such as

$$\begin{aligned} &\{h_{j,l} : l = 0, \cdots, L-1\} \;\; j = 1 \cdots, J \\ &\{g_{J,l} : l = 0, \cdots, L-1\} \end{aligned} \tag{21}$$

where the sequences $\{h_{j,l}\}$ and $\{g_{J,l}\}$ are called the jth level wavelet filter and the Jth level scaling filter, respectively. The jth level wavelet filter $\{h_{j,l}\}$ is derived by convoluting together the following j filters

$$\begin{aligned}
&\text{filter 1: } (g_0, g_1, \cdots, g_{L-2}, g_{L-1}) \\
&\text{filter 2: } (g_0, 0, g_1, 0, \cdots, g_{L-2}, 0, g_{L-1}) \\
&\text{filter 3: } (g_0, 0, 0, 0, g_1, 0, 0, 0, \cdots, g_{L-2}, 0, 0, 0, g_{L-1}) \\
&\vdots \\
&\text{filter j-1: } \left(g_0, 0_{(1\times 2^{j-2}-1)}, g_1, 0_{(1\times 2^{j-2}-1)}, \cdots, g_{L-2}, 0_{(1\times 2^{j-2}-1)}, g_{L-1}\right) \\
&\text{filter j: } \left(h_0, 0_{(1\times 2^{j-1}-1)}, h_1, 0_{(1\times 2^{j-1}-1)}, \cdots, h_{L-2}, 0_{(1\times 2^{j-1}-1)}, h_{L-1}\right)
\end{aligned}$$

To get the Jth level scaling filter $\{g_{J,l}\}$, one just needs to replace the h's by g's in the filter $j = J$ above. For instance, with $J = 3$ and $L = 2$, the first three levels of wavelet

[19] The width of the filter L gives the number of the different filters that share the same squared gain function.

[20] The procedure spectral factorization is used to compute the roots of $|G(f)|$. The scaling filter coefficients are associated to these roots.

[21] The extremal phase filters are associated to the roots of $|G(f)|$ that are all inside the unit circle.

filters $\{h_{j,l}\}$ are calculated as follows

$$\begin{aligned}\{h_{1,l}\} &= (h_{1,0}, h_{1,1})\\ &= (h_0, h_1)\\ \{h_{2,l}\} &= (h_{2,0}, h_{2,1})\\ &= (g_0, g_1) * (h_0, 0, h_1)\\ \{h_{3,l}\} &= (h_{3,0}, h_{3,1})\\ &= (g_0, g_1) * (g_0, 0, g_1) * (h_0, 0, 0, 0, h_1)\end{aligned}$$

In turn, the 3rd level scaling filter is given by

$$\begin{aligned}\{g_{3,l}\} &= (g_{3,0}, g_{3,1})\\ &= (g_0, g_1) * (g_0, 0, g_1) * (g_0, 0, 0, 0, g_1)\end{aligned}$$

As seen above, the sequences of the jth level wavelet filter $\{h_{j,l}\}$ and Jth level scaling filter $\{g_{J,l}\}$ are used to form the submatrices defining the MODWT, (15) and (16) respectively. Note that $\{h_{j,l}\}$ and $\{g_{J,l}\}$ are periodized to length N in (15) and (16).

3.2.4 Practical choices in performing the MODWT

The main practical issue the researcher faces in implementing a wavelet analysis is how to choose an appropriate wavelet filter. Usually, researchers consider three criteria when choosing the wavelet filter. The first criterion is the width L of the wavelet filter. In particular, with a short width ($L = 2, 4$ or 6), the wavelet filter will be a poor approximation of an ideal bandpass filter[22]. In that case, the filter might lead to the construction of wavelet coefficients that cannot be correctly interpreted. Therefore, the Haar wavelet filter is usually not an appropriate wavelet filter. Thus, one should go for a wavelet filter with a large width. However, the larger is L, the larger will be the number of wavelet coefficients affected by the boundaries (see below). The second criterion is the ability of the wavelet filter to mimic the key features of the data. It is difficult to meet this criteria when data display different features over different times and scales. The last criterion is the property of symmetry in filters. Symmetric filters make it easier to align wavelet and scaling coefficients with the data on the time dimension. Hence, according to these criteria, we choose the use the least asymmetric wavelet filter of width 8. Indeed, $LA\,(8)$ is a good approximation of an ideal bandpass filter and is nearly symmetric. The width $L = 8$ is large enough to get a filter close to the ideal bandpass filter and is small enough to minimize the number of wavelet coefficients affected by the boundaries.

[22]An ideal bandpass filter is a filter that preserves the dynamics of the data within a given band of frequencies.

The choice of the number of scales to consider is also crucial as it defines the scale level of the long term oscillations of the time series x_t. For instance, if one is interested in long wave cycles in prices or production such as the Kondratieff cycles (with a duration of 40 to 60 years), using annual data, s/he needs to consider $J = 5$[23]. However, if one has no prior on the scale level, s/he can use one of the following rules of thumb

$$\text{conservative rule of thumb: } J = \log_2\left(\frac{N}{L-1}+1\right)$$
$$\text{maximum rule of thumb: } J = \log_2(N)$$
$$\text{supermaximum rule of thumb: } J = \log_2(1.5N)$$

where L is the width of the filter. In our empirical work we choose to use the conservative rule of thumb. Indeed, the size of our dataset is not large enough to use the two other rules of thumb. In the four countries we have 4 scaling levels. We get the following frequencies: detail 1: 2-4 annual frequency, detail 2: 4-8 annual frequency, detail 3: 8-16 annual frequency and detail 4: 16-32 annual frequency. The smoothed series is $S4$ referring to more than 32 annual frequency.

Another issue in performing the MODWT is the handling of the boundaries. Indeed, the wavelet transforms are defined on a space of infinite length $L^2(\mathbb{R})$. However, the data, on which the wavelet transform is applied to, are usually defined in a finite interval I and therefore have discontinuities at the boundaries of I. Thus, the original data need to be expanded before being transformed. One way to deal with the interval's endpoints, is to assume the data to be periodic[24] and to expand the data accordingly. This technique is called periodic boundary conditions. This assumption is reasonable if the data start and end on the same level, such as data displaying seasonality. However, if the data are not truly periodic, the implementation of the wavelet transform on the periodically replicated data[25] might lead to abnormally large wavelet coefficients in the neighborhood of the boundaries. Another way to deal with the boundary issue is first to expand the data by reflection[26], then to consider the reflected data as being periodic when performing the wavelet transform. This technique is called the reflection boundary conditions. If the data are not truly periodic, reflection boundary conditions might be more appropriate than periodic boundary conditions because the former provide continuity at the boundaries. Besides, the reflection boundary conditions do not affect the sample mean nor the sample variance of the original data.

[23]Gallegati et al. (2014) are currently analyzing the long wave cycles in prices and economic activity using the wavelet methodology.

[24]A periodic data is a data that repeats its oscillations after a fixed interval.

[25]Data are periodically replicated as follows: $x_t = x_{t-N}\ \forall t = N+1, \ldots, 2N$.

[26]Data are replicated by reflection as follows: $x_0, x_1, \ldots, x_{N-2}, x_{N-1}, x_N, x_{N-1}, x_{N-2}, \ldots, x_1, x_0$.

4 Empirical results

Figures 2, 3 and 4 display the power spectrum for growth (upper left corner) and the labour share (upper right corner) as well as the wavelet coherency (lower left corner) and the cross-wavelet power (lower right corner).

The power spectrum analysis describes at which period of time and for which frequencies the information is concentrated. The wavelet power spectrum is an energy density (or variance distribution) in the time-frequency plane. In the upper two plots of Figures 2, 3 and 4, the horizontal axis denotes the time and the vertical axis the scale. The concentration of the information is represented by colors intensity: the warmer colors standing for higher power. The regions surrounded by a bold line are the region significant at 10% against the null hypothesis that the data generating process is stationary. The cone of influence is represented by the shaded area and identifies the region affected by the edge effects.

The power spectrum analysis for growth (upper right hand corner) has similarities in the three countries considered. The information is concentrated before World War II pointing that growth fluctuations have dampened since the second half of the XXth century. Additionally, power is decreasing with the scale, which can be expected as taking the growth rate of GDP removes the trend. This description is very relevant for France and the United-States. The power spectrum of growth in the UK differs slightly to the extent that information is more evenly concentrated across time and the concentration at higher frequencies is more pronounced than in France and in the United-States.

[Figure 2 about here.]

Similarly to growth, the power spectrum analysis for the labour share (upper left hand corner) shows that power declines after World War II. However, there are regions with high power in the 1970s, which captures the large increase in the labour share that took place as a result of low unemployment and wage indexation mechanisms. Additionally, information concentration increases with the scale as illustrated by the red colours that characterized the detail D_3, D_4 and the smooth component S_4. In the United-States, the information is less concentrated in large scales as in the other two countries. However, the power increases across all scales in the decade 2000s.

[Figure 3 about here.]

The purpose of the cross-wavelet power is to analyse the time-frequency dependencies between two time series. The cross-wavelet power captures the co-variance between two variables in the time-frequency domain. The wavelet coherency is the cross-wavelet power normalized by the power spectrum of both series. The coherency analysis between growth and the labour share (lower left hand corner) shows that there are strong correlations at certain point in time at the highest frequencies. The region of significant coherency expands with the scale to cover the entire period for the smooth component S_4. This last point is less true in the case of the United-States as indicated by the blue colour at the 32 years horizon. Overall the coherency analysis tends to show that there is strong local correlation between growth and the labour share.

[Figure 4 about here.]

The cross-wavelet power is illustrated in the lower right hand corner. The cross-wavelet power represents regions of common high power in the time-frequency space. Contrastingly, the coherency analysis shows significant coherence although the common power might be low. The cross-wavelet power also represents the relative phase. The direction in the arrow can be interpreted as follow. An arrow pointing right indicates that the two variables are in co-movement. The arrow pointing left indicates that the two variables are in anti-phase. The arrow pointing up indicates that the labour share is leading growth by 90 degrees; while the arrow pointing down means that the labour share is lagging growth by 90 degrees.

In France and the UK, in areas characterized by high common power (the red colour in the figure), the arrows are pointing up indicating that the labour share is leading growth by 90 degrees. This is especially true at the low frequency S_4. The relative phase sheds a new light on the issue of endogeneity between the labour share and growth. The labour share leading growth is an indication that the labour share is not endogenous to growth. However, the relative phase should be interpreted carefully as a lead by 90 degrees could also mean a lag by 270 degrees. At higher frequencies, the arrows tend to point left in areas of low common power (in blue) indicating an anti-phase movement between the labour share and growth. In the United-States, the arrows are pointing down at the 32 years frequencies, while arrows are pointing left at the 8-16 years and 16-32 years frequencies.

Figure 14 in the appendix reproduce the continuous wavelet analysis for the US using the Piketty and Zucman (2014) database over the period 1930-2010 and for the definition of the labour share ls. The power spectrum for growth and the labour share are similar to those produced with the Groth and Madsen (2013) database. The coherency analysis differs slightly to the extent that regions with common information are wider using the Piketty and Zucman (2014) database. The relative phase at low frequencies (16 years and beyond) displays arrow pointing upwards relative to figure 5(d). An explanation may be that the number of scale is smaller given the shorter samples.

5 Regression across time scale

In this section, we explore the relation between functional income distribution and growth at different frequencies for France 1897-2010, UK 1857-2010 and the US 1899-2010. This section makes use of discrete wavelet transform rather than continuous wavelet transform as presented in section ??.

[Figure 5 about here.]

Figures 5, 6 and 7 display the outcome of the wavelet analysis for growth (the blue line) and the labour share (the red line) for each frequencies and for each countries. To simplify the presentation, the highest frequency D_1 is excluded from the figure (but is available upon request). In each country, visual inspection tends to show that at highest frequencies D_2, growth and the labour share are in anti-phase. However, when the scaling level increases the two series seem to become gradually in-phase with the labour share leading growth. This visual impression must be confirmed by more elaborate statistical tests.

[Figure 6 about here.]

It is interesting to look at the long term trend in the labour share through the smoothed series S_4. In the case of France, the smoothed series of the labour share declines from 1896 to the mid 1920's. The labour share then increases through both the Great Depression and World War II. The labour share reaches a maximum of 82%. It then declines gradually with the exception of the large bump in the 1970s. In the case of the United-Kingdom, the smooth component of the labour share increases between 1870 and 1890 and subsequently declines until 1910. World War I marks the beginning of an increase in the labour share that lasts until the early 1980s. The labour share is on a declining trend since then. In the United-States, the labour share declines until the 1930s and reaches a pick in the aftermath of World War II. The labour share then stabilizes around 64% until the late 1970s before to follow a continuous decline until the Great Recession.

[Figure 7 about here.]

Correlations are illustrated in Table 1 showing that the sign associated with the labour share and growth is negative at the frequencies D_1 to D_4 and then turn positive in the long run S_4. Small differences exist across countries. The sign of the correlation is not significant at the frequencies D_3 and D_4 in France. In the UK, the sign is positive in S_4 but not significant. The correlations tends to confirm the visual inspection described above, which points to a change in the relation between growth and the labour share across frequencies. In the Appendix, Table 4 displays similar correlation for the ls_{pz} labour share definition in France and the United-Kingdom. The table also displays the correlation using both the ls and ls_{pz} definition using the Piketty and Zucman (2014) database for the US over the period 1930-2010. The results confirm that the sign of the correlation changes at high and low frequencies from negative to positive. The change of definition hardly impact the correlation in France. In the UK, the correlation associated with the smoothed component S_4 is positive and significant, while the same coefficient is no longer significant in the US.

[Table 1 about here.]

We further explore the relation between functional income distribution and growth using LOESS methods.[27]. LOESS fits the data using localized subsets of the data. LOESS assesses the relation between two variables using a locally weighted polynomial regression. It therefore gives us information on the non-linear relation between growth and distribution. Figures 11, 12 and 13 display the labour share on the horizontal axis and growth on the vertical axis as well as the LOESS curve fit for each time scale in France, the UK and the US respectively.

[Figure 8 about here.]

The LOESS fit confirms the signs of the correlation and explains why some correlations were not statistically different from zero. In France, the negative signs at D_1 and D_2 and the positive sign at S_4 appear clearly. At D_3 and D_4 the relation seems to be non-linear: overall positive but with some negative sections. In the UK, the relation between growth and the labour share appears almost linear at D_1 and D_4, while the relation is non-linear at D_2, D_3 and S_4. The strong non-linearity at S_4 may explain why the sign of the correlation is not significant in the long-run. In the US, the LOESS fits confirm the signs given by the correlation. The relation appears to be quite linear across frequencies. Interestingly, at frequency S_4 the positive relation between the two variables seems to be taking place after World War II. The non-linearities are further explored below performing rolling window regressions in section 6.

[Figure 9 about here.]

[Figure 10 about here.]

We complement the correlation and the LOESS fit by performing the regression described in eq 22. The dependent variable is the growth rate of real GDP per capita and the

[27] LOESS stands for locally weighted scatter plot smoothing Cleveland (1979)

independent variable is the labour share of income. This regression is performed for each time scale $j = [D_1, D_2, D_3, D_4, S_4]$. Regressions using wavelets differ from regressions with time series. The objective is not to fit as well as possible raw data but to show whether the sign of the relation changes cross time scales. It follows that the lagged dependent variable is not added as a regressor. The details and smooth components are sinusoid functions that display strong auto-correlation especially at low frequencies. A lagged dependent variable as a regressor would appear strongly significant and may overshadow the relation existing with other explanatory variables. In the absence of a lagged dependent variable the R^2 is mechanically lower.

Eq 22 is likely to have auto-correlated errors especially at low frequency since the wavelet analysis uses a combination of sinusoid functions. Auto-correlation in the residual generates a non-consistent estimate of the variance of the OLS estimates. To overcome this issue, Eq 22 is estimated using an OLS regression with heteroscedastic and auto-correlation consistent estimator (HAC). The HAC estimator adjusts the covariance matrix by applying weight to account for auto-correlation. The HAC estimator in this paper uses pre-whitening of the error term and the weights are chosen following Newey and West (1987).

$$\Delta y_{j,t} = \alpha_j + \hat{\beta}_j \omega_{j,t} + \epsilon_{j,t} \tag{22}$$

The independent variable enters the regression contemporaneously. As indicated by the relative phase, the labour share is a leading indicator of growth in the areas of common power. This may be interpreted as pointing that endogeneity is not an issue when estimating eq 22 at t. Additionally, wavelet analysis greatly reduces the problem of endogeneity as the filtered series can be viewed as an instrumental variable of the original series (see Ramsey et al. (2010) for a discussion of the properties of wavelet). A second motivation is that using annual data it seems realistic to assume that the impact of changes in labour income affects growth within a year.

[Table 2 about here.]

The regressions results for France, the UK and the US are displayed in Table 2. Across the three countries, the regressions confirm that the sign of the relation between growth and the labour share changes across frequencies from negative at high frequencies to positive at low frequencies. At each country level, the coefficient estimated is in line with the correlation presented above. In France, the sign associated with the labour share is negative at the frequency D_1 and D_2, not significant at frequencies D_3 and D_4 and positive and significant at frequency S_4. The coefficient is increasing with the scale considered from -1.4, to -0.4 and 0.03. In the UK, the sign associated with the labour share is negative from D_1 to D_4 and not significant for S_4. Similarly to France, the coefficient increases with the time scale considered from -1.3, -0.4, -0.3 and -0.2. In the US, the labour share has a negative impact on growth at frequencies D_1 to D_4 the sign turning positive at frequency S_4. Here as well the sign increases with the scale considered to the exception of D_4. In the Appendix, Table 5 presents sensitivity analysis with respect to alternative labour share definitions, data source and year coverage. Results are only affected at the margin. In France, the results are identical regardless of the definition chosen. In the UK, only the coefficients associated with D_3 and D_4 are significant (the p-values for D_1 and D_2 are just above 10%), while in the US the coefficient using the smoothed series S_4 is not significant anymore.

[Table 3 about here.]

Table 3 displays of the estimation of eq 22 augmented with dummies for World War I and World War II. Controlling for World Wars is necessary as economic structure have been deeply affected by both events. In particular, figure 1 shows that the labour share tends to get closer to unity during war periods. The main result is that adding dummies does not modify the results described above. The slight difference concerns France for which the coefficients using component D_3 and D_4 are now positive and significant, while they were not significantly different from zero previously. The coefficients using component D_3 and D_4 are still positive and significant when substituting the definition ls_{pz} for ls in Table 6 in the Appendix. The coefficient turns positive and significant in the UK for the smoothed component S_4 under the definition ls_{pz} when dummies are included.

6 Stability of the relation over time

The advantage of using historical data is that it allows to perform single country estimation, while existing studies using data from the 1970s rely on pooled estimations. The shortcoming of using historical data is that the sign of the relation may change over time as the countries considered have experienced profound changes over the past 100 years. The instability of the relation over time may also explain some of the results described in the previous section as for instance the non-significant coefficient for the frequency S_4 in the UK. This section studies whether the relation between functional income distribution and growth has changed over time by using both rolling window correlations and rolling window regressions for the smoothed component S_4. Figure 11 displays the 50 years rolling window correlation for France, the UK and the US.

[Figure 11 about here.]

In France, the rolling window correlation is positive for most of the sample. The correlation declines to become negative over the window 1940-1990. This change in the sign of the relation may be related to the 1970s which where characterized by full employment, wage indexation and inflationary expectations. In the UK, the correlation is negative over the second half of the XIXth century before turning positive in the first half of the XXth century. The correlation then changes sign again twice: negative after World War II and positive in last part of the time series. In the US, the rolling correlation are not significantly different from zero in the first half of the XXth century. The correlations are then negative reaching a low point for the window 1935-1985. The correlation then increases and turns positive beyond the window 1945-1995.

[Figure 12 about here.]

In order to better capture the instability in the relation between the labour share and growth, we perform the estimation described in eq 22 using 75 years rolling window as well as 100 years rolling window. The results are described in Figures 13(a), 13(b) and 13(c). In France, the 75 years rolling window estimation shows a positive coefficient comprised between 0.3 and 0.44. The coefficient is quite stable around 0.3 for the 100 years rolling window correlation. For both windows the coefficient is significantly different from zero. In the US, the coefficient fluctuates between 0.05 and 0.25 for the 75 years window and between 0.1 and 0.25 for the 100 years window. Contrastingly to France, the coefficients are not significant at the beginning of the sample. Lastly, in the UK the coefficient associated with the labour share is negative and significant when regressions are performed using data for the XIXth and early XXth centuries. The coefficient then turns positive and significant. This may explain that the coefficient is not significant for S_4 in the regression using the entire sample presented in Table 2. A last interesting point is that the sign associated with the labour share is positive and increasing after World Ward II in all three countries. This section illustrates that a second advantage of using historical data is that the impact of income distribution on growth has changed over time. In the Appendix, Figure 15 displays the rolling window regression for alternative labour share definition. In France and the UK, the rolling regression are reproduced using the ls_{pz} definition for both windows (75 years and 100 years). For the US, 50 years rolling window regressions are performed using the ls and ls_{pz} definition of the Piketty and Zucman (2014) database over the period 1931-2010. The results described in Figure 12 are robust to a change in the labour share definition. A slight difference concerns the UK for which the coefficient is always positive for the ls_{pzu} definition and applying the 100 years windows. For the US, the coefficient associated with the labour share was not significant over the period 1930-2010 at the $S4$ frequency as displayed in Table 5. The rolling window regression shows that the coefficient is negative over the beginning of the period and is then turning positive in more recent year. Overall, it appears that regardless of the country and the labour share definition considered, the impact of the labour share on growth tends to be positive and increasing in size when estimated on the most recent data.

7 Conclusion

This paper addresses the question of the link between factor shares and growth. The attempt is to combine a new source of information that constitutes labour share measures over a long period of time with a new methodology the wavelet analysis. This new data source, which provides time series of roughly a 100 points (and more) enable to run single country estimation while existing papers rely exclusively on pooled data. The time-frequency analysis shades a new light on economic relation as it tests for the sign of the effect across frequencies.

The time frequency analysis shows that there are large regions of common information between growth and the labour share especially at low frequencies. Additionally, the labour share tends to lead growth by 90 degree in the medium to long run. This is especially true in France and in the United-Kingdom.

The correlation and regression analysis point to changing signs of the relationship between functional income distribution and growth at different frequencies from negative in the short and medium run to positive in the long run. This result is consistent across countries, across specifications and across labour share's definitions. Lastly, rolling window regression using the low frequency component indicates that the sign of the coefficient associated with the labour share is increasing when using more recent data. In the United-Kingdom, this means that the sign of the coefficient becomes positive and significant when excluding the XIXth century data.

This leaves open the question of the interpretation of the results. While the negative sign at the business cycle frequency is consistent with a DSGE model with search and matching in the labour market for instance, it is more challenging to account for the positive sign in the longer run. This result is consistent with the model by Kumhof et al. (2015) in which lower labour income produces a Minsky moment over a 50 years horizon. An alternative interpretation would be a growth model where human capital accumulation depends on labour income.

References

Aguiar-Conraria, L. and M. Soares (2011). The continuous wavelet transform: A primer. *NIPE WP 16/2011*.

Aguiar-Conraria, Luis and, M. M. M. and M. J. Soares (2012). The yield curve and the macro-economy across time and frequencies. *Journal of Economic Dynamics and Control 36*, 1950–1970.

Alesina, A. and D. Rodrik (1994). Distributive politics and economic growth. *Quarterly Journal of Economics 109(2)*, 465–90.

Andrews, D., C. Jencks, and A. Leigh (2011). Do rising top incomes lift all boats? *B.E. Journal of Economic Analysis & Policy 1*, 1–45.

Atkinson, A. B. (1969). The timescale of economic models: How long is the long run? *The Review of Economic Studies 36(2)*, 137–152.

Banerjee, A. and E. Duflo (2003). Inequality and growth: What can the data say? *Journal of Economic Growth 8(3)*, 267–99.

Bentolila, S. and G. Saint-Paul (2003). Explaining movements in the labor share. *Contributions to Macroeconomics 3*.

Berg, J. G. and J. D. Ostry (2011). Inequality and unsustainable growth: Two sides of the same coin? *IMF Staff Papers 08*, 1–21.

Blanchard, O. J. and F. Giavazzi (2003). Macroeconomic effects of regulation and deregulation in goods and labor markets. *The Quarterly Journal of Economics 118(3)*, 879–907.

Canova, F. (1998). Detrending and business cycle facts. *Journal of Monetary Economics 41*, 475–512.

Checchi, D. and C. Garcia-Penalosa (2010). Labour market institutions and the personal distribution of income in the oecd. *Economica 77(307)*, 413–450.

Chiarella, C. and P. Flaschel (2000). *The Dynamic of Keynesian Monetary Growth*. Cambridge University Press.

Cleveland, W. S. (1979). Robust locally-weighted regression and scatterplot smoothing. *Journal of the American Statistical Association 74*, 829–836.

Dominicisn, L. d., R. J. G. M. Floraxnn, and H. L. F. d. Grootnnn (2008). A meta-analysis on the relationship between income inequality and economic growth. *Scottish Journal of Political Economy 55(5)*, 654–682.

Elsby, M., B. Hobijn, and A. Sahin (2013). The decline of the u.s. labor share. *Brookings Papers on Economic Activity 47(2 (Fall))*, 1–63.

Galí, J., J. D. López-Salido, and J. Vallés (2007, March). Understanding the effects of government spending on consumption. *Journal of the European Economic Association 5*(1), 227–270.

Gali, J., F. Smets, and R. Wouters (2011). Unemployment in an estimated new keynesian model. *NBER Working Papers 17084*.

Gallegati, M., M. Gallegati, J. Ramsey, and W. Semmler (2014). Wavelet nonparametric estimation of long wave cycles. Ongoing research.

Gallegati, M., M. Gallegati, J. B. Ramsey, and W. Semmler (2011). The us wage phillips curve across frequencies and over time. *Oxford Bulletin of Economics and Statistics 73(4)*, 489–50.

Gallegati, M. and J. B. Ramsey (2013). Bond vs stock market's q: Testing for stability across frequencies and over time. *Journal of Empirical Finance 24(C)*, 138–150.

Gençay, R., F. Seleçuk, and B. Whitcher (2002). *An Introduction to Wavelets and Other Filtering Methods in Finance and Economics*. Academic Press.

Gertler, M. and A. Trigari (2009). Unemployment fluctuations with staggered nash wage bargaining. *Journal of Political Economy 117*(1), 38–86.

Gollin, D. (2002). Getting income shares right. *Journal of Political Economy 110(2)*, 458–74.

Goodwin, R. (1967). *Socialism, Capitalism and Economic Growth*, Chapter A Growth Cycle. Cambridge: Cambridge University Press.

Grinsted, A., J. Moore, and S. Jevrejeva (2004). Application of the cross wavelet transform and wavelet coherence to geophysical time series. *Nonlinear Processes in Geophysics 11*, 561–566.

Groth, C. and J. B. Madsen (2013). Medium-term fluctuations and the "great ratios" of economic growth. *Discussion Papers, University of Copenhagen. Department of Economics 13-16*.

Guscina, A. (2006). Effects of globalization on labor's share in national income. *IMF Working Paper 294*, 1–35.

Halter, D., M. Oechslin, and J. Zweimüller (2014). Inequality and growth: the neglected time dimension. *Journal of Economic Growth 19*, 81–104.

Harrison, A. (2002). Has globalization eroded labor's share? some cross-country evidence. *mimeo*.

Jaumotte, F. and I. Tytell (2007). How has the globalization of labor affected the labor income share in advanced countries? *IMF Working Paper 7-298.*

Jayadev, A. (2007). Capital account openness and the labour share of income. *Cambridge Journal of Economics 31(3)*, 423.

Kaldor, N. (1955). Alternative theories of distribution. *Review of Economic Studies 23*, 209–230.

Kauermann, G., T. Teuber, and P. Flaschel (2012). Exploring us business cycles with bivariate loops using penalized spline regression. *Computational Economics, Society for Computational Economics 39(4)*, 409–427.

Kramer, H. M. (2011). Bowley's law: The diffusion of an empirical supposition into economic theory. *Cahiers d'économie Politique / Papers in Political Economy 61*, 19–49.

Kumhof, M., R. Ranciere, and P. Winant (2015). Inequality, leverage and crises. *American Economic Review 105(3)*, 1217–45.

Liesner, T. (1989). *One Hundred Years of Economic Statistics.* Facts on File Publications.

Lilly, J. and S. Olhede (2010). On the analytic wavelet transform. *IEEE Transactions on Information Theory 56*(8), 4135–4156.

Mohun, S. and R. Veneziani (2006). Goodwin cycles and the u.s. economy, 1948-2004. *University Library of Munich, Germany MPRA Paper 30444.*

Newey, W. K. and K. D. West (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica 55 (3)*, 703–708.

Ogden, T. (2012). *Essential Wavelets for Statistical Applications and Data Analysis.* Springer Science & Business Media.

Ott, R. (2012). A Wavelet Packet Transform based Stock Index Prediction Algorithm. Master's thesis, Swiss Federal Institute of Technology Zurich, Switzerland.

Percival, D. (2014). Wavelets: Data analysis, algorithms and theory. Lecture presentations, Spring quarter 2014.

Percival, D. and A. Walden (2006). *Wavelet Methods for Time Series Analysis.* Cambridge University Press.

Piketty, T. (2013). *Le Capital au XXi Siècle*. Seuil.

Piketty, T. and G. Zucman (2014). Capital is back: Wealth-income ratios in rich countries, 1700-2010. *Quarterly Journal of Economics forthcoming.*

Ramsey, J. B., M. Gallegati, M. Gallegati, and W. Semmler (2010). Instrumental variables and wavelet decompositions. *Economic Modelling 27(6)*, 1498–1513.

Solow, R. (1958). A skeptical note on the constancy of relative shares. *The American Economic Review 48(4)*, 618–631.

Stockhammer, E., . Onaran, and S. Ederer (2009). Functional income distribution and aggregate demand in the euro area. *Cambridge Journal of Economics 33 (1)*, 139–159.

Young, A. T. (2010). One of the things we know that ain't so: Is us labor's share relatively stable? *Journal of Macroeconomics 32(1).*

Appendix 8

9 Self-employment imputation for the U.K.

This appendix shortly details the computation of self-employment income in Piketty and Zucman for the U.K.. ce_{hh} is wages and social contributions paid by non-corporate business (and households). According to PZ (2014): "No decomposition of wage payment by non-corporate vs corporate business is available in BB series 1948-1986 (nor in Martin) nor in Feinstein 1855-1948; the decomposition given here is based upon the assumption of a fixed ratio between wage payments by non-corporate businesses and mixed income of non-corporate businesses. Ideally we would like to know the number of wage-earners in the non-corporate business sector vs. in the corporte business sector, but this info does not appear to be available (neither in Mitchell 1988, nor in MFOS 1982)". ce_{hh} is then computed as follow:

$$ce_{hh} = gmi_{hh} * \frac{w_{nc}}{gmi_{hh}}$$

with gmi_{hh}: gross mixed income (self-employment), w_{nc}: non-corp. wage. w_{nc} is largely imputed. According to PZ (2014): "From 1876 to 1986 we keep this fixed to the 1987 value (55%). From 1855 to 1875 we assume a gradual declines from 90% to 55%, which is consistent with roughly stable factor shares in the corporate sector (keeping the 55% share fixed through to 1855 would result in too little wages in the non-corporate sector and too much in the corporate sector)"

The labor share of agricultural self-employment net income ce_{sea} is computed as follow:

$$ce_{sea} = \frac{ce_{farm}}{n^{\%}_{farm} * n} \frac{n^{\%}_{sefarm} * n}{(ndp + nfi)}$$

with ce_{farm}: farm wages (Wages and social contributions paid by farm sector), $n^{\%}_{farm}$: farm salaried workers (% total employed population) , n: employed population, $n^{\%}_{sefarm}$: farm self-employed workers (% total employed population), ndp: net domestic product and nfi: net foreign income.

The labor share of non-agricultural self-employment net income ce_{sena} is defined as follow:

$$ce_{sena} = w^{\%}_{sena} * w_{ees} \frac{n^{\%}_{senfarm} n}{(ndp + nfi)}$$

with $w^{\%}_{sena}$: imputed wage of non-agricultural self-employed workers (in percent), w_{ees}: average wage of salaried workers , $n^{\%}_{senfarm}$: non-farm self-employed workers (% total employed population).

$w^{\%}_{sena}$ is assumed to start from 50 percent in 1855 and to increase gradually to 100 percent in 1885. Average wage of salaried workers is defined as $w_{ees} = \frac{ce_c + ce_{hh} + ce_g}{n * n^{\%}_{ees}} 1000$. Salaried workers $n^{\%}_{ees}$ is defined as $n^{\%}_{ees} = 1 - n^{\%}_{se} = 1 - (n^{\%}_{sefarm} + n^{\%}_{senfarm}) = 1 - (\frac{n_{sea}}{n_{eesa}} n_{farm} + n^{\%}_{senfarm})$. $\frac{n_{sea}}{n_{eesa}}$ is assumed 30% in 1855 and increasing up to 72% in 1965 and then decline to 68% in 1973. The source is MFOS 1982 p.170. n_{farm} is total farm employment (imputed from Mitchell 2003).

10 Alternative labour share definition and time coverage

This appendix section presents results using alternative definitions of the labour share (ls_{pz}) in France and the UK. This section also reproduce the results for the US using the Piketty and Zucman (2014) data between 1930 and 2010.

[Table 4 about here.]

[Table 5 about here.]

[Table 6 about here.]

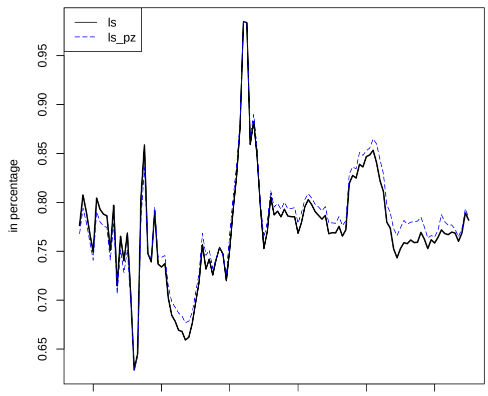
[Figure 13 about here.]

[Figure 14 about here.]

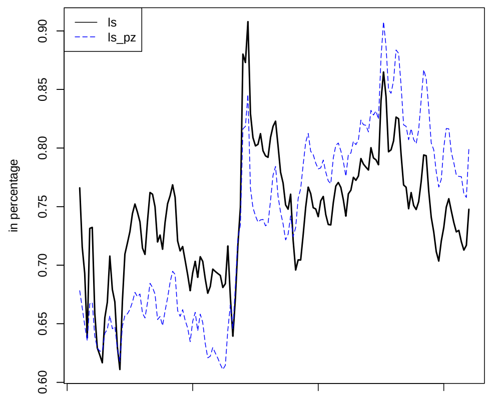
[Figure 15 about here.]

Figures

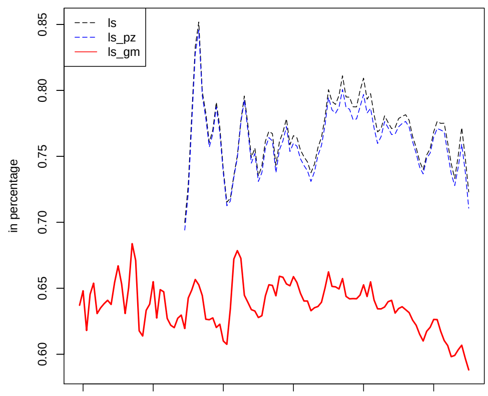
Figure 1: The labour share of income



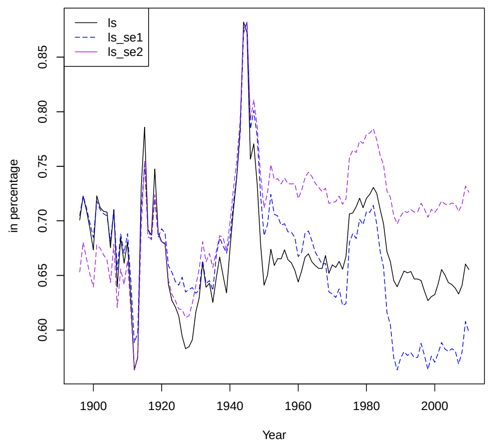
(a) France



(b) UK



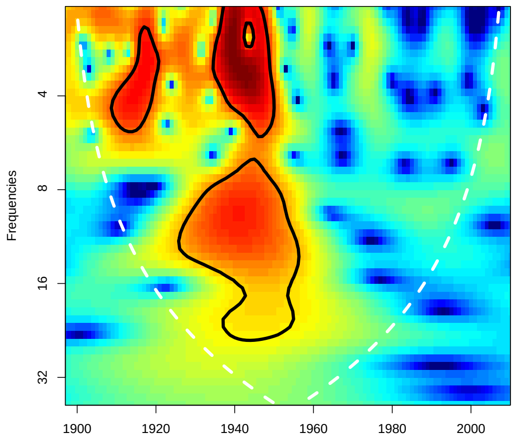
(c) USA



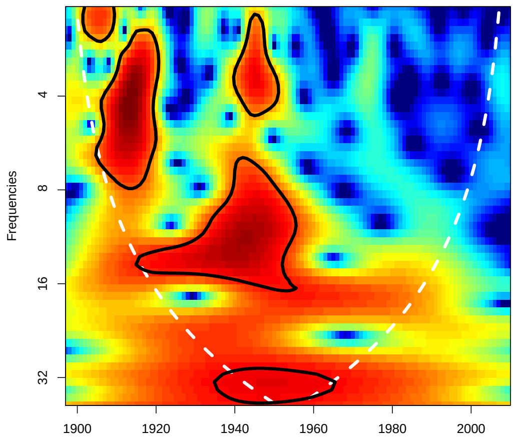
(d) Alternative self-employment adj. (FR)

These figures display the different labour share definitions in the three countries considered. The figure in the lower right hand corner displays different imputation of self-employment labour income in the case of France.

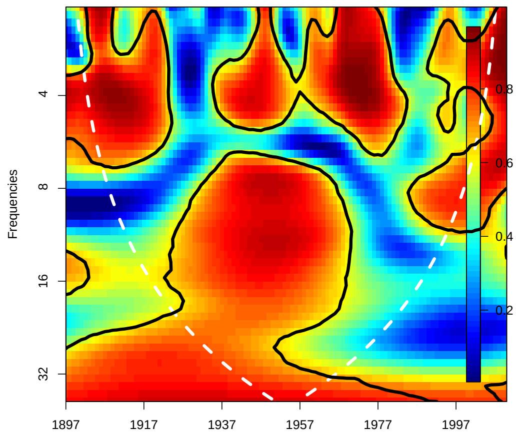
Figure 2: FR: Wavelet analysis



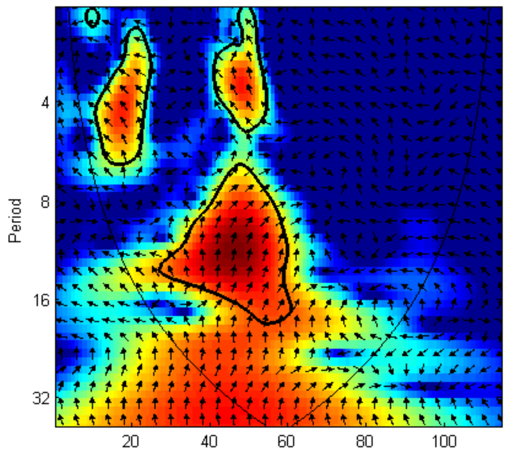
(a) Power spectrum: growth



(b) Power spectrum: labour share



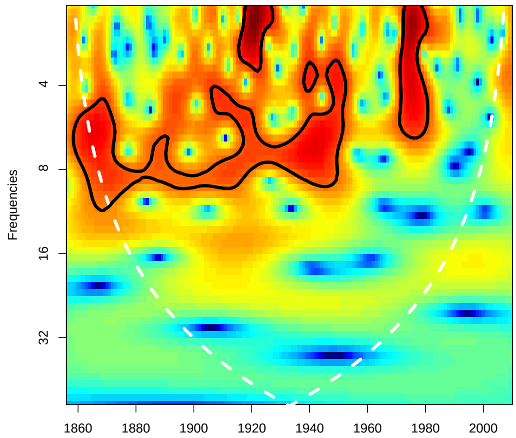
(c) Wavelet coherency



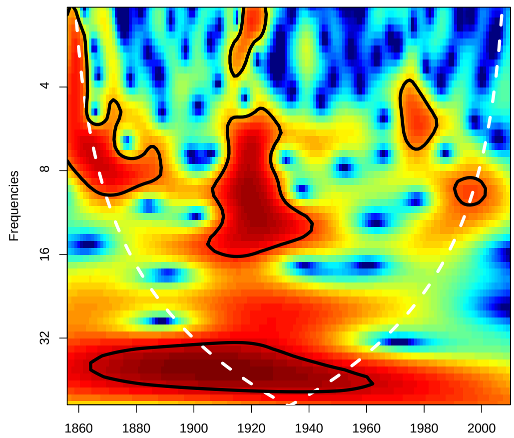
(d) Cross wavelet power

The four figures display time on the horizontal axis and frequencies (in years) on the vertical axis. The wavelet power spectrum is an energy density (or variance distribution) in the time-frequency plane. The cross-wavelet power captures the co-variance between two variables in the time-frequency domain. The wavelet coherency is the cross-wavelet power normalized by the power spectrum of both series. The warmer colors stand for high power, or high coherency. An arrow pointing right (left) means that both series are in (anti) phase. An arrow pointing up(down) means that labour share is leading(lagging) growth by 90 degrees.

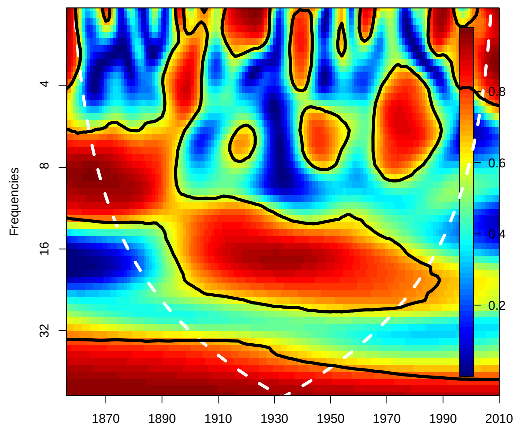
Figure 3: UK: Wavelet analysis



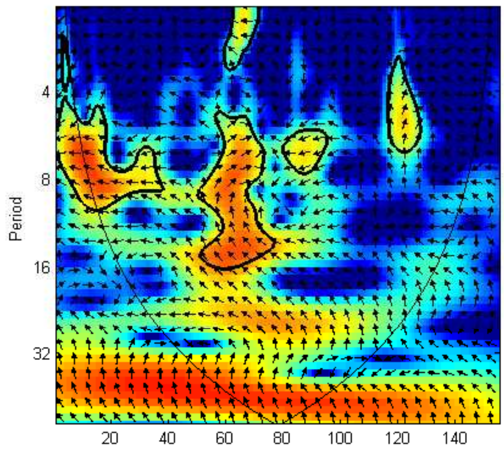
(a) Power spectrum: growth



(b) Power spectrum: labour share



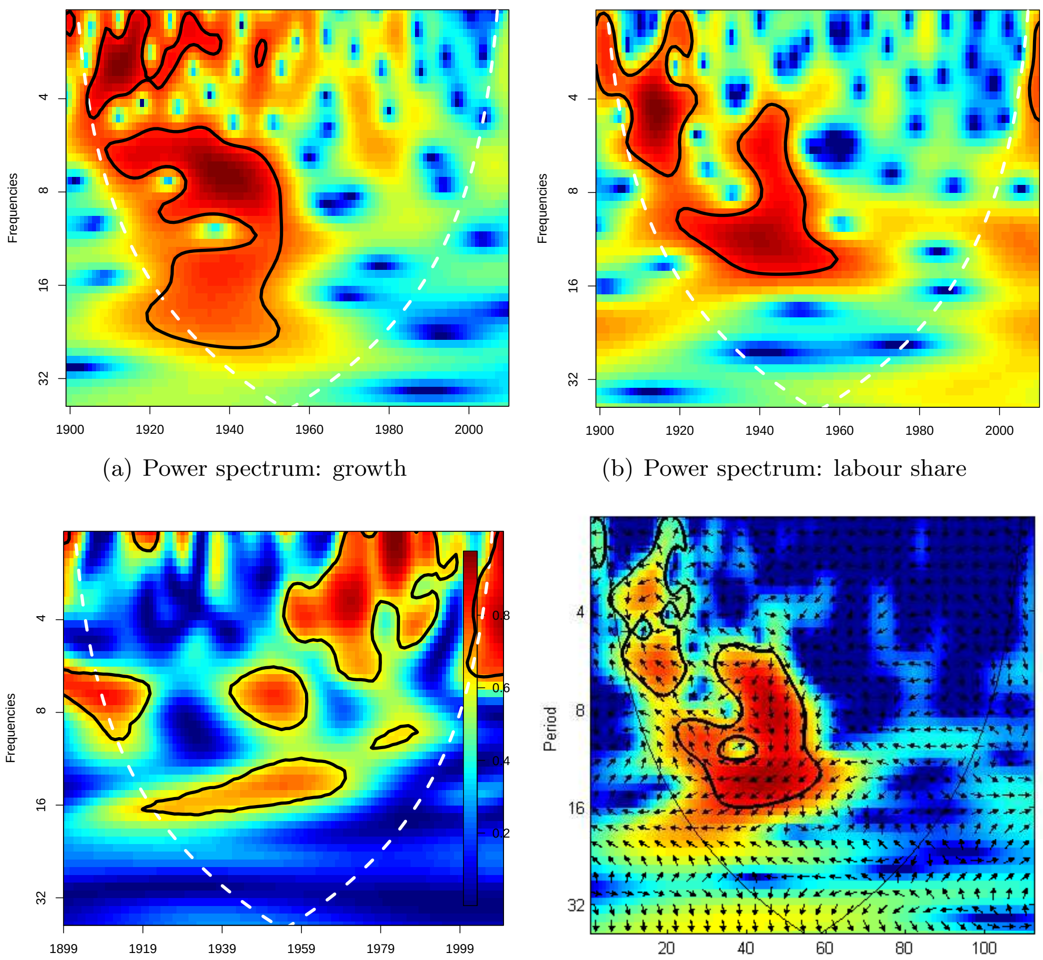
(c) Wavelet coherency



(d) Cross wavelet power

The four figures display time on the horizontal axis and frequencies (in years) on the vertical axis. The wavelet power spectrum is an energy density (or variance distribution) in the time-frequency plane. The cross-wavelet power captures the co-variance between two variables in the time-frequency domain. The wavelet coherency is the cross-wavelet power normalized by the power spectrum of both series. The warmer colors stand for high power, or high coherency. An arrow pointing right (left) means that both series are in (anti) phase. An arrow pointing up(down) means that labour share is leading(lagging) growth by 90 degrees.

Figure 4: US: Wavelet analysis



(a) Power spectrum: growth

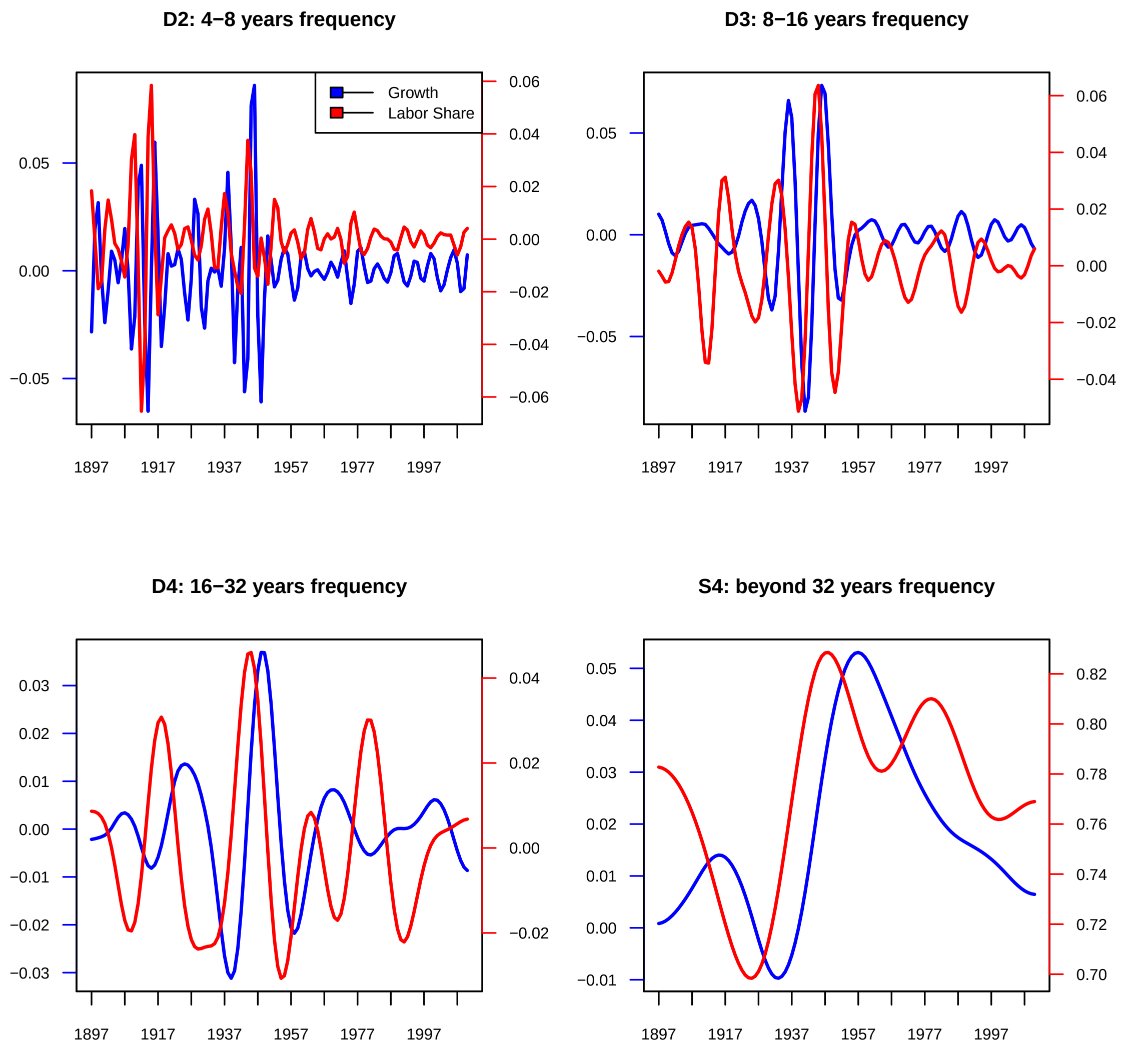
(b) Power spectrum: labour share

(c) Wavelet coherency

(d) Cross wavelet power

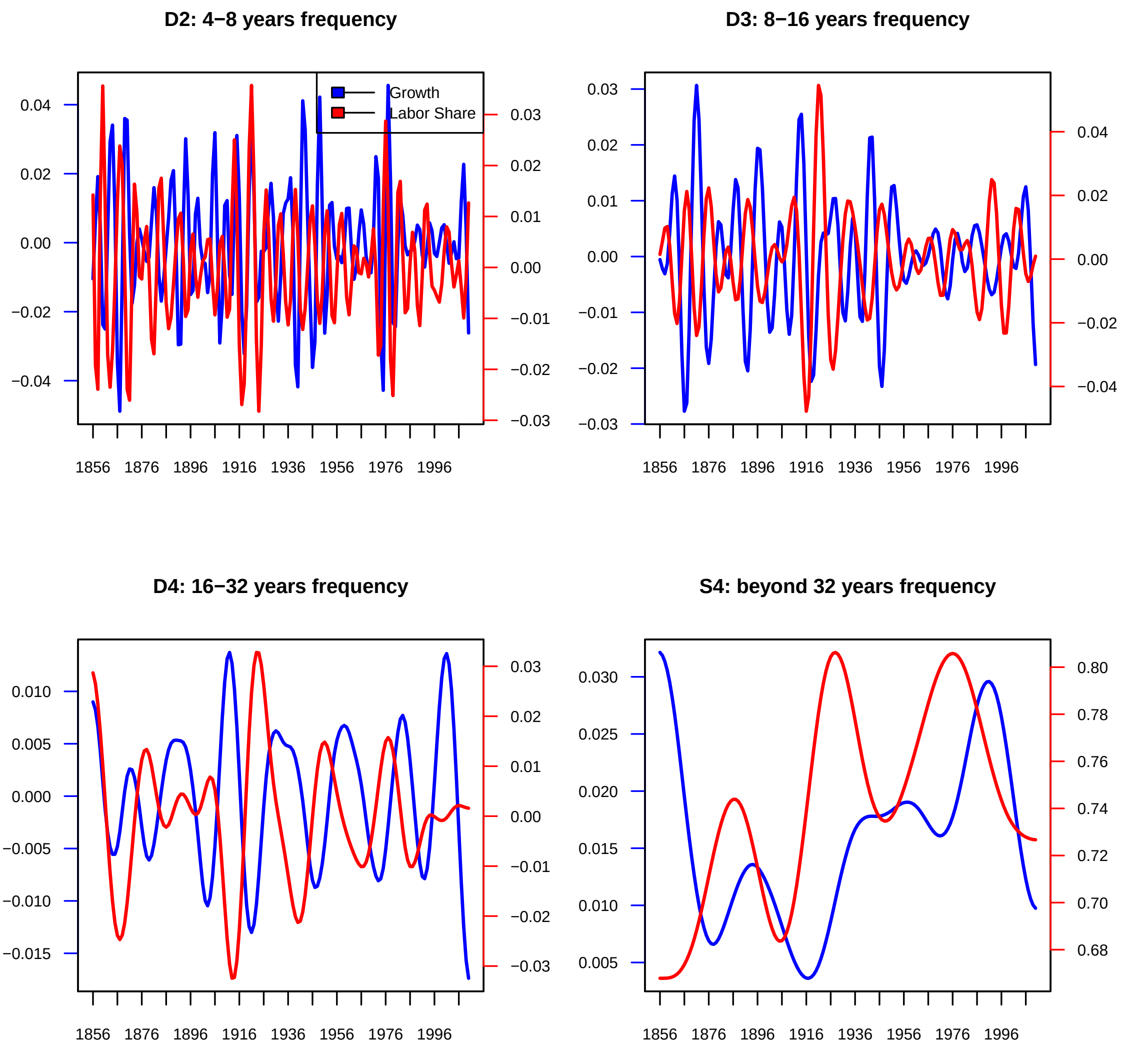
The four figures display time on the horizontal axis and frequencies (in years) on the vertical axis. The wavelet power spectrum is an energy density (or variance distribution) in the time-frequency plane. The cross-wavelet power captures the co-variance between two variables in the time-frequency domain. The wavelet coherency is the cross-wavelet power normalized by the power spectrum of both series. The warmer colors stand for high power, or high coherency. An arrow pointing right (left) means that both series are in (anti) phase. An arrow pointing up(down) means that labour share is leading(lagging) growth by 90 degrees.

Figure 5: Discret wavelet analysis of the labour share in France



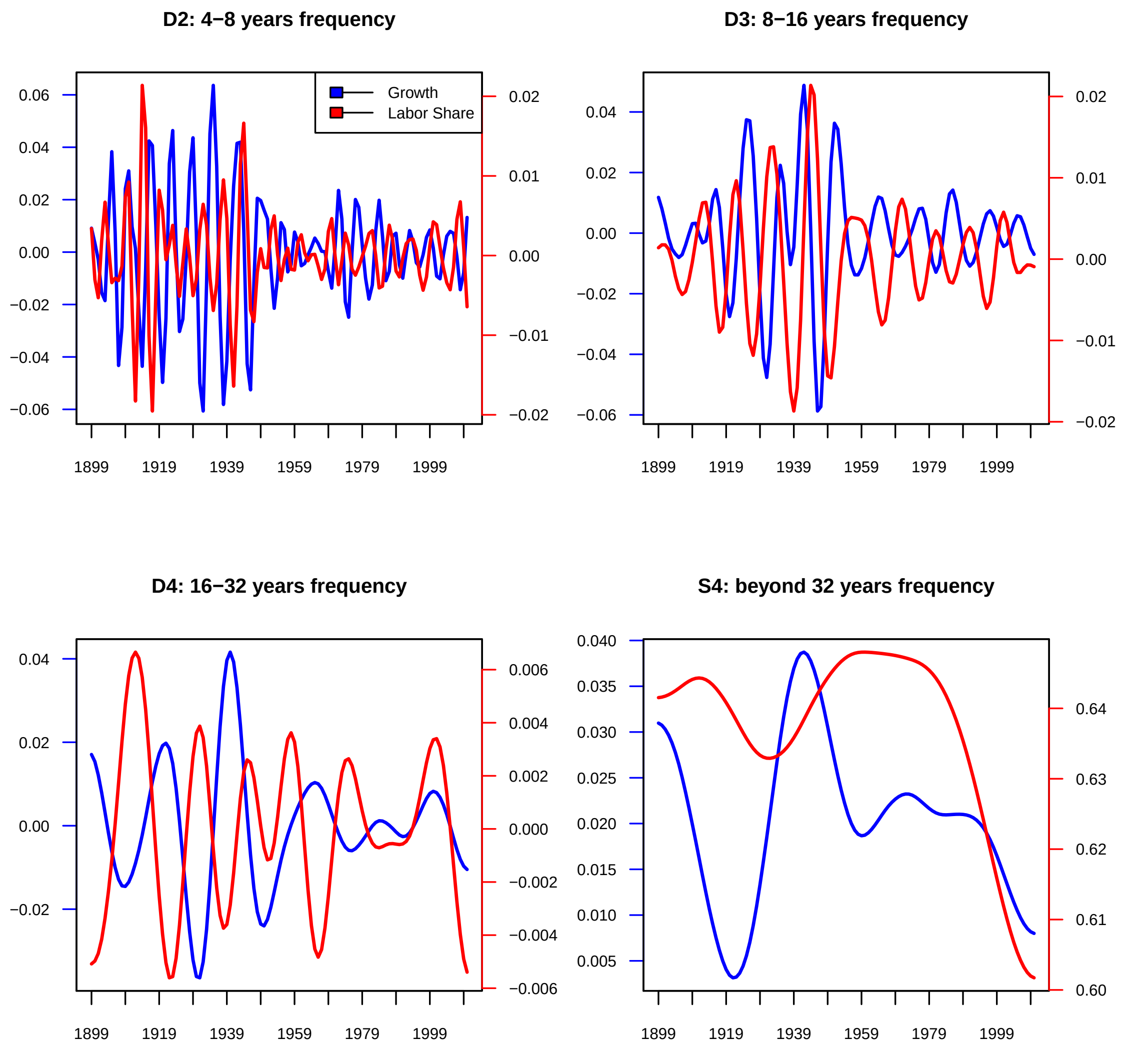
This figure displays the discrete wavelet filter for labour share (ls) and growth for the following frequencies: D_2 4-8 years, D_3 8-16 years, D_4 16-32 years and S_4 Beyond 32 years. The highest frequency D_1 2-4 years is not displayed for presentation purpose.

Figure 6: Discret wavelet analysis of the labour share in UK



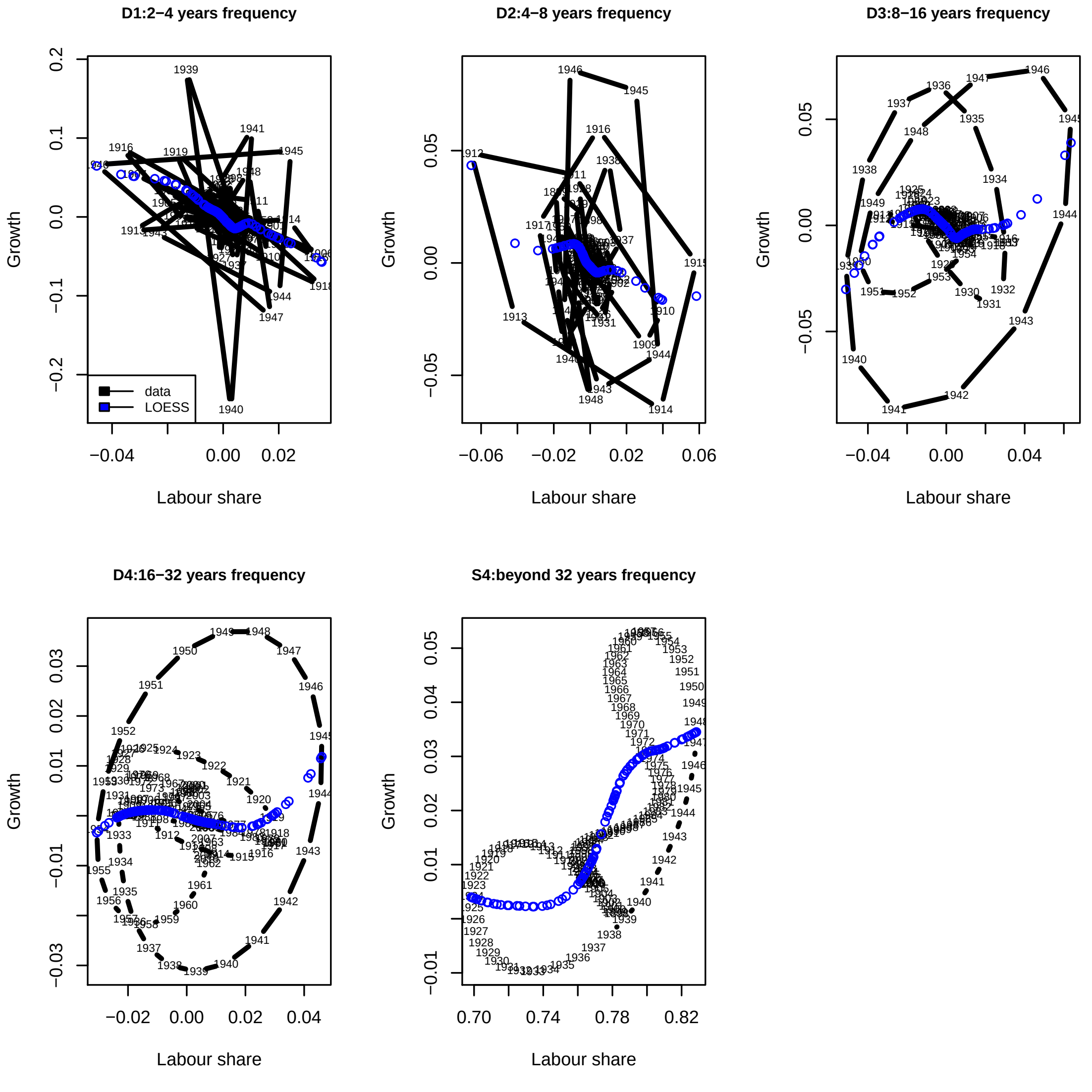
This figure displays the discrete wavelet filter for labour share (ls) and growth for the following frequencies: D_2 4-8 years, D_3 8-16 years, D_4 16-32 years and S_4 Beyond 32 years. The highest frequency D_1 2-4 years is not displayed for presentation purpose.

Figure 7: Discret wavelet analysis of the labour share in United-States



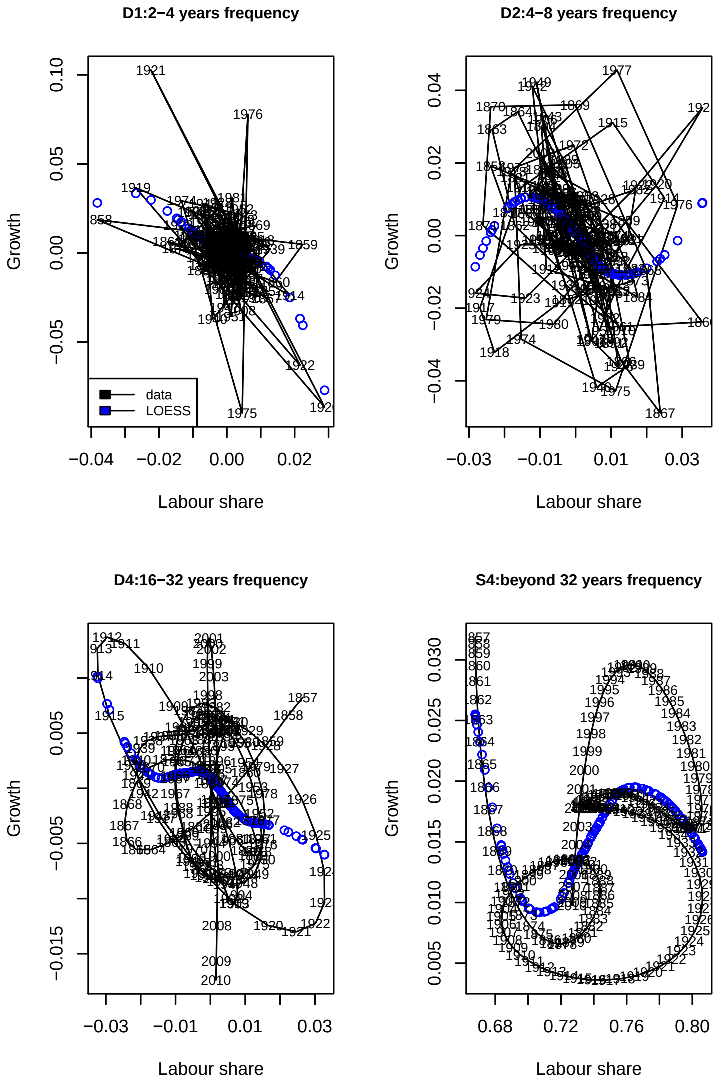
This figure displays the discrete wavelet filter for labour share (ls_{gm}) and growth for the following frequencies: D_2 4-8 years, D_3 8-16 years, D_4 16-32 years and S_4 Beyond 32 years. The highest frequency D_1 2-4 years is not displayed for presentation purpose.

Figure 8: LOESS fit in France



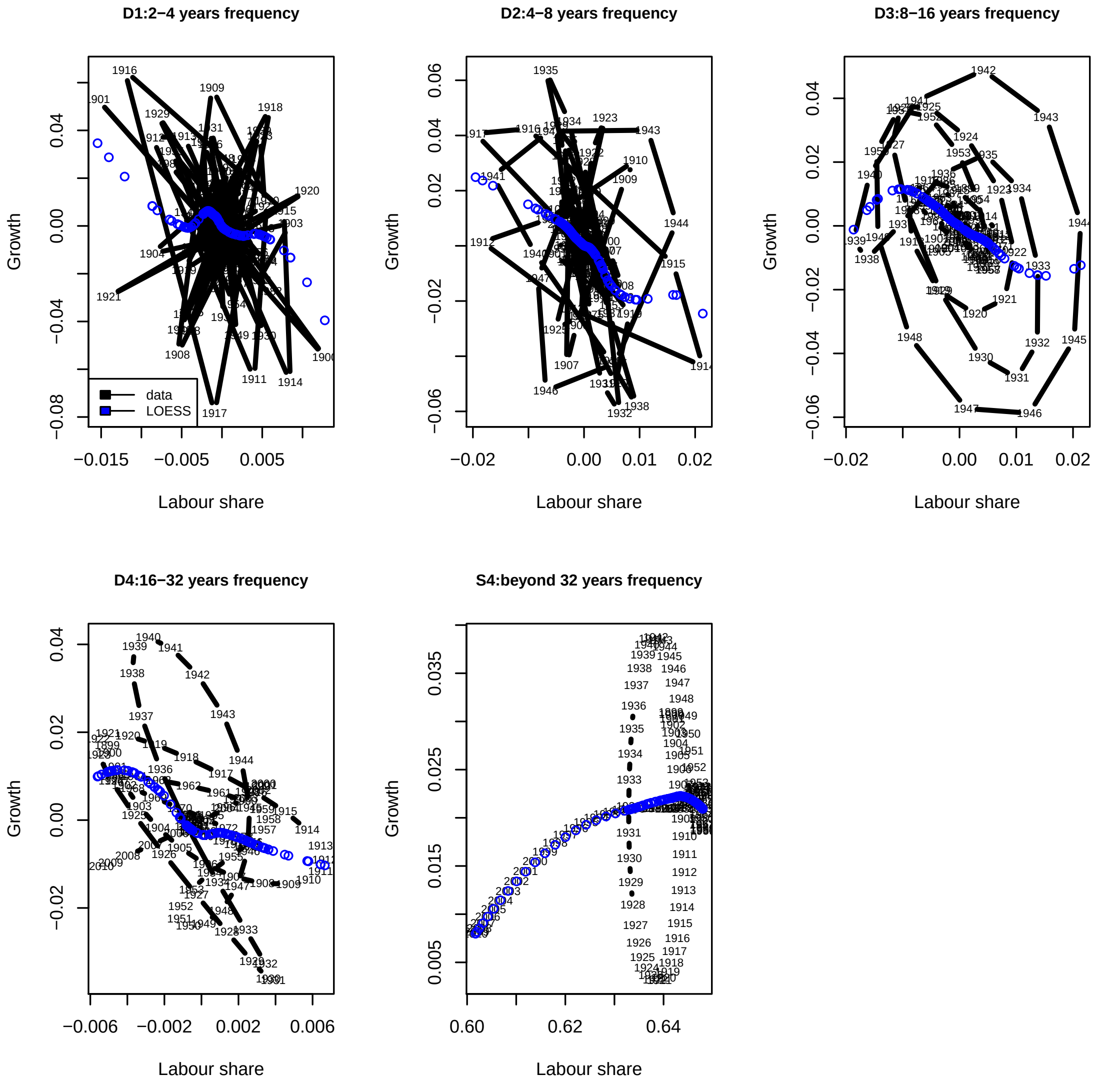
This figure displays the labour share (ls) on the horizontal axis and growth on the vertical axis. The blue line corresponds to the LOESS fit. Each subfigure corresponds to a given frequency: D_1 2-4 years, D_2 4-8 years, D_3 8-16 years, D_4 16-32 years and S_4 Beyond 32 years.

Figure 9: LOESS fit in UK



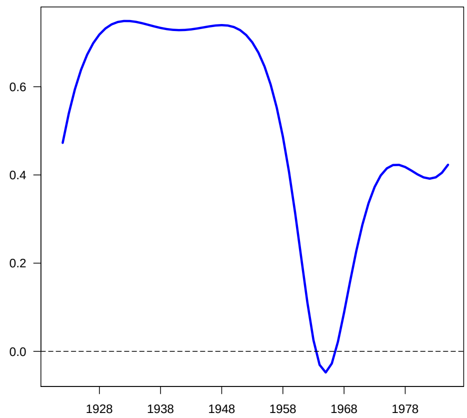
This figure displays the labour share (ls) on the horizontal axis and growth on the vertical axis. The blue line corresponds to the LOESS fit. Each subfigure corresponds to a given frequency: D_1 2-4 years, D_2 4-8 years, D_3 8-16 years, D_4 16-32 years and S_4 Beyond 32 years.

Figure 10: LOESS fit in US

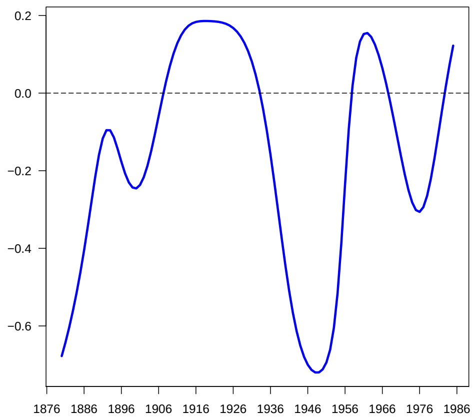


This figure displays the labour share (ls_{gm}) on the horizontal axis and growth on the vertical axis. The blue line corresponds to the LOESS fit. Each subfigure corresponds to a given frequency: D_1 2-4 years, D_2 4-8 years, D_3 8-16 years, D_4 16-32 years and S_4 Beyond 32 years.

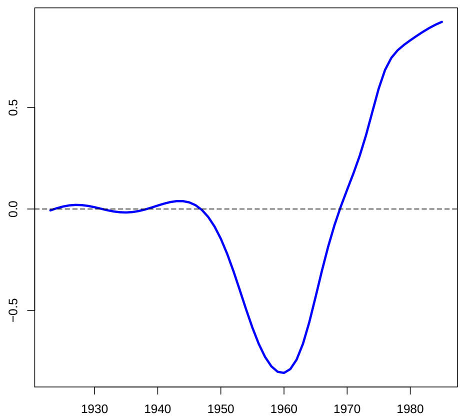
Figure 11: 50 years - Rolling correlation - S4



(a) France



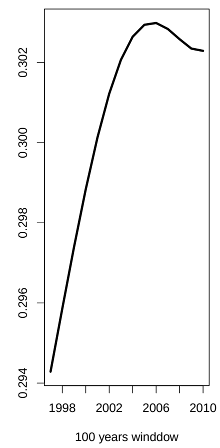
(b) UK



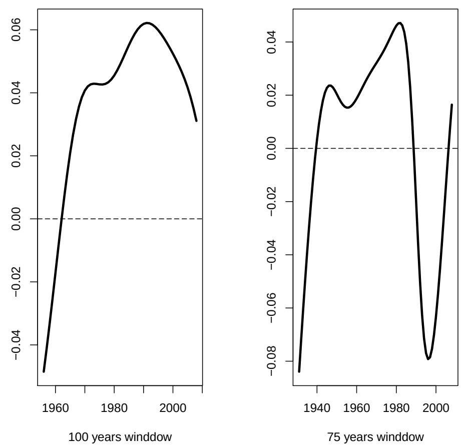
(c) USA

These figures display the 50 years rolling window correlation for France, the UK and the USA using the smoothed component $S4$. The dashed line is the zero line. The horizontal axis displays the median year of the period over which the correlation is performed.

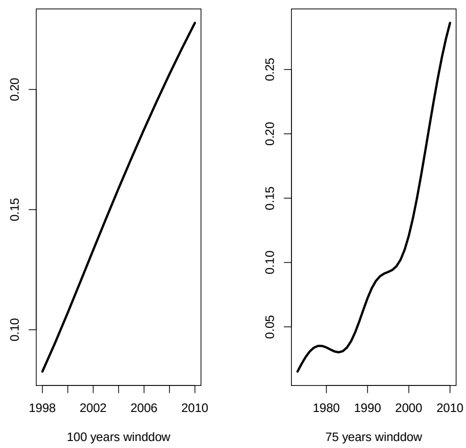
Figure 12: Rolling regression 100 years and 75 years window OLS HAC - S4



(a) France



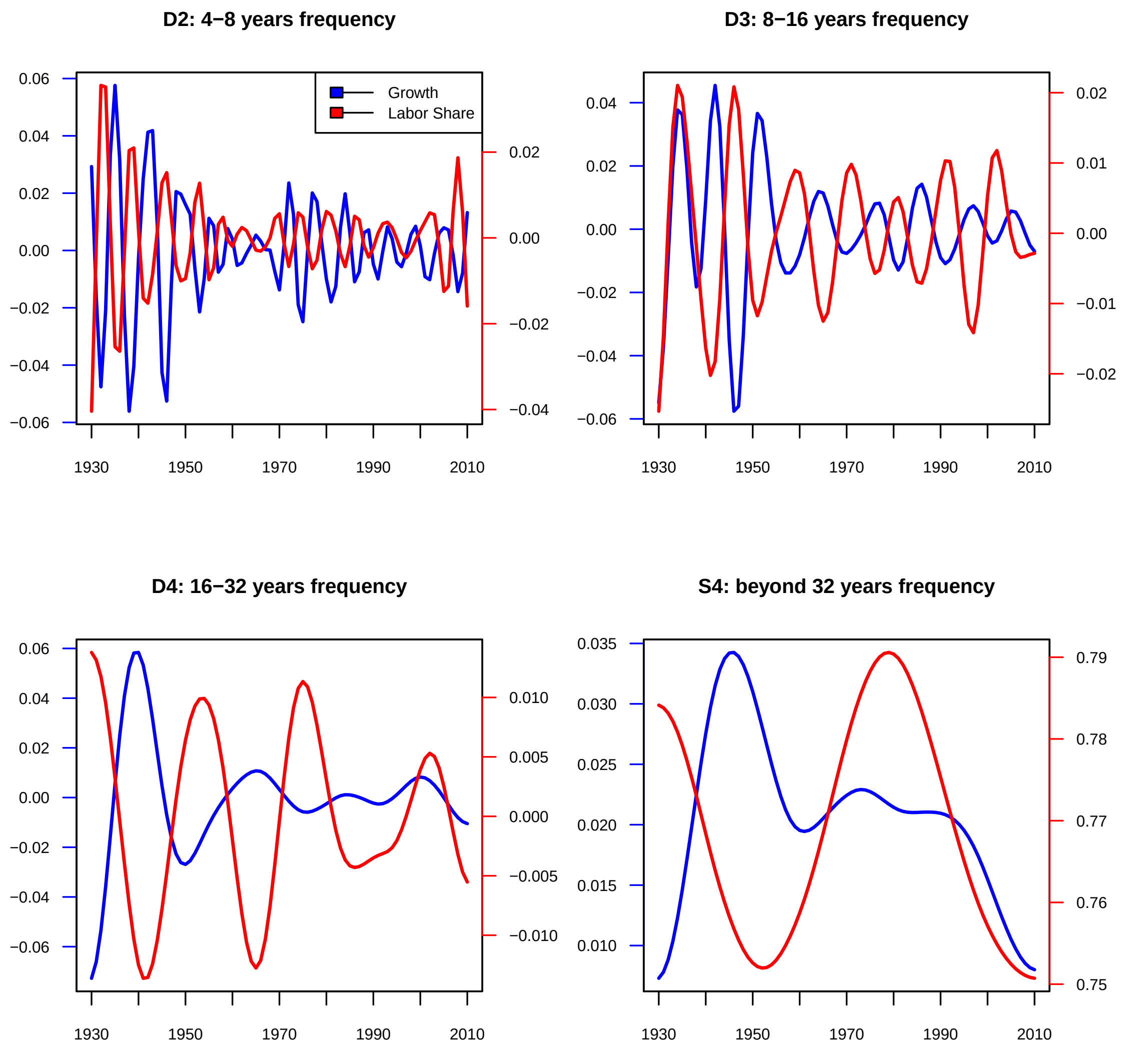
(b) UK



(c) USA

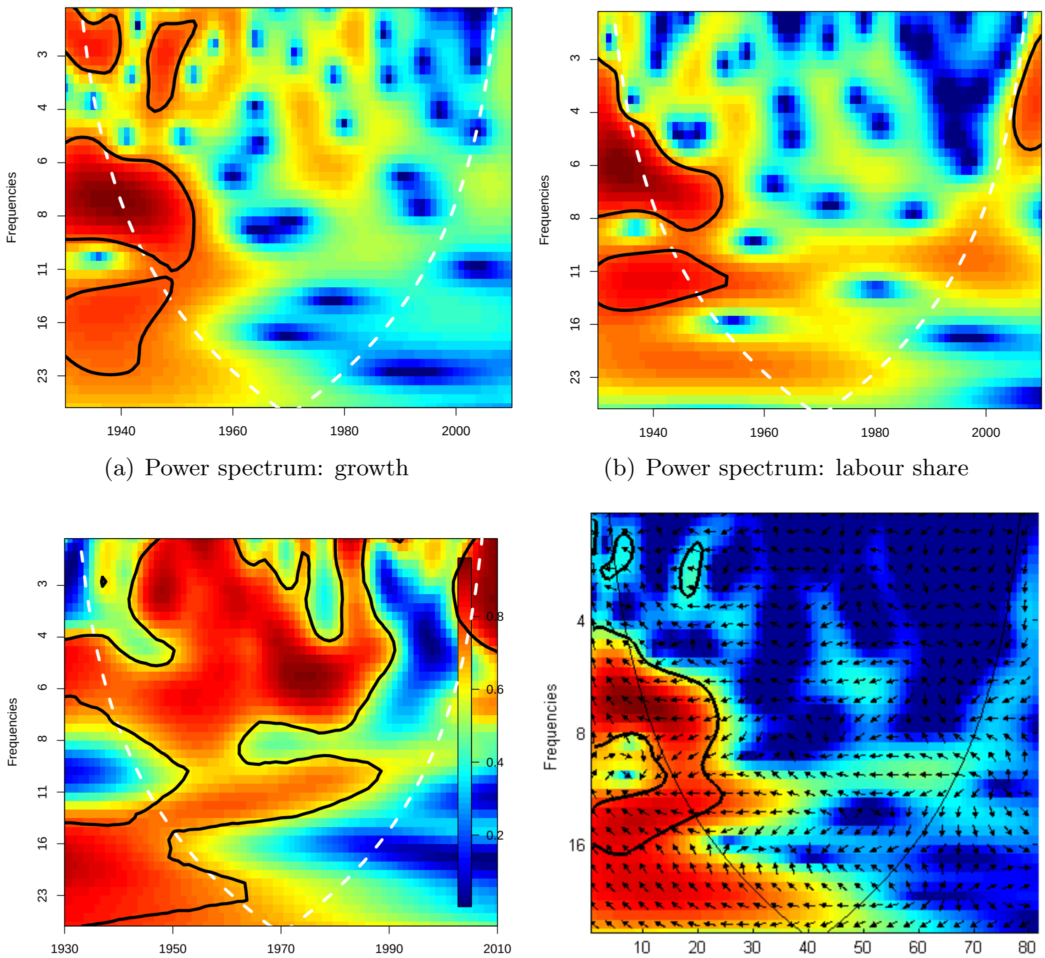
These figures display the 100 years and 75 years rolling window regression for France, the UK and the USA using the smoothed component $S4$. The estimation method is OLS-HAC. The definition of the labour share used are ls for France and the UK and ls_{gm} for the USA. The dashed line is the zero line. The horizontal axis displays the median year of the period over which the regression is performed.

Figure 13: Discrete wavelet analysis of the labour share in United-States - 1930-2010



This figure displays the discrete wavelet filter for labour share (ls) and growth for the following frequencies: D_1 2-4 years, D_2 4-8 years, D_3 8-16 years and S_3 Beyond 16 years. As the number of points is smaller there is one less frequency than in Figure7. The data source for the labour share is Piketty and Zucman (2014).

Figure 14: Continuous wavelet analysis of the labour share in United-States - 1930-2010



(a) Power spectrum: growth

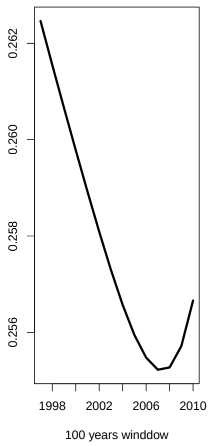
(b) Power spectrum: labour share

(c) Wavelet coherency

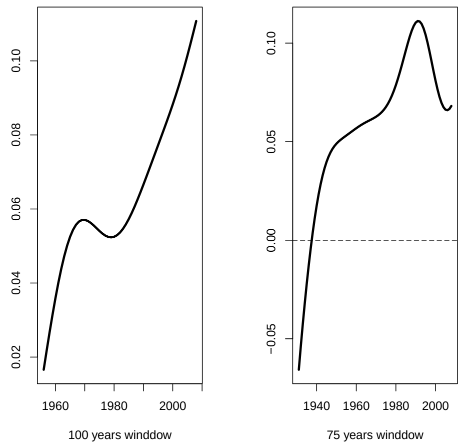
(d) Cross wavelet power

The four figures display time on the horizontal axis and frequencies (in years) on the vertical axis. The wavelet power spectrum is an energy density (or variance distribution) in the time-frequency plane. The cross-wavelet power captures the co-variance between two variables in the time-frequency domain. The wavelet coherency is the cross-wavelet power normalized by the power spectrum of both series. The warmer colors stand for high power, or high coherency. An arrow pointing right (left) means that both series are in (anti) phase. An arrow pointing up(down) means that labour share is leading(lagging) growth by 90 degrees. The data source is Piketty and Zucman (2014) and the labour share definition is ls.

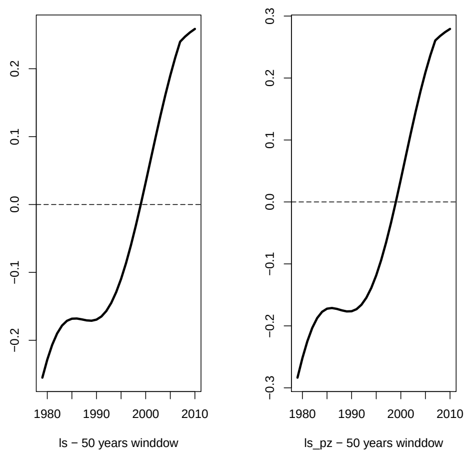
Figure 15: Rolling regression - $S4$ - OLS HAC



(a) France



(b) UK



(c) USA

This figure displays the rolling window regression using alternative labour share definition for the smoothed component $S4$. The labour share definition is ls_{pz} in France and the UK. The regression's results are displayed for both a 75 years window and a 100 years window. The labour share definition are both ls and ls_{pz} for the USA using the Piketty and Zucman (2014) over the period 1931-2010. In the USA a single 50 years window is used. The estimation method is OLS-HAC. The dashed line is the zero line. The horizontal axis displays the median year of the period over which the regression is performed.

Tables

Table 1: Correlation by frequency scale

	FR	UK	US
	1897-2010	1856-2010	1899-2010
definition	ls		ls_{gm}
D1	-0.40***	-0.48***	-0.24**
D2	-0.29***	-0.29***	-0.36***
D3	0.13	-0.47***	-0.37***
D4	0.03	-0.30***	-0.45***
S4	0.60***	0.11	0.34***

Table 2: Regression across frequencies

Dependent variable $\Delta y_{j,t}$						
Estimation method - HAC OLS						
France						
Year coverage 1898-2010						
Labour share definition ls						
	D_1	D_2	D_3	D_4	S_4	Raw data
cst	0.000	0.000	0.000	0.000	-0.213^{***}	-0.003
	(0.002)	(0.003)	(0.010)	(0.004)	(0.075)	(0.154)
ω_t	-1.446^{***}	-0.422^{***}	0.155	0.020	0.299^{***}	0.027
	(0.295)	(0.107)	(0.177)	(0.128)	(0.108)	(0.205)
R^2	0.165	0.075	0.017	0.01	0.366	0
N. obs.	113	113	113	113	113	113
U.K.						
Year coverage 1857-2010						
Labour share definition ls						
	D_1	D_2	D_3	D_4	S_4	Raw data
cst	0.000	0.000	0.000	0.000	0.006	0.102^{**}
	(0.000)	(0.003)	(0.003)	(0.005)	(0.033)	(0.049)
ω_t	-1.263^{**}	-0.421^{**}	-0.327^{***}	-0.169^{**}	0.013	-0.117^{*}
	(0.592)	(0.213)	(0.123)	(0.083)	(0.050)	(0.064)
R^2	0.23	0.08	0.217	0.11	0.005	0.029
N. obs.	154	154	154	154	154	154
United-States						
Year coverage 1899-2010						
Labour share definition ls_{gm}						
	D_1	D_2	D_3	D_4	S_4	Raw data
cst	0.000	0.000	0.000	0.000	-0.135^{***}	0.349
	(0.002)	(0.002)	(0.001)	(0.001)	(0.041)	(0.242)
ω_t	-1.439^{**}	-1.427^{***}	-0.937^{***}	-2.192^{***}	0.245^{***}	-0.516
	(0.550)	(0.357)	(0.226)	(0.419)	(0.064)	(0.382)
R^2	0.06	0.13	0.13	0.20	0.11	0.033
N. obs.	112	112	112	112	112	112

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

This table presents the regressions results across each time scale and for each country. The table also presents the regression results using raw data for comparison. The equation estimated is eq 22. The estimation method is HAC-OLS. The weights follows Newey-West

Table 3: Regression across frequencies with dummies for World Wars I and II

	Dependent variable $\Delta y_{j,t}$				
	Estimation method - HAC OLS				
	France				
	Year coverage 1898-2010				
	Labour share definition ls				
	D_1	D_2	D_3	D_4	S_4
Intercept	-0.001	0.001	0.003	0.002	-0.257^{***}
	(0.004)	(0.002)	(0.002)	(0.001)	(0.029)
ω_t	-1.461^{***}	-0.417^{***}	0.229^{**}	0.161^{**}	0.356^{***}
	(0.315)	(0.144)	(0.105)	(0.069)	(0.038)
D_{WWI}	0.000	0.000	-0.014	-0.012^{**}	0.012^{*}
	(0.019)	(0.010)	(0.010)	(0.006)	(0.006)
D_{WWII}	0.013	-0.007	-0.038^{***}	-0.019^{***}	-0.020^{***}
	(0.016)	(0.008)	(0.008)	(0.005)	(0.005)
R^2	0.147	0.056	0.171	0.108	0.441
Num. obs.	113	113	113	113	113
	U.K.				
	Year coverage 1857-2010				
	Labour share definition ls				
	D_1	D_2	D_3	D_4	S_4
Intercept	0.000	0.000	0.000	0.000	0.007
	(0.002)	(0.001)	(0.001)	(0.001)	(0.010)
ω_t	-1.269^{***}	-0.424^{***}	-0.377^{***}	-0.199^{***}	0.011
	(0.187)	(0.117)	(0.055)	(0.043)	(0.014)
D_{WWI}	0.003	-0.002	-0.012^{**}	-0.002	-0.012^{***}
	(0.008)	(0.008)	(0.005)	(0.003)	(0.003)
D_{WWII}	-0.001	-0.001	0.002	-0.004^{*}	0.002
	(0.007)	(0.007)	(0.004)	(0.002)	(0.003)
R^2	0.219	0.062	0.236	0.113	0.081
Num. obs.	154	154	154	154	154
	United-States				
	Year coverage 1899-2010				
	Labour share definition ls_{gm}				
	D_1	D_2	D_3	D_4	S_4
ω_t	-1.451^{**}	-1.448^{***}	-0.987^{***}	-2.320^{***}	0.258^{***}
	(0.557)	(0.358)	(0.222)	(0.351)	(0.051)
D_{WWI}	-0.001	0.010	0.004	0.015^{***}	-0.014^{***}
	(0.012)	(0.010)	(0.007)	(0.005)	(0.003)
D_{WWII}	0.008	0.007	0.018^{***}	0.029^{***}	0.018^{***}
	(0.010)	(0.009)	(0.006)	(0.004)	(0.003)
R^2	0.038	0.116	0.175	0.455	0.455
Num. obs.	112	112	112	112	112

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

This table presents the regressions results across each time scale and for each country. The equation estimated is eq 22 augmented with dummies for World War I and II. The estimation method is HAC-OLS. The weights follows Newey-West.

Table 4: Correlations by frequency scale using alternative definition and year coverage

Country	FR	UK	US	
Labour share definition	ls_{pz}		ls	ls_{pz}
Year coverage	1897-2010	1856-2010	1930-2010	1930-2010
D1	-0.42***	-0.34***	-0.65***	-0.67***
D2	-0.29***	-0.16*	-0.79***	-0.79***
D3	0.09	-0.44***	-0.30***	-0.28**
D4	0.08	-0.54***	-0.71***	-0.68***
S4	0.61***	0.45***	-0.13	-0.09

Table 5: Regression across frequencies

Dependent variable $\Delta y_{j,t}$					
Estimation method - HAC OLS					
France					
Labour share definition ls_{pz}					
Year coverage 1898-2010					
	D_1	D_2	D_3	D_4	S_4
cst	0.000	0.000	0.000	0.000	-0.186^{***}
	(0.002)	(0.003)	(0.010)	(0.004)	(0.053)
ω_t	-1.543^{***}	-0.451^{***}	0.100	0.052	0.266^{***}
	(0.328)	(0.106)	(0.172)	(0.116)	(0.082)
R^2	0.179	0.076	0.008	0.007	0.376
N. obs.	113	113	113	113	113
UK					
Labour share definition ls_{pz}					
Year coverage 1857-2010					
	D_1	D_2	D_3	D_4	S_4
cst	0.000	0.000	0.000	0.000	-0.019
	(0.000)	(0.003)	(0.002)	(0.006)	(0.144)
ω_t	-1.270	-0.322	-0.377^{***}	-0.277^{**}	0.047
	(0.858)	(0.250)	(0.103)	(0.135)	(0.220)
R^2	0.120	0.024	0.197	0.307	0.230
N. obs.	154	154	154	154	154
United-States					
Labour share definition ls					
Year coverage 1931-2010					
	D_1	D_2	D_3	D_4	S_4
cst	0.000	0.001	0.001	0.001	0.059
	(0.002)	(0.001)	(0.002)	(0.002)	(0.042)
ω_t	-2.773^{***}	-1.597^{***}	-0.859^{***}	-1.970^{***}	-0.049
	(0.366)	(0.134)	(0.201)	(0.235)	(0.055)
R^2	0.43	0.65	0.19	0.48	0.01
N. obs.	78	78	78	78	78

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

This table presents the regressions results across each time scale and for each country using alternative labour share definition for France and the UK (ls). For the US, the data source is Piketty and Zucman (2014). The equation estimated is eq 22. The estimation method is HAC-OLS. The weights follows Newey-West.

Table 6: Regression across frequencies with dummies for World Wars I and II

	Dependent variable $\Delta y_{j,t}$				
	Estimation method - HAC OLS				
	France				
	Labour share definition ls_{pz}				
	Year coverage 1898-2010				
	D_1	D_2	D_3	D_4	S_4
Intercept	-0.001	0.001	0.003	0.002	-0.226^{***}
	(0.004)	(0.002)	(0.002)	(0.001)	(0.025)
ω_t	-1.561^{***}	-0.445^{***}	0.174^{*}	0.186^{***}	0.321^{***}
	(0.320)	(0.153)	(0.101)	(0.063)	(0.033)
D_{WWI}	0.001	0.000	-0.012	-0.012^{**}	0.014^{**}
	(0.019)	(0.010)	(0.010)	(0.006)	(0.006)
D_{WWII}	0.013	-0.007	-0.038^{***}	-0.020^{***}	-0.020^{***}
	(0.016)	(0.008)	(0.008)	(0.005)	(0.005)
R^2	0.162	0.057	0.158	0.132	0.456
Num. obs.	113	113	113	113	113
	UK				
	Labour share definition ls_{pz}				
	Year coverage 1857-2010				
	D_1	D_2	D_3	D_4	S_4
Intercept	0.000	0.000	0.000	0.000	-0.016^{***}
	(0.002)	(0.001)	(0.001)	(0.000)	(0.005)
ω_t	-1.273^{***}	-0.322^{*}	-0.388^{***}	-0.277^{***}	0.043^{***}
	(0.283)	(0.169)	(0.063)	(0.034)	(0.007)
D_{WWI}	0.001	0.000	-0.005	0.001	-0.010^{***}
	(0.009)	(0.008)	(0.004)	(0.002)	(0.003)
D_{WWII}	-0.001	-0.001	0.005	-0.002	0.001
	(0.007)	(0.007)	(0.004)	(0.002)	(0.002)
R^2	0.102	0.004	0.197	0.297	0.278
Num. obs.	154	154	154	154	154
	United-States				
	Labour share definition ls				
	Year coverage 1931-2010				
	D_1	D_2	D_3	D_4	S_4
Intercept	0.000	0.000	0.001	-0.001	0.038
	(0.002)	(0.001)	(0.002)	(0.002)	(0.038)
ω_t	-2.755^{***}	-1.594^{***}	-0.831^{***}	-1.538^{***}	-0.024
	(0.368)	(0.134)	(0.210)	(0.256)	(0.049)
D_{WWII}	0.004	0.004	0.003	0.021^{***}	0.010^{***}
	(0.006)	(0.005)	(0.007)	(0.006)	(0.002)
R^2	0.419	0.646	0.175	0.535	0.206
Num. obs.	78	78	78	78	78

$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

This table presents the regressions results across each time scale and for each country using alternative labour share definition for France and the UK (ls). For the US, the data source is Piketty and Zucman (2014). The equation estimated is eq 22 augmented with dummies for World War I and II. The estimation method is HAC-OLS. The weights follows Newey-West.