Mho Testing Techniques & Math Lecture Summary Notes
Lecture with diagrams, test equipment and examples will finalize concepts.

The goal of this lecture is to learn how to select the voltage and current phasors to properly test the characteristic of phase to phase and 3 phase mho distance units with no offset. With the tools we have available now for testing mho distance units, the math behind how we test these characteristics is becoming less apparent. Hopefully this lecture with real examples will enhance our skills of phasor diagrams and vector math and you will be able to manually test the most common distance relay characteristics. This characteristic testing is necessary to be able to say if any particular distance element is in calibration or not. Concepts learned here can apply to other characteristics.

Distance elements are a measurement of impedance and the impedance is calculated using the faulted phase voltages and currents.

During a phase to phase fault the faulted phase voltages collapse and their phasors swing towards each other. The faulted phase currents increase in magnitude equally and lag the respective faulted line to line voltage by an angle defined by the fault impedance and are 180 degrees out from each other. The phase to phase mho units work off of the faulted line to line voltage and the delta (difference), faulted phase currents.
This relationship is; **Z= Line-Line Voltage / 2 x Line Current, for a phase to phase fault.** The two times line current is because it is the delta (difference), of the faulted phase currents and they are 180 degrees out from each other.

During a three phase fault the phase to neutral voltages collapse in on themselves and the respective phase currents increase in magnitude and lag their respective phase to neutral voltage by an angle defined by the fault impedance.
This relationship is; **Z= Line-Neut Voltage / Line Current, for a three phase fault.**

Old test gear like described in the KD-10 I.L. (41-490J) uses Line to Line voltage applied to the phase angle meter and therefore they expressed the relationship for a three phase fault as; **Z= Line-Line Voltage /sqrt of 3 x Line Current, for a three phase fault.** And they tell you to subtract 30 degrees from the phase angle meter.

You do not have to do this with modern test gear. For modern test gear it is easier by far to use the first relationship; **Z= Line-Neut Voltage / Line Current, for a three phase fault.**

I use this symbol ~ to represent lagging angle in degrees. 75 ~ means 75 degrees lagging.

A mho unit impedance characteristic with no offset is a circle that goes thru the origin. The origin is zero resistance and zero inductance. Mho unit characteristic diagrams will be posted in the lecture room. Also see figure 5 on page 31 in the CEY51A I.L. GEK-1275K. To define a mho unit impedance circle that goes thru the origin you set the diameter angle commonly referred to as the maximum torque angle (MTA), and you set the impedance reach at the MTA. This impedance reach at the MTA is the diameter of the circle and the furthest impedance reach of the mho unit impedance characteristic. Now on a R/X diagram and with a protractor and compass you can draw the impedance characteristic. You can now calculate the reach at any other angle with simple trigonometry and Thales theorem. Explanation is following.
Example: MTA = 75 degrees (~), Reach @ MTA = 12 ohms. What is the reach at 30 ~?
Z @ 30 ~ = Reach @ MTA * cosine of |MTA – 30 ~|
Z @ 30 ~ = 12 ohms * cos of 45 ~ = 12 ohms * 0.707 = 8.49 ohms, **Z @ 30 ~ = 8.49 ohms**

Thales' theorem: if AC is a diameter, then the angle at B is a right angle. In geometry, **Thales' theorem** states that if A, B and C are points on a circle where the line AC is a diameter of the circle, then the angle ∠ABC is a right angle.

Point A is the origin. Point C is the reach at the MTA. Point B is the reach under calculation. Line AC is the diameter and the furthest reach of the mho characteristic and forms the hypotenuse of this right triangle. Line AB is the reach under calculation and is the adjacent side of this right triangle. Therefore the cosine of the absolute value of the difference of the MTA and the angle under calculation * the reach at the MTA equals the reach at the angle under calculation. Cosine in a right triangle = the ratio of the side adjacent to an acute angle (in a right-angled triangle) to the hypotenuse. **We will diagram this in the lecture.**

Another Example: MTA =75 degrees (~), Reach @ MTA = 12 ohms. What is the reach at 85 ~?
Z @ 85 ~ = Reach @ MTA * cosine of |MTA – 85 ~|
Z @ 85 ~ =12 ohms * cos of 10 ~ =12 ohms * 0.985 11.82 ohms, **Z @ 30 ~ = 11.82 ohms**

Testing Techniques for phase to phase mho units using 3 phase modern test gear connections:
Note; using lagging angles on the test gear like with Epoch's. Doble uses leading angles.

1) Always keep the voltage on the un-faulted phase @ nominal voltage and correct angle.
2) Line to Neutral nominal voltage will be 69.28 volts, therefore Line to Line = 120 volts.
3) Use equal impedance test leads for current into relay and back out to test gear.
4) Leave current on at zero amps on un-faulted phase.
5) The Line to Line voltage is equal to the sqrt of 3 * Line to Neutral Voltage and leads the Line to Neutral voltage by 30 ~. Line to Line voltage is the vector difference of the two faulted Line to Neutral Voltages.
6) Never test below an expected current of 5 amps. You do this by picking a test voltage that the expected current calculates out to 5 amps at the MTA(The highest impedance reach). Now for all other test angles leave your test voltage at this calculated test voltage. Test Values Calculation Example to follow.
7) Lag the fault current from the Line to Line Voltage by the desired test angle. If the Line to Line Voltage is at 330 ~ and you are testing the reach at 75 ~ then lag the lead phase current angle from 330 ~ by 75 ~ which would be 45 ~. The other current angle would be 180~ + 45 ~ which would be 225 ~. Example for an A-B phase fault @ 75 ~ : A-Gnd voltage @ 0 ~, B-Gnd voltage @ 120 ~, Aph curr @ 45 ~ & Bph curr @ 225 ~.
8) To prove the mho unit characteristic test 5 points on the circle. For checking calibration & accuracy reasons do not test points further than 30 degrees off the MTA. Example, if the MTA is 75 ~ test the characteristic at 75 ~, 75 ~ + & - 30 ~, 75 ~ + & - 15 ~.
9) Before proving the mho unit characteristic make sure the MTA is within + or – 2 ~ of ideal and the reach at the MTA is within + or – 5 % of ideal. You might have to make adjustments and re-check both MTA and reach at MTA for electro-mechanical until both are within tolerance. Once both are within tolerance characteristic checks are almost always within tolerance.
10) Never test the MTA by swinging angles beyond 30 degrees from the MTA. I generally test MTA at 20 ~ off from the MTA. I calculate the expected current for 20 ~ off from the MTA using the test voltage calculated earlier. I apply the test voltage and this current to the mho unit at the MTA, check that the contact closed (if not raise the current up slightly until the contact is closed. Then I swing the angle one direction away from the MTA until the contact just opens and record this angle. Then swing the angle back to the MTA and keep swinging the other direction away from the MTA until the contact just opens again and record this angle. Add these two recorded angles and divide this sum by 2. This is the MTA of this phase to phase mho unit.

Test Values Calculation Example for a 75 ~ MTA Phase to Phase Mho Unit & reach at 75 ~ is 9.0 ohms:
Seeing how we want to be above 5 amps for pickup current at the MTA we will solve for test voltage phase to phase first. **V_{L-L} = Z * 2 * I**, V_{L-L} = 9 * 2 * 5, V_{L-L} = 90 volts. This is an okay test voltage because it does not exceed the rated voltage of the mho unit. V_{L-N} = V_{L-L}/sqrt of 3. V_{L-N} = 51.96 volts.

Now for an A-B phase fault @ 75 ~: A-Gnd voltage @ 0 ~ & 51.96 volts, B-Gnd voltage @ 120 ~ & 51.96 volts, C-Gnd voltage @ 240 ~ & 69.28 volts, Aph curr @ 45 ~ & Bph curr @ 225 ~. Then ramp up Aph & Bph current equally and if the mho unit is close to being in calibration the contact goes closed @ 5amps or so. This is because we are applying 90 volts @ 330 ~ for the phase to phase test voltage calculated above. You now keep this test voltage the same and move on to testing other points or testing the MTA.

Values to know for finishing testing & calibrating this phase to phase mho unit:

Z_{Reach} @ + & - 30 ~ of MTA = Z_{Reach} @ MTA * cos of 30 ~ = 9.0 ohms * .866 = **7.79 ohms**
Thus: Z_{Reach} @ 105 ~ = **7.79 ohms** and Z_{Reach} @ 45 ~ = **7.79 ohms**

Z_{Reach} @ + & - 20 ~ of MTA = Z_{Reach} @ MTA * cos of 20 ~ = 9.0 ohms * .940 = **8.46 ohms**
Thus: Z_{Reach} @ 95 ~ = **8.46 ohms** and Z_{Reach} @ 55 ~ = **8.46 ohms**

Z_{Reach} @ + & - 15 ~ of MTA = Z_{Reach} @ MTA * cos of 15 ~ = 9.0 ohms * .966 = **8.69 ohms**
Thus: Z_{Reach} @ 90 ~ = **8.69 ohms** and Z_{Reach} @ 60 ~ = **8.69 ohms**

$I_{expected}$ @ 105 ~ = V_{L-L} / (Z_{Reach} @ 105 ~ * 2) = 90volts / (7.79 ohms*2) = **5.78 amps**
$I_{expected}$ @ 45 ~ = V_{L-L} / (Z_{Reach} @ 45 ~ * 2) = 90volts / (7.79 ohms*2) = **5.78 amps**

$I_{expected}$ @ 95 ~ = V_{L-L} / (Z_{Reach} @ 95 ~ * 2) = 90volts / (8.46 ohms*2) = **5.32 amps**
$I_{expected}$ @ 55 ~ = V_{L-L} / (Z_{Reach} @ 55 ~ * 2) = 90volts / (8.46 ohms*2) = **5.32 amps**

$I_{expected}$ @ 90 ~ = V_{L-L} / (Z_{Reach} @ 90 ~ * 2) = 90volts / (8.69 ohms*2) = **5.18 amps**
$I_{expected}$ @ 60 ~ = V_{L-L} / (Z_{Reach} @ 60 ~ * 2) = 90volts / (8.69 ohms*2) = **5.18 amps**

Again for this example A-B phase mho unit with the V_{L-L} Voltage @ 90 volts & 330 ~ to test the:

Z_{reach} @ 105 ~, Aph current would be @ 75 ~ and Bph current would be @ 255 ~
Z_{reach} @ 95 ~, Aph current would be @ 65 ~ and Bph current would be @ 245 ~
Z_{reach} @ 90 ~, Aph current would be @ 60 ~ and Bph current would be @ 240 ~
Z_{reach} @ 75 ~, Aph current would be @ 45 ~ and Bph current would be @ 225 ~
Z_{reach} @ 60 ~, Aph current would be @ 30 ~ and Bph current would be @ 210 ~
Z_{reach} @ 55 ~, Aph current would be @ 25 ~ and Bph current would be @ 205 ~
Z_{reach} @ 45 ~, Aph current would be @ 15 ~ and Bph current would be @ 195 ~

We will do a B-C phase mho unit and a C-A phase mho unit with different settings during the lecture to finalize concepts.

Testing Techniques for 3 phase mho units using 3 phase modern test gear connections:
Note; using lagging angles on the test gear like with Epoch's. Doble uses leading angles.

1) Use equal impedance test leads for current into relay and back out to test gear.
2) Never test below an expected current of 5 amps. You do this by picking a test voltage that the expected current calculates out to 5 amps at the MTA(The highest impedance reach). Now for all other test angles leave your test voltage at this calculated test voltage. Test Values Calculation Example to follow.
3) To prove the mho unit characteristic test 5 points on the circle. For checking calibration & accuracy reasons do not test points further than 30 degrees off the MTA. Example, if the MTA is 75 ~ test the characteristic at 75 ~, 75 ~ + & - 30 ~, 75 ~ + & - 15 ~.
4) Before proving the mho unit characteristic make sure the MTA is within + or – 2 ~ of ideal and the reach at the MTA is within + or – 5 % of ideal. You might have to make adjustments and re-check both MTA and reach at MTA for electro-mechanical until both are within tolerance. Once both are within tolerance characteristic checks are almost always within tolerance.
5) Never test the MTA by swinging angles beyond 30 degrees from the MTA. I generally test MTA at 20 ~ off from the MTA. I calculate the expected current for 20 ~ off from the MTA using the test voltage calculated earlier. I apply the test voltage and this current to the mho unit at the MTA, check that the contact closed (if not raise the current up slightly until the contact is closed. Then I swing the angle one direction away from the MTA until the contact just opens and record this angle. Then swing the angle back to the MTA and keep swinging the other direction away from the MTA until the contact just opens again and record this angle. Add these two recorded angles and divide this sum by 2. This is the MTA of this phase to phase mho unit.

Test Values Calculation Example for a 75 ~ MTA 3 phase Mho Unit & reach at 75 ~ is 9.0 ohms:
Seeing how we want to be above 5 amps for pickup current at the MTA we will solve for test voltage 3 phase first. $\mathbf{V_{L-N} = Z * I}$, $V_{L-N} = 9 * 5$, $V_{L-N} = 45$ volts. This is an okay test voltage because it does not exceed the rated voltage of the mho unit.

Now for a 3 phase fault @ 75 ~: A-Gnd voltage @ 0 ~ & 45.0 volts, B-Gnd voltage @ 120 ~ & 45.0 volts, C-Gnd voltage @ 240 ~ & 45.0 volts, Aph curr @ 75 ~ & Bph curr @ 195 ~ & Cph curr @ 315 ~. Then ramp up Aph & Bph & Cph current equally and if the mho unit is close to being in calibration the contact goes closed @ 5amps or so. You now keep this test voltage the same and move on to testing other points or testing the MTA.

Values to know for finishing testing & calibrating this 3 phase mho unit:

Z_{Reach} @ + & - 30 ~ of MTA = Z_{Reach} @ MTA * cos of 30 ~ = 9.0 ohms * .866 = **7.79 ohms**
Thus: Z_{Reach} @ 105 ~ = **7.79 ohms** and Z_{Reach} @ 45 ~ = **7.79 ohms**

Z_{Reach} @ + & - 20 ~ of MTA = Z_{Reach} @ MTA * cos of 20 ~ = 9.0 ohms * .940 = **8.46 ohms**
Thus: Z_{Reach} @ 95 ~ = **8.46 ohms** and Z_{Reach} @ 55 ~ = **8.46 ohms**

Z_{Reach} @ + & - 15 ~ of MTA = Z_{Reach} @ MTA * cos of 15 ~ = 9.0 ohms * .966 = **8.69 ohms**
Thus: Z_{Reach} @ 90 ~ = **8.69 ohms** and Z_{Reach} @ 60 ~ = **8.69 ohms**

$I_{expected}$ @ 105 ~ = V_{L-N} / Z_{Reach} @ 105 ~ = 45volts / 7.79 ohms = **5.78 amps**
$I_{expected}$ @ 45 ~ = V_{L-N} / Z_{Reach} @ 45 ~ = 45volts / 7.79 ohms = **5.78 amps**

$\mathbf{I_{expected}}$ **@ 95 ~** = V_{L-N} / Z_{Reach} @ 95 ~ = 45volts / 8.46 ohms = **5.32 amps**
$\mathbf{I_{expected}}$ **@ 55 ~** = V_{L-N} / Z_{Reach} @ 55 ~ = 45volts / 8.46 ohms = **5.32 amps**

$\mathbf{I_{expected}}$ **@ 90 ~** = V_{L-N} / Z_{Reach} @ 90 ~ = 45volts / 8.69 ohms = **5.18 amps**
$\mathbf{I_{expected}}$ **@ 60 ~** = V_{L-N} / Z_{Reach} @ 60 ~ = 45volts / 8.69 ohms = **5.18 amps**

Again for this example 3 phase mho unit with the A-Gnd voltage @ 0 ~ & 45.0 volts, B-Gnd voltage @ 120 ~ & 45.0 volts, C-Gnd voltage @ 240 ~ & 45.0 volts the test current angles are:

For Z_{reach} @ 105 ~; Aph current @ 105 ~ & Bph current @ 225 ~ & Cph current @ 345 ~
For Z_{reach} @ 95 ~; Aph current @ 95 ~ & Bph current @ 215 ~ & Cph current @ 335 ~
For Z_{reach} @ 90 ~; Aph current @ 90 ~ & Bph current @ 210 ~ & Cph current @ 330 ~
For Z_{reach} @ 75 ~; Aph current @ 75 ~ & Bph current @ 195 ~ & Cph current @ 315 ~
For Z_{reach} @ 60 ~; Aph current @ 60 ~ & Bph current @ 180 ~ & Cph current @ 300 ~
For Z_{reach} @ 55 ~; Aph current @ 55 ~ & Bph current @ 175 ~ & Cph current @ 295 ~
For Z_{reach} @ 45 ~; Aph current @ 45 ~ & Bph current @ 165 ~ & Cph current @ 285 ~

We will use a CEY51A relay for demonstration of 3 phase reach testing @ the MTA of 75 ~. This should strengthen the math given is correct for phase to phase mho units & 3 phase mho units. A CEY51A has three phase to phase mho units for complete phase to phase & 3 phase distance protection.

Question: With the relay set to pick up at 9.0 ohms at the MTA of 75 ~ for each phase to phase mho unit, will the phase to phase mho units pick up using the three phase values given earlier for testing reach at 75 ~**???** **Yes they will.** The phase to phase mho units work off of the faulted line to line voltages and the delta (difference), faulted phase currents.

We will apply: A-Gnd voltage @ 0 ~ & 45.0 volts, B-Gnd voltage @ 120 ~ & 45.0 volts, C-Gnd voltage @ 240 ~ & 45.0 volts, Aph curr @ 75 ~ & 5 amps, Bph curr @ 195 ~ & 5 amps, Cph curr @ 315 ~ & 5 amps.

The Aph to Bph mho unit will pick up when (V_{AB} / I_{AB}) is 9 ohms or lower @ 75 ~ MTA
V_{AB} = the vector difference of V_{AN} – V_{BN} = 45volts@0~ - 45volts@120~ = 77.94 volts @ 330 ~
I_{AB} = the vector difference of I_A – I_B = 5amps@75~ - 5amps @195~ = 8.66amps @ 45~
This 8.66amps @ 45~ lags the 77.94volts @ 330~ by 75~
77.94 volts / 8.66 amps = 9.0 ohms and so the Aph to Bph mho unit will pick up.

The Bph to Cph mho unit will pick up when (V_{BC} / I_{BC}) is 9 ohms or lower @ 75 ~ MTA
V_{BC} = the vector difference of V_{BN} – V_{CN} = 45volts@120~ - 45volts@240~ = 77.94 volts @ 90 ~
I_{BC} = the vector difference of I_B – I_C = 5amps@195~ - 5amps @315~ = 8.66amps @ 165~
This 8.66amps @ 165~ lags the 77.94volts @ 90~ by 75~
77.94 volts / 8.66 amps = 9.0 ohms and so the Bph to Cph mho unit will pick up.

The Cph to Aph mho unit will pick up when (V_{CA} / I_{CA}) is 9 ohms or lower @ 75 ~ MTA
V_{CA} = the vector difference of V_{CN} – V_{AN} = 45volts@240~ - 45volts@0~ = 77.94 volts @ 210 ~
I_{CA} = the vector difference of I_C – I_A = 5amps@315~ - 5amps @75~ = 8.66amps @ 285~
This 8.66amps @ 285~ lags the 77.94volts @ 210~ by 75~
77.94 volts / 8.66 amps = 9.0 ohms and so the Cph to Aph mho unit will pick up.

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