Ground Distance Relays – Understanding the Various Methods of Residual Compensation, Setting the Resistive Reach of Polygon Characteristics, and Ways of Modeling and Testing the Relay

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Abstract –

The correct operation of ground distance relays is highly dependent on the correct application of the residual or zero-sequence compensation factor. But what are these factors? Various relays vendors have different forms of naming, defining and applying these factors and this confuses relay application engineers. Names like KN, K0, KE, KG, KZN, RE/RL and XE/XL, Z0/Z1, and others are used; some factors are simple scalar numbers and others are in vector form. A factor K0 in one relay can have a different definition from that of another relay that uses exactly the same name, and this may be true whether the relays come from different vendors or even from the same vendor. An incorrect compensation setting can result in either misoperation of the relay or its failure to operate for ground faults. This paper provides some derivations of residual compensation and a survey of the various forms of compensation, their definitions and how to convert from one form to another.

Another source of confusion in the application of ground distance relays that have polygon-shaped characteristics is the way the resistive reach is defined. In some relays the relay resistive reach setting is defined as a per phase resistance, which can be affected by the residual compensation factor; while in another relay it could be the ground loop resistance, which is not affected by the residual compensation factor. In addition the angle of the resistive blinder may or may not be influenced by the residual compensation angle. Hence, there is a need to accurately model and visualize the relay characteristic consistently, especially when comparing relays.

In order to ensure that the relay is set correctly and operates as expected it is necessary to test the relay characteristic for ground faults and verify that it matches the relay settings and expected characteristics. Modeling the ground relay characteristic and using the correct residual compensation setting can be a complex subject for the test technician. Depending on how the relay resistive settings are entered, questions arise on how to model the relay polygon characteristics – should it be modeled in the per phase plane, or in the loop plane, or should the reactance line be modeled per phase and the resistance line modeled per loop since many distance relays enter the settings as such? Even mho ground characteristics can be a challenge when the residual compensation setting is a vector with angles far away from zero.

When testing the ground distance relay the capability of the protective relay test system becomes another important factor since the test system may or may not have the capability to exactly model the relay characteristic and residual compensation factor. One may have to convert the compensation to another form and model the characteristics in another way when testing and viewing the results of the test. This paper describes some available methods to do this and offers suggestions for testing ground distance relays.

A good understanding of these concepts is essential for relaying personnel in order to properly set the relays and test them. Post mortem analysis of relay operations also requires a good understanding of these concepts.

Conventions used in this paper:

Complex variables are shown in bold letters. For example, **Z** is a complex impedance that consists of the resistance R and reactance X, or the magnitude Zmag and angle Zang, and may be written as:

$$\mathbf{Z} = R + jX = Zmag\ \underline{/Zang.}$$

Residual Compensation and Zero-sequence Current Compensation

Most ground distance relays are designed so that they measure, and can be set, in terms of positive-sequence impedance. Consider the system shown in Figure 1 where a relay 21G is required to measure the impedance, n•**Z1**, from the relaying point to the phase A to ground fault at F. Quantities that are readily available or that can be derived include the relay voltage **VaR**, the current **Ia**, the residual current **In**, and zero-sequence current **I0**.

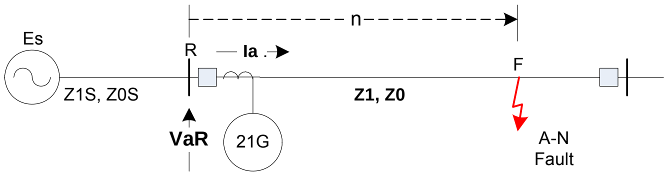


Figure 1

In order to accomplish this goal to measure the distance to the fault in terms of positive sequence some form of compensation are used. Appendix A includes the derivation of some of these forms or techniques of compensation. In order to measure the fault impedance n**Z1,** ground distance relays modify the phase current **Ia** by adding to it a portion of the residual current **In**.

One technique known as *residual compensation* adds a portion **KN** (called *residual compensation factor*) of the residual current **In**, to the phase current **Ia**. This modified current is used in the measuring circuit of the relay so that it measures the impedance as.

$$\mathbf{Zrelay} = \mathbf{VaR}/(\mathbf{Ia} + \mathbf{KN}\bullet\mathbf{In}) = n\mathbf{Z1} \qquad (1)$$

$$\text{Where} \quad \mathbf{KN} = (\mathbf{Z0}/\mathbf{Z1} - 1)/3 \qquad (2)$$

Figure 2 shows a simplified network model equivalent to Figure 1, where it can be seen (see Appendix) that the ground return impedance is

$$n\bullet\mathbf{ZN} = n\bullet(\mathbf{Z0}\text{-}\mathbf{Z1})/3 \text{ and } \mathbf{ZN} = \mathbf{KN}\bullet\mathbf{Z1}$$

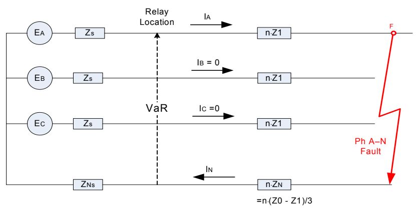


Figure 2. Simplified network model

The concept of residual compensation is implemented in some relays as shown in Figure 3, where the phase current **Ia** (for a phase A to ground distance element) passes through a phase or positive-sequence replica impedance **Z1** and the residual current **In** passes through a neutral replica impedance **ZN**. A residual compensation factor **KN** is used to adjust the value of **ZN** so that **ZN** = **KN • Z1**. The net effect is the same as equation (1).

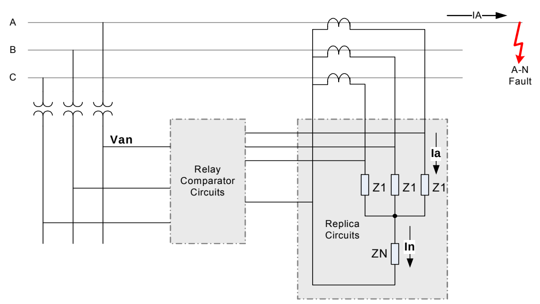


Figure 3. An implementation of residual compensation in a ground distance relay

Another technique is zero-sequence current compensation. Here the phase current **Ia** is modified by adding to it a portion **K0** (called *zero-sequence current compensation factor*) of the zero-sequence current **I0**. The relay measures the impedance

$$\mathbf{Zrelay} = \mathbf{VaR}/(\mathbf{Ia} + \mathbf{K0}\bullet\mathbf{I0}) = n\bullet\mathbf{Z1} \qquad (3)$$

$$\mathbf{K0} = \mathbf{Z0}/\mathbf{Z1} - 1 \qquad (4)$$

Clearly both factors are related as **K0** = 3•**KN**, which is not surprising since **In** = 3•**I0**.

In some relays the zero-sequence current compensation factor **K0** is defined as in equation (2), which is actually the definition of **KN**; while in other relays **K0** is defined as the ratio **Z0**/**Z1**, although internally it may be used and substituted into equation (4).

Other compensation factor symbols used in some relays include **KE**, **KG**. And these could be defined using either equations (2) or (4). The factors **KN**, **K0**, **KE** or **KG** are also referred to as earth-return compensation factor, ground-return compensation factor, and neutral impedance correction factor.

Other relays use only the reactance part of the impedances, X0/X1, and compensate for the angles of **Z1** and **ZN** or **Z0** but some relays make an approximation by assuming that the angles of **Z1** and **ZN** are almost the same and that the reactance values are almost equal to the impedance magnitudes. In other relays the ratio RE/RL is used to compensate the resistance part and the ratio XE/XL is uses to compensate the reactance part separately. Yet, some relays are simply set in terms positive-sequence impedance **Z1** (magnitude and angle) and zero-sequence impedance **Z0** (magnitude and angle). And still other relays set the relay using R1, X1, R0 and X0. In both cases the residual compensation is automatically calculated internally by the relay using these set values.

A list of formulas, compensation type names, and symbols, including other related variables to completely define the type of compensation, used in various distance relays is shown in Table 1. This is not a complete list of all terms used in all relays but it represents a majority of common relays. Other terms that are very similar are not included for brevity and clarity. Prefixes or suffixes that indicate the zone that the setting applies to have been removed.

Table 1. Compensation Formulas, Names and Symbols

Formula for calculating factors	This form of compensation is known as	Factor symbol or name (As set in the relay)	Other settings directly related to factor	Remarks
$\frac{1}{3}\left(\frac{\boldsymbol{Z}0}{\boldsymbol{Z}1}-1\right)$	Residual compensation Zero-sequence current compensation Earth-return compensation Ground-return compensation Earth impedance compensation Neutral impedance correction	**KN** magnitude and angle or **K0** magnitude and angle or **KE** magnitude and angle or **KG** magnitude and angle or KZN res comp & KZN angle		
$\frac{\boldsymbol{Z}0}{\boldsymbol{Z}1}-1$	Zero-sequence current compensation	**K0** magnitude and angle		
$\frac{\boldsymbol{Z}0}{\boldsymbol{Z}1}$	Zero-sequence current compensation	**Z0**/**Z1** magnitude and angle		Relay internally calculates the correct complex factor using (|**Z0**/**Z1**| /angle) - 1
		K0 or ZerSeqK0 Z0/Z1 ratio	Z1 angle Z0 angle	Relay internally calculates the correct complex factor using K0 /Z0angle – Z1angle
$\frac{1}{3}\left(\frac{X0}{X1}-1\right)$	Residual compensation Neutral impedance correction	**KN**	Z1 angle ZN angle	ZN angle=Z0angle-Z1angle Some relays use only the magnitude (scalar), angle=0.
		KN	Line L/R matching (ms) TauK=X1/ωR TauN=(X0-X1)/(ω(R0-R1))	Z1 angle=ArcTan(tauK) ZN angle=ArcTan(tauN)
	Residual compensation		ZPh magnitude and angle ZN magnitude and angle	Relay uses **ZN** directly in measurements
			KZPh , KZN, KZ1 Z1 angle, ZN angle	Relay internally calculates the **ZPh**=KZ1•KZPh=**Z1** and **ZN**=KZ1•KZN/KZPh
$\frac{1}{3}\left(\frac{R0}{R1}-1\right)$ $\frac{1}{3}\left(\frac{X0}{X1}-1\right)$	Earth impedance (residual) compensation	RE/RL and XE/XL		
	Earth-return compensation Residual compensation Zero-sequence current compensation		R1, X1, R0, X0 or Z1 magnitude and angle Z0 magnitude and angle	Relay internally calculates the complex compensation factor $=(\boldsymbol{Z}0/\boldsymbol{Z}1-1)/3$ where **Z1**=R1+jX1, **Z0**=R0+jX0

Conversion between Forms of Compensation

It is sometimes necessary to convert the values from one form of compensation to another. For example when replacing an existing relay one can use the existing settings and convert some setting as necessary. Of course, it is best to recalculate everything whenever possible. It may also be necessary to convert when testing since the relay may have some form of compensation that the test system does not support.

In order to avoid ambiguity for the rest of the discussion, we shall use the following terms. These terms refer to the ground distance relay settings. It is possible that the positive-sequence ground element impedance settings may be different from that of the phase-phase elements both in magnitude and angle.

Z1 = positive-sequence impedance reach setting of the ground distance element
Z1mag, Z1ang: magnitude and phase angle of **Z1**
R1, X1: resistance and reactance part of **Z1**
RL, XL: same as R1 and X1

Z0 = zero-sequence impedance (is used for calculating other compensation settings or is set directly)
Z0mag, Z0ang: magnitude and phase angle of **Z0**
R0, X0: resistance and reactance part of **Z0**
ZN = ground- or earth-return impedance = (**Z0** – **Z1**) / 3
ZNmag, ZNang: magnitude and phase angle of **ZN**
RN, XN: resistance and reactance part of **ZN**

ZE = **ZN**
RE, XL: same as RN and XN

KN = residual compensation factor = **ZN/Z1** = $\frac{1}{3}\left(\frac{Z0}{Z1} - 1\right)$

KNmag, KNang: magnitude and phase angle of **KN**

K0 = zero-sequence current compensation factor = $\left(\frac{Z0}{Z1} - 1\right)$

K0mag, K0ang: magnitude and phase angle of **K0**

K0ratio = zero-sequence current compensation ratio = |**Z0**|/|**Z1**|
KNx = residual compensation factor using reactance part only

$$= \frac{1}{3}\left(\frac{X0}{X1} - 1\right)$$

RE/RL = earth-impedance compensation resistance ratio = $\frac{1}{3}\left(\frac{R0}{R1} - 1\right)$

XE/XL = earth-impedance compensation reactance ratio = $\frac{1}{3}\left(\frac{X0}{X1} - 1\right)$

ZLoop = Phase-to-ground loop impedance (excluding the arc resistance, which will be dealt with separately)
= **Z1** + **ZN**

ZLoopMag, ZLoopAng: magnitude and phase angle of **ZLoop**
RLoop, XLoop: resistance and reactance parts of **ZLoop**

Zp = positive-sequence impedance of the fault point from the relay location
Rp = positive-sequence resistance of the fault point from the relay location
Xp = positive-sequence reactance of the fault point from the relay location

Table 2 includes a list of formulas on how to convert one form of compensation to another.

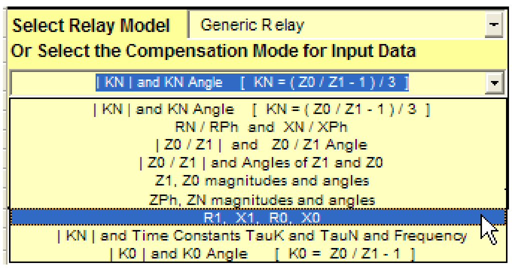
Table 2. Conversion from one form of compensation to another

From	**To**	**Conversion** [1]
KNmag and KNang	K0mag and K0ang	K0mag = KNmag•3 K0ang = Knmag
K0mag and K0ang	KNmag and KNang	KNmag = K0mag/3 KNang = K0mag
K0ratio and Z0ang	K0mag and K0ang	**K0** = K0ratio /(Z0ang - Z1ang) – 1
K0ratio and Z0ang	KNmag and KNang	**K0** = (K0ratio /(Z0ang - Z1ang) – 1) / 3
K0mag and K0ang	K0ratio and Z0ang	K0ratio /Angle01 = **K0** + 1 Z0ang = Angle01 + Z1Ang
KNmag and KNang	K0ratio and Z0ang	K0ratio /Angle01 = (3•**KN** + 1) Z0ang = Angle01 + Z1Ang
RE/RL and XE/XL	KNmag and KNang	KNr = (RE/RL)•Cos(Z1ang) KNx = (XE/XL)•Sin(Z1ang) $KNmag = \sqrt{KNr^2 + KNx^2}$ $KNang = ArcTan(KNx/KNr) - Z1ang$
KNmag and KNang	RE/RL and XE/XL	RE/RL=KNmag•Cos(KNang+Z1ang)/Cos(Z1ang) XE/XL=KNmag•Sin(KNang+Z1ang)/Sin(Z1ang)
Z1 and **ZN**	KNmag and KNang	**KN** = (**ZN/Z1**)
Z1 and **Z0**	KNmag and KNang	**KN** = $\frac{1}{3}\left(\frac{Z0}{Z1} - 1\right)$
R1, X1, R0, X0	KNmag and KNang	**KN** = [(R0+jX0)/(R1+jX1) – 1)] / 3
R1, X1, R0, X0	RE/RL and XE/XL	RL/RL = $\frac{1}{3}\left(\frac{R0}{R1} - 1\right)$ XE/XL = $\frac{1}{3}\left(\frac{X0}{X1} - 1\right)$

[1] It is assumed that the characteristic angle Z1ang is known. Z1ang and set Z0ang are not necessarily equal to the line impedance angles.

For fault points that lie on the characteristic setting angle Z1ang, all the above methods of compensation are equivalent and will have exactly the same measurement when applied correctly.

These formulas have been implemented in a spreadsheet as shown in the images Figure 4. This spreadsheet allows one to select the type of compensation, enter the setting values, and perform the conversion. All the values in the tables are equivalent to one another. A graph of the phase or positive-sequence impedance, ground-return impedance and loop impedance together with a mho characteristic and the equivalent loop characteristic, helps in visualizing their relationships.



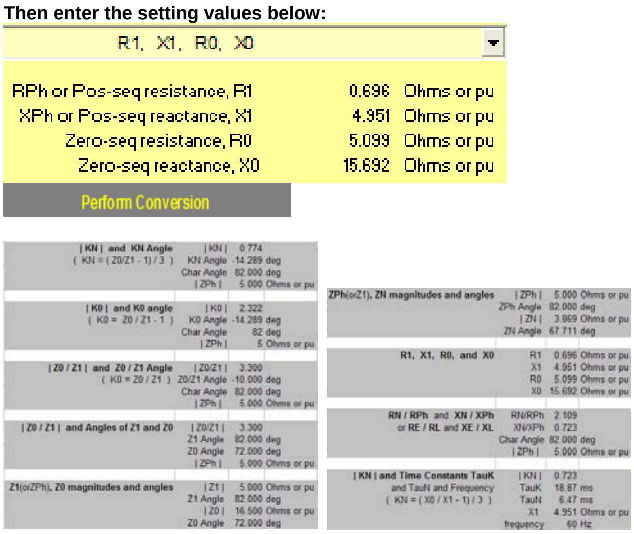


Figure 4. An MS Excel implementation of conversion compensation formulas

Loop Impedance, ZLoop

Although the ground distance relay may be set in terms of positive-sequence or per-phase impedance, in reality the loop impedance is measured. Figure 2 (loop network) shows that relay voltage **VaR** is the voltage drop across the loop impedance **Ia•** (**Z1**+**ZN**); or the voltage to current ratio is

VaR/Ia = Z1 + ZN = ZLoop

Figure 5 shows this relationship in the RX diagram, including their resistive and reactive components.

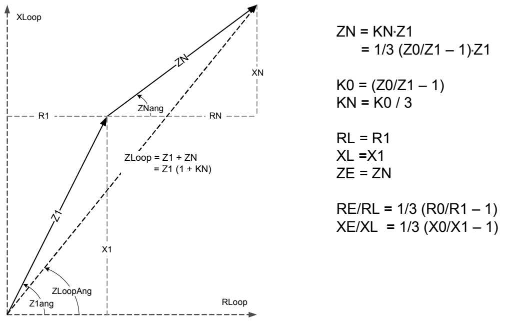


Figure 5. Relationships of Phase, Ground-return and Loop Impedances

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Converting to Loop Impedance.

Given the fault point, **Zp** = Rp + j Xp in the positive-sequence impedance plane, Table 3 provides the formulas needed to calculate the impedances in the loop impedance plane. It is assumed that the characteristic angle Phi1 is always known for the factors that use angles.

Table 3 Formulas for Converting Fault Impedances along the Characteristic Angle to Loop Impedances

Compensation type or Setting Type	Related settings	ZpLoop	RpLoop	XpLoop
KN		**ZpLoop=Zp•** (1+**KN**)	RpLoop=Re{**ZLoop**}	XpLoop=Im{**ZLoop**}
K0		**ZpLoop=Zp•** (1+**K0**/3)	RpLoop=Re{**ZLoop**}	XpLoop=Im{**ZLoop**}
K0ratio	Z0ang	**ZpLoop=Zp•** [1+(K0ratio /(Z0ang-Z1ang)-1)/3]	RpLoop=Re{**ZLoop**}	XpLoop=Im{**ZLoop**}
KNx	ZNang	**ZpLoop** = RpLoop+jXpLoop	RpLoop=Rf+Xfptan(ZNang)	XpLoop=Xp• (1+KNx)
	no ZNang	**ZpLoop** = RpLoop+jXpLoop	RpLoop=Rp• (1+KNx)	XpLoop=Xp• (1+KNx)
RE/RL XE/XL		**ZpLoop** = RpLoop+jXpLoop	RpLoop=Rp• (1+RE/RL)	XpLoop =Xp• (1+XE/XL)
Z1 and **ZN**		**ZpLoop=Zp•** (1+**ZN**/**Z1**)	RpLoop=Re{**ZLoop**}	XpLoop=Im{**ZLoop**}
R1, X1, R0, X0		**ZpLoop=Zp•**{1+ [(R0+jX0)/(R1+jX1)-1]/3}	RpLoop=Re{**ZLoop**}	XpLoop=Im{**ZLoop**}
Note: The table applies only to points that lie on the characteristic line angle setting. The exception is for those that use scalar compensation factors like KNx and RE/RL, XE/XL.				

Until now we have not yet included the fault resistance; this includes the arc resistance and the tower footing resistance, and is shown in Figure 6. The ratio V/I now becomes

VaR / **Ia** = **Z1** + **ZN** + Rarc + Rtf
= **ZLoop** + RFLoop

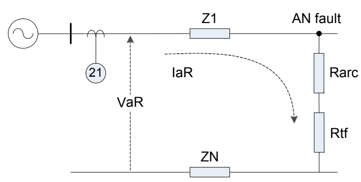


Figure 6

Where ZLoop is the same as before. RFloop is the fault loop resistance and does not include the line resistance.

Figure 7 shows the addition of the fault resistance plotted in the RX diagram. The RFLoop resistance is usually shown at the base of the RX diagram in relay manuals.

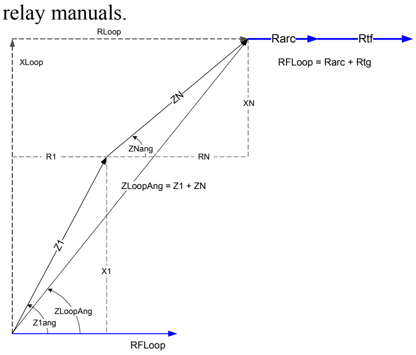


Figure 7

Characteristic Shapes and Resistive Reach

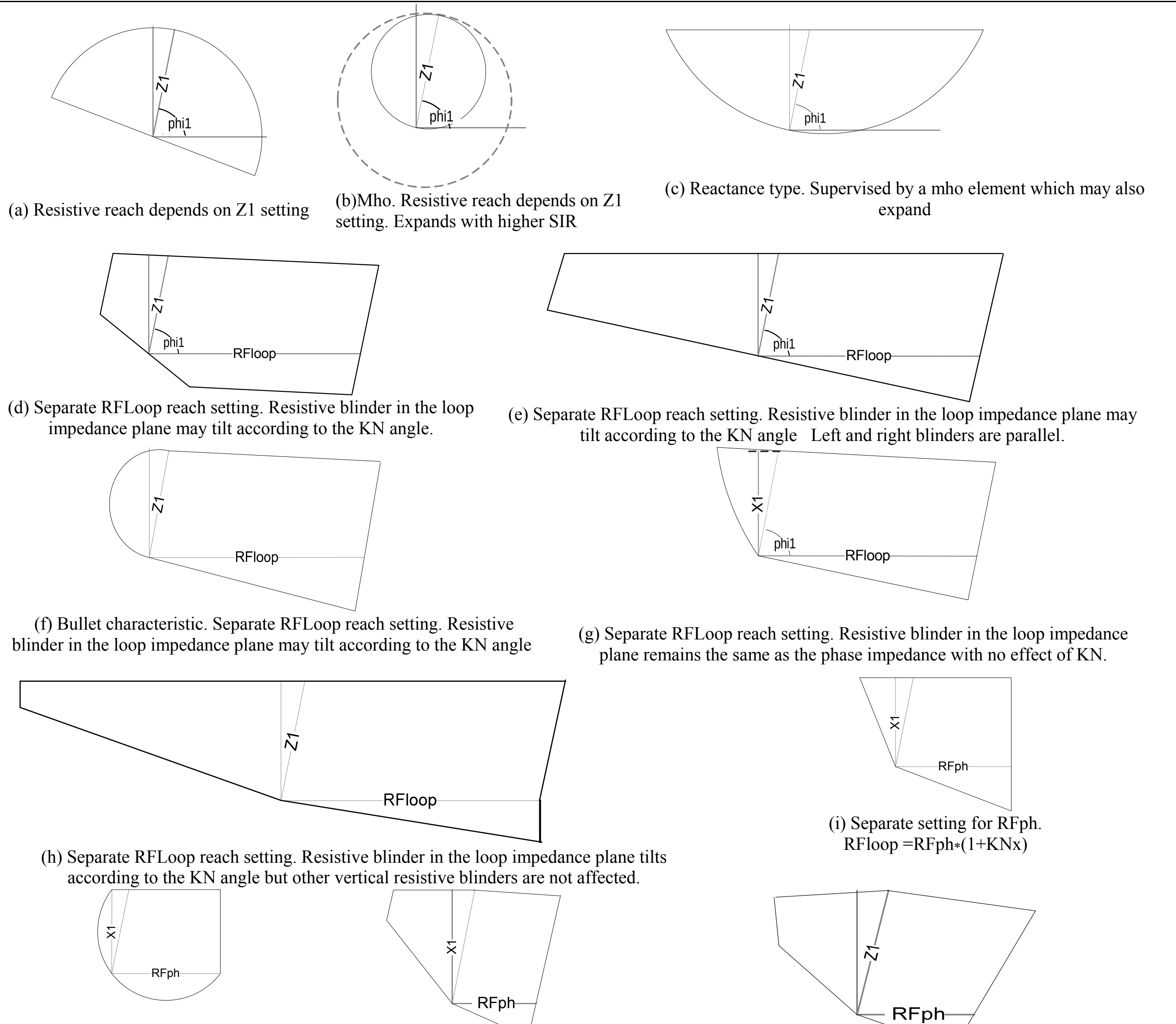
Figure 8 shows a survey of some of the characteristic shapes used in ground distance relay including the influence of the residual compensation on the resistive reach. Other shapes that are not shown exist but are not included here. The first shape (a) either use an impedance relay that is supervised by a directional element; shape (b) is a mho circular characteristic and shape (c) uses a reactance element supervised by a mho element. Two general terms used for shapes (d) through (j) are *quadrilateral* (four sides) and *polygon* (multiple angles). In this paper we shall refer to these shapes as polygons even though some have circular arcs.

For the purpose of general comparison the shapes are drawn to have the same reactive reach (Z1 or X1) at the impedance set point along the protected line angle; and for shapes that have a specific resistive reach setting, they are set to measure the *same loop resistance* along the R-axis.

The first three do not have a specific resistive reach setting. Shape (a) has less resistive coverage for shorter lines. Shape (b) mho characteristic has less resistive coverage especially for self-polarized elements; but for a cross-polarized mho element, it expands with increasing source impedance ratio (SIR), thereby increasing the resistive coverage. Shape (c) uses a reactance element that is supervised by a higher zone mho element and has more resistive coverage especially if the mho characteristic expands with higher SIR.

The next four shapes (d) through (h) have a separate and specific arc and fault resistance reach setting, which we shall call *RFLoop* in this paper. The term *ohms per loop* is usually used in some relays to indicate this fact. The resistive setting is the desired reach to cover the fault arc resistance *Rarc* and the tower footing resistance *Rtf*. (See Figure 6). An extra margin of 10% to 20% is usually added to cover more fault resistance. Thus,

RFloop = (1.1 to 1.2) • (Rarc + Rtf)



(a) Resistive reach depends on Z1 setting

(b)Mho. Resistive reach depends on Z1 setting. Expands with higher SIR

(c) Reactance type. Supervised by a mho element which may also expand

(d) Separate RFLoop reach setting. Resistive blinder in the loop impedance plane may tilt according to the KN angle.

(e) Separate RFLoop reach setting. Resistive blinder in the loop impedance plane may tilt according to the KN angle Left and right blinders are parallel.

(f) Bullet characteristic. Separate RFLoop reach setting. Resistive blinder in the loop impedance plane may tilt according to the KN angle

(g) Separate RFLoop reach setting. Resistive blinder in the loop impedance plane remains the same as the phase impedance with no effect of KN.

(h) Separate RFLoop reach setting. Resistive blinder in the loop impedance plane tilts according to the KN angle but other vertical resistive blinders are not affected.

(i) Separate setting for RFph.
RFloop =RFph∗(1+KNx)

(j) Separate setting for RFph.
RFloop =RFph∗(1+KNx)

(k) Separate setting for RFph.
RFloop =RFph∗(1+RE/RL). Left and right blinders are parallel.

(l) Separate setting for RFph.
RFloop ≈ RFph∗(1+K0/3). Resistive blinders can have different inclination angles

Note: Other compensation forms like K0ratio, (Z1 and ZN), and (R1, X1, R0, X0) are used but are not shown. These are usually equivalent to KN

Figure 8. Samples of characteristic shapes and the relation of compensation and resistive reach settings

The above RFLoop setting formula is valid for radial lines or when the infeed from the remote end to the fault is negligible; but this is not the case in most transmission line applications. For non-radial lines, the setting should be further increased by an additional ratio factor to handle remote infeed as shown in Figure 9.

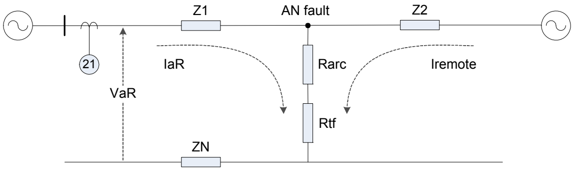


Figure 9. Resistance measurement with fault resistance and remote infeed current

Thus, the setting is usually calculated as

RFloop = (1.1 to 1.2) • (1 + Iremote/IaR) • (Rarc + Rtf)

The next four characteristics have a per-phase resistive reach setting *RFph* (in this paper) since the reach setting is calculated and referred to the positive-sequence part of the fault loop. In some relays the term, *ohms per phase*, is used to indicate this. The actual resistive reach is affected by the residual compensation factor. In calculating the resistive setting, the desired loop resistance coverage is divided by (1 plus the compensation factor). Most but not all relays of these types usually have a scalar compensation factor.

For shapes (i) and (j), which use residual compensation KNx =(X0/X1-1)/3, the resistive setting is

RFph = (1.1 to 1.2) • (1 + Iremote/IaR) • (Rarc + Rtf)/(1+KNx)

For shape (k), which uses ratio compensation RE/RL, the resistive setting is

RFph = (1.1 to 1.2) • (1 + Iremote/IaR) • (Rarc + Rtf)/(1+RE/RL).

For shape (l), which uses residual compensation K0 = (Z0/Z1-1), the resistive setting is

RFph ≈ (1.1 to 1.2) • (1 + Iremote/IaR) • (Rarc + Rtf)/(1+ K0/3)

Note that for faults on the line and assuming zero fault resistance the point will fall on the ZLoop line. Most numerical relays adjust the inclination of the resistive blinder to be the same as that of the angle of ZLoop, in order to maintain a constant resistive reach over the entire impedance reach setting.

There is a practical limit in setting the resistive reach of the polygon characteristic since a very long reach will affect security of the measurement and may encroach into the load area.

One thing that needs to be pointed out is that for some shapes the effective resistive reach may be less than the setting for faults away from the relay location. For shapes like (i) and (j) which have *vertical* resistive blinders, and shape (g) where the angle of the resistive blinder remains equal to that of the positive sequence impedance angle setting, it can be seen in Figures 10 and 11 that the resistive reach decreases towards the end of the line; most modern shapes have a constant resistive reach coverage.
The resistive reach may need to be increased further if needed. However, the maximum resistive reach that can be set on the relay is limited or affected by other factors – relay maximum resistive setting, maximum load, use of load encroachment feature, relay current sensitivity, and the tilting effect of remote infeed current. These aspects are not covered in this paper.

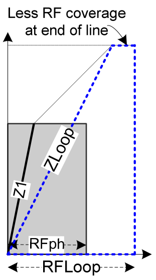


Figure 10

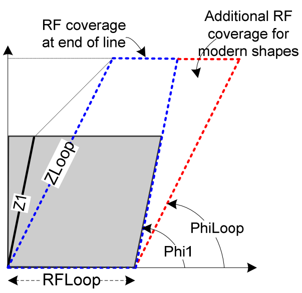


Figure 11

Modeling & Testing

In order to properly test the relay one has to know the expected relay characteristic that conforms to the settings; we call this the reference characteristic. In this paper we shall focus our testing discussion mainly on the first quadrant of the R-X diagram for the following reasons:

1. A huge majority of faults fall in the first quadrant.
2. The settings and characteristics in the first quadrant are more well-defined and easier to model and test.
3. The directional lines affect the fourth quadrant as well as the second quadrant in most of the relays. For test methods that consider the source impedance the directional line moves down into the third quadrant. The characteristics and behavior of the directional lines varies to a high degree and can be the subject of another paper. Most test methods do not handle the directional lines very well.

Although the focus as regards testing is on the first quadrant, some issues need to be considered when modeling the reference characteristics:

4. The form of residual compensation.
5. Relay impedance or reactance reach settings are in ohms per phase, i.e., positive-sequence values, but the resistive reach could be set either in ohms per loop or ohms per phase. One needs to be careful, as it may be difficult to find this information in some relay manuals.
6. The tilt or droop of the reactance characteristic affects the accuracy of some testing methods.
7. The inclination angles of the resistive blinders which may be fixed or set.
8. Whether or not the resistive blinder angle inclination is affected by the residual compensation angle.
9. Based on the above information, should we model and test the relay using a per-phase impedance plane or a loop impedance plane?

Before discussing the polygon characteristics, let us first consider the simpler mho characteristic shown in Figure 12 having a reach setting Z1mag ohms/phase, characteristic angle Z1ang, and a residual compensation factor **KN**.

The reference characteristic is usually modeled directly in the per-phase plane as shown by the smaller mho circle. The characteristic looks exactly as set. From equation (1) the impedance measured is **Zrelay** = **VaR**/(**Ia**+**KN·In**). But since In=Ia for a single line to ground fault and replacing Zrelay with ZFault, we can write the voltage equation as **VaR = ZFault (Ia + KN·Ia)** = **Ia · ZFault** (1+**KN**). The ratio of the relay voltage to the current is then **VaR/Ia** = **ZFault** (1+**KN**). Any point on the diagram (Z1, P2, P3, P4) is a **ZFault** point on the per phase plane. Multiplying **ZFault** by (1+**KN**) gives the equivalent points (Z1L, P2L, P3L and P4L) in the per loop plane.

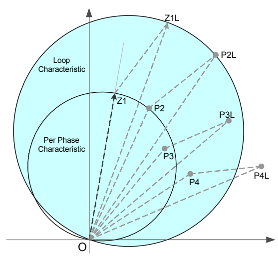


Figure 12. Mho per-phase and loop characteristics

Per-phase model

Constant test current method

VaR = **Ia** · **ZFault** · (1+**KN**)

Constant test voltage method

Ia = **VaR** / (**ZFault** · (1+**KN**))

Loop model

Constant test current method

VaR = Ia · ZFltLoop

Constant test voltage method

Ia = VaR / ZFltLoop.

So given any point **ZFault** and the test current **Ia**, the test voltage **VaR** can be calculated. This is called constant test current method.

VaR = **Ia** · **ZFault** · (1+**KN**)

If the test voltage is given the test current can be calculated, this is called constant test voltage method

Ia = **VaR** / (**ZFault** · (1+**KN**))

When the relay is modeled per phase the **KN** factor needs to be considered in the calculation.

The relay can also be modeled in per loop plane as shown by the larger circle. The loop model has a diameter O to Z1L at the ZLoopAngle. Since the effect of KN is already included in the loop model, it should not be included again in the calculation of the test voltages and currents. Each of the test points shown is tested using the equations **VaR = Ia · ZFltLoop** and **Ia = VaR / ZFltLoop**.

The polygon characteristic shown in Figure 13 has setting X1 and RFph, both in ohms/phase as shown in the smaller figure below. The residual compensation factor is a scalar KNx = (X0/X1-1)/3. Because the compensation factor is scalar (KN angle=0) it is even simpler than the mho characteristic. To get the loop impedance for testing, one only needs to multiply the per-phase fault point by (1+KNx)

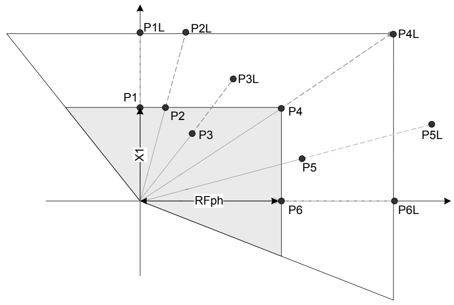


Figure 13. Polygon characteristic with residual compensation KNx = (X0/X1 - 1) / 3

We shall now take the case where the resistance reach setting is in ohms/loop while the reactance line setting is in ohms/phase. Figure 14 on the right shows the how a typical ground distance zone characteristic would be depicted in the relay manual. Z1mag/Phi1 is the per-phase reach setting in ohms/phase and RFLoop is the resistance setting in ohms/loop. The residual compensation factor KNmag/KNang is another setting. In this relay the resistive reach is constant. Only the ground-return impedance is compensated.

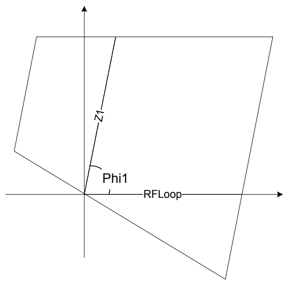


Figure 14. Polygon characteristic with RFLoop resistance setting and compensation factor KN

Figure 15 on the right shows a simplified model (for a phase A-to-ground fault) of a ground distance element having a per-phase reach setting **Z1** and residual compensation **KN** and a resistance setting RFLoop. The distance to the fault along the setting angle is represented by n. Note that the resistance RFLoop is not affected by the location of the fault.

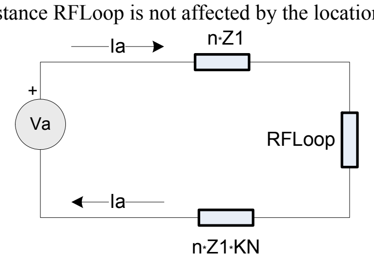


Figure 15. Equivalent circuit diagram

If we consider the setting reach point Z1 in Figure 16 and multiply it by (1+**KN**) we get the loop impedance **ZLoop**, from the origin to Z1L. Now we can add the loop resistance RFLoop to **ZLoop** and we get the upper right-hand corner of the loop characteristic, Q. Drawing horizontal lines of length RFLoop starting from the ZLoop impedance line shows this constant resistive reach also in the loop impedance plane. Note that the resistive blinder is parallel to the ZLoop line, i.e., the angle of the resistive blinder in the loop characteristic is PhiLoop.

This constant resistive reach in both per-phase and loop impedance planes allows us to treat the fault resistance separately from ZLoop.

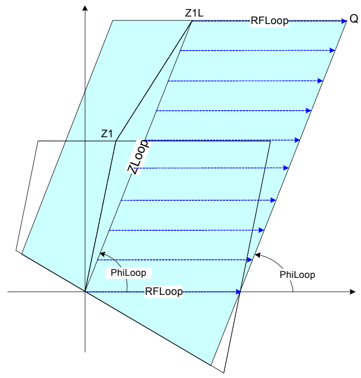


Figure 16. Characteristic loop impedance with constant resistive reach

Consider point P2 in Figure 17 on the resistive blinder of the per-phase characteristic. What would be the equivalent point on the loop characteristic? We can draw a horizontal line from P2 until it interests the per-phase Z1 line at point P'2. The length of this line is RF2. Now multiply the value of point P'2 by (1+**KN**) and we get P'2L, which lies on the ZLoop line. This is equivalent to drawing a line parallel to **ZN** (=**KN·Z1**) until it intersects the ZLoop line. If we now add the resistance RF2 to P'2L we get the point P2L which should lie on the loop characteristic resistive blinder. This is the loop impedance that we use for testing. We can do the same process for the other points P3 though P5 to get the loop impedances P3L through P5L. To test each point we substitute these loop impedances into the formula **VaR = Ia · ZLoop** in order to calculate either the voltage or current.

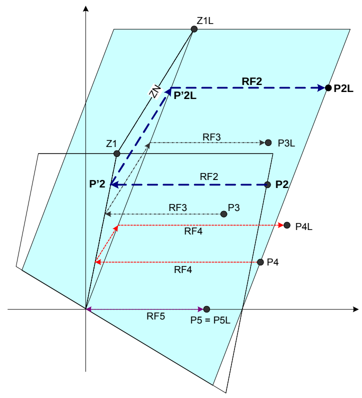


Figure 17. Determining total fault loop impedance when modeling separate fault resistance.

When testing such characteristic in the per-phase plane, a test method that treats the fault resistance separately has been implemented. In Figure 18 on the right ticking the checkbox +R direction will treat the fault resistance of all test points to the right of the characteristic angle as a separate resistance, and will not be affected by the KN factor.

The –R direction checkbox, when not checked, allows simultaneous testing a circular part of a bullet characteristic as well as the separate arc resistance.

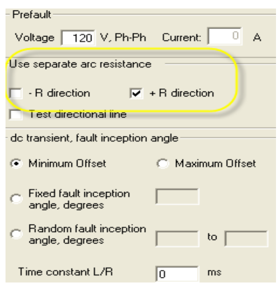
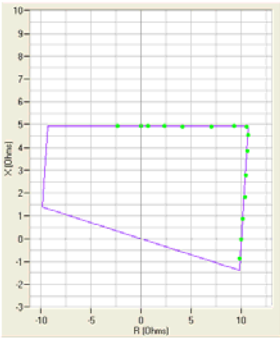
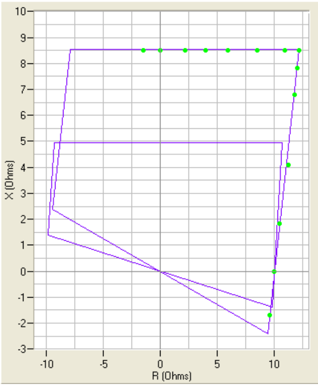


Figure 18. Testing with separate fault resistance

Figure 19 shows results of modeling and testing such a relay in both the per-phase plane as well as the loop impedance plane. The calculations to get the loop impedance models reference characteristic can be found in Appendix B.



Per-phase test



Loop testing result with per-phase reference characteristic superimposed

Figure 19. Modeling and testing a polygon characteristic using separate fault resistance in the per-phase impedance plane, compared with loop modeling and testing.

For characteristics having drooped or tilted reactance lines and with complex number compensation factors, testing modeling, testing with this method results in error for test points on the reactance line, especially for large angle tilt settings, as shown in Figure 20. Testing for a point P outside the reactance line will result in a point PL that is well inside the characteristic in the loop impedance plane. This results in the relay operating for a fault outside of the characteristic.
For a small tilt angle less than 2º, the error is hardly noticeable. For tilt angles larger than ± 3º it is recommended to model and test the characteristic in the loop impedance plane.
This is also one of the problems when testing the directional lines.

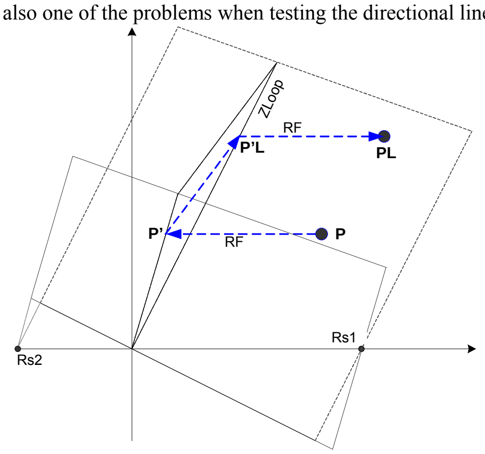


Figure 20. Error when using separate fault resistance with reactance lines having large tilt angles.

To model the characteristic of this type of distance element on the loop impedance plane, one has to draw the loop reactance line parallel to the original per-phase reactance line and passing through the ZLoop reach point. The blinders are drawn parallel to the ZLoop line and passing through the RLoop resistive setting points, Rs1 and Rs2 in this diagram. The angle of the directional line depends on the relay design. In some relays the angle remains the same as the per-phase characteristic, while in others it may change depending on the KN angle.

If the angle of the residual compensation KN is equal to zero or very near to zero, it is also possible to test this characteristic in the per-phase plane without treating the arc resistance as separate. The new per-phase resistance is then RFph = |ZFLoop| / (1+KNmag). This is equivalent to modeling the relay using as scalar compensation KNx. The loop impedance used for calculation of voltages and currents would then use **VaR**/**IaR** = **Zp**•(1+**KN**). This method of modeling and testing should not be used if the angle of KN is more than 2º away from zero.

Some relays have fixed vertical blinders in the 2nd and 4th quadrants; i.e., they are not affected by the KN angle in the same way are the resistive blinder angle in the first quadrant is when calculating the loop characteristic. The left-hand resistive blinder may behave similarly but is not apparent when modeled in per phase but appears in the loop model as shown in Figure 21. This kind of characteristic, if tested only for the first quadrant, can be tested per phase with a separate arc resistance. However, if the entire characteristic needs to be tested, the only way to test it is to model the complete loop characteristic.

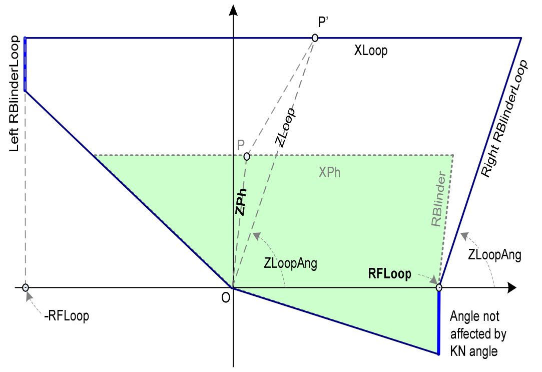


Figure 21. Modeling polygon loop characteristics with vertical blinders in the 2nd and 4th quadrants

Yet some relay characteristics have left-side blinders that are never allowed to enter into the first quadrant as shown in the Figure 22; the reactance axis becomes the limit.

The first quadrant can be tested using a per-phase model; but if the left side needs to be tested, a loop impedance model should be used for the entire characteristic.

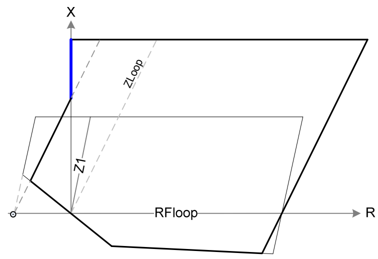


Figure 22. Modeling polygon loop characteristics where the left blinder not allowed to enter into the 1st quadrant.

For other ground distance elements that have complex number residual compensation factors and at the same time with characteristics shapes that have the resistance blinder angle different from the ZLoop angle it is difficult to test them in the per-phase plane. One would have to model these elements in the loop impedance plane. Figure 23 shows an example where the resistive blinder remains the same as the positive-sequence impedance or per-phase angle, phi1.

There are also some relays that allow the user to set the resistive blinder angle values that those mentioned above.

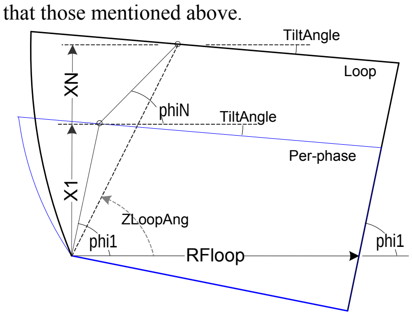


Figure 23. Loop modeling of characteristic with resistive blinder that remains parallel to the phase impedance.

For relays that use compensation setting factors RE/RL and XE/XL the loop impedance characteristic is modeled simply by multiplying the resistive and reactive components separately. Thus,

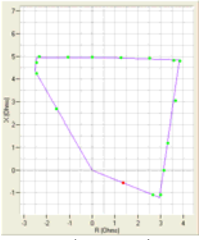
The resistive part is: PRLoop = PRPh • (1 + RE/RL)

The reactive part is: PXLoop = PXPh • (1 + XE/XL)

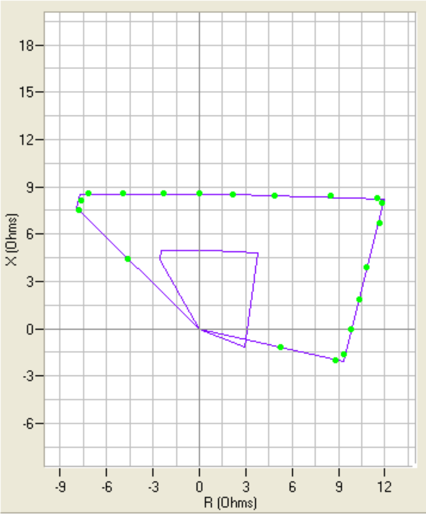
For most relays that use these constants, these equations apply to the entire characteristic, i.e. to all quadrants. The equations also apply to directional lines as well as tilted reactance lines. The problems described above in relation to Figures 20 through 23 do not usually exist here.

The left-hand image in Figure 24 below shows the result of testing on the per-phase plane using compensation factors RE/RL and XE/XL. The figure on the right shows the loop characteristics and the result of testing using loop impedance, with the per-phase model (smaller one) superimposed. The loop calculations for this characteristic can be found in Appendix B.

If the test software supports this form of residual compensation it is better and straightforward to simply model and test in the per-phase impedance plane and avoid unnecessary extra work of converting to loop impedance.



Per-phase testing



Loop testing with per-phase characteristic superimposed

Figure 24. Modeling and testing characteristic that use compensation factors RE/RL and XE/XL

A protection test system software with graphical interface and automated calculation of characteristics has been implemented that makes it easy for test personnel to model the relay by simply entering the settings. The equivalent reference characteristics are automatically modeled and plotted.

In Figure 25 the following are entered: the type of characteristic, the characteristic angle, residual compensation type and values. Several modes or forms of residual compensation can be selected depending on what the relay is using. If it is not available one has to convert to a form that is available.

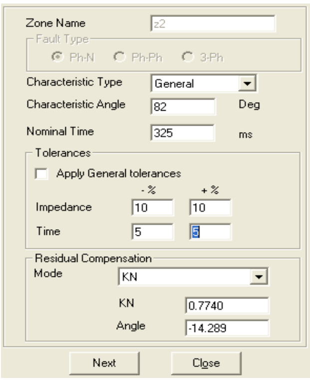


Figure 25. Selecting the residual compensation form and entering relay parameters

Figure 26 shows an example of a graphical user interface where one can enter and define straight lines and circular arcs that comprise the characteristic.

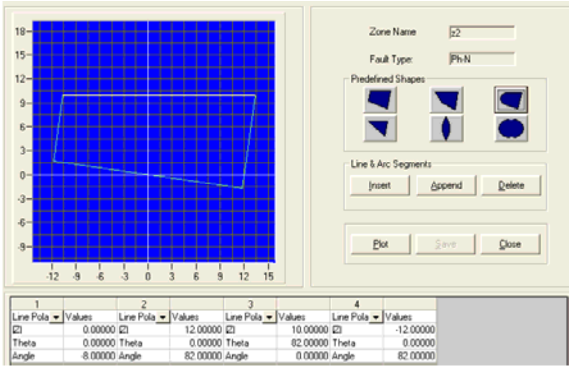
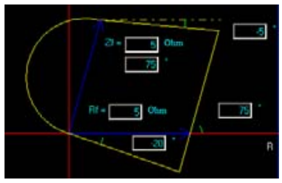


Figure 26. Graphical and tabular interface for modeling and editing distance relay characteristics

Selecting a pre-defined shape allows direct entry of the settings, which then automatically adds the lines and arcs to the table. Examples of such interfaces are shown in Figure 27.



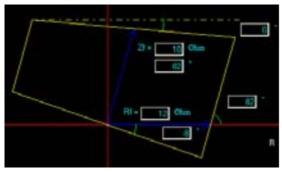


Figure 27. Pre-defined shapes for entering relay settings to automatically model relay characteristics

Even with these modeling tools one has still to decide in some cases whether to model in per-phase or loop. If the characteristic and compensation form allows correct testing in the per-phase impedance plane, it is preferred to model it that way. Modeling in loop plane always works but requires some auxiliary calculations to get the new reach points and angles. When in doubt, model and test using both loop as well as per phase impedance planes to see the effects of the compensation on the resistive blinder as well as the resistive reach. Appendix B includes worked examples on modeling polygon characteristics in the loop impedance plane.

Advanced test systems include a relay library where one simply enters the relay configuration and settings, or import setting and modeling files exported from relay vendors software, to automatically create the impedance functions for all zones of the distance relay. This eliminates the necessity to do tedious and error prone manual calculations, especially when it needs to be modeled in the loop plane. Figure 28 shows such characteristics that have been automatically created.

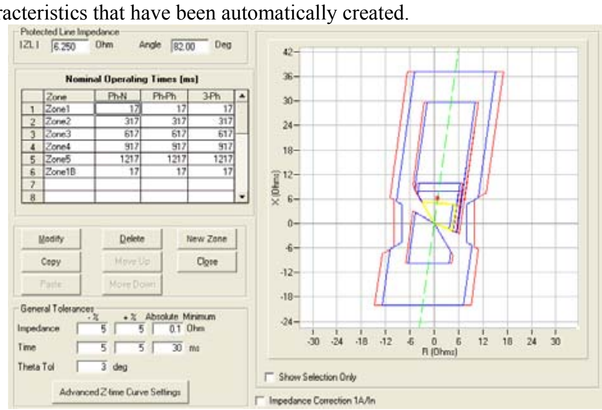


Figure 28. Modeling distance relay zones

If one wants to test the polygon elements together with load encroachment elements that have circular arcs, modeling in the loop impedance plane will avoid the load circular arc part from looking like an elliptical arc.

Figure 29 shows the result of simultaneous testing of all ground distance zones together with load encroachment. The directional lines and reverse zones are also tested.

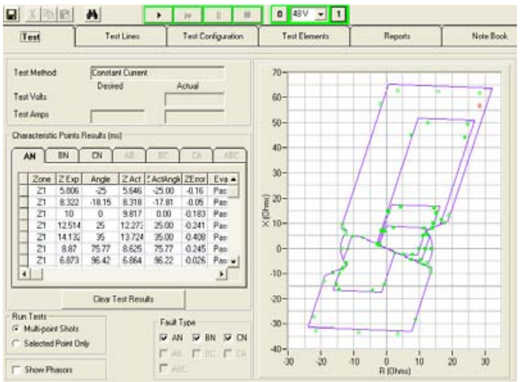


Figure 29. Simultaneous testing multiple ground distance zones having load encroachment elements

Conclusions

1. Ground distance relays employ some form of compensation of the ground-return impedance in order to allow the relay to be set and to measure in terms of positive-sequence impedance. The paper presents these forms of compensation including some derivation.

2. The many names, symbols and formulas that are in use for residual or ground-return compensation pose a challenge to personnel who set and test the relays. Moreover, some forms of compensation that use different formulas are called by the same name and symbol. This can result in applying the wrong setting if one is not careful and may result in either relay misoperation or failure to trip.

3. The fault resistance reach, for polygon-shaped characteristics, is set in *ohms per phase* in some relays while in other relays it is set in *ohms per loop*. In some relay manuals this fact is not explicitly indicated and relaying personnel will need to find this out.

4. The ground-return compensation affects the fault resistance reach and the angle of the resistive blinder in different ways, depending on the design of the relay. In some case the line remains vertical and in other cases it remains equal to the positive-sequence impedance angle.

5. Each form of ground-return impedance compensation can be converted to another form. Formulas are derived to perform this conversion. These formulas are handy when a relay being tested has a compensation setting that is not supported the relay test system. A spreadsheet that implements these formulas makes conversion easy and avoids calculation errors.

6. In addition to testing both reactance line and résistance blinder in the per-phase impedance plane, and both in the loop impedance plane, a third test method that treats the fault resistance separately from the main loop impedance is described for testing polygon-shaped characteristics. The third method models the reactance line in per-phase and the resistive reach in ohms per loop.

7. Selecting the most suitable model for testing depends on assessment of item 4 above as well as the tilt angle of the reactance line. Testing points for a reactance line that has a large tilt angle, using a separate fault resistance, will result in test errors.

8. Personnel who set relays and those who test them must have a good understanding of the methods of residual compensation, how the resistive reach is set and affected by the compensation and how the relay characteristics are modeled. Cooperation between these personnel is very important to actually verify their understanding of the settings and relay behavior and that the models are suitable.

9. The operation in the second and fourth quadrants of polygon characteristics is affected by additional factors – including the behavior of the directional lines, the type of characteristic lines (straight lines of circular arcs), and the source impedance.

10. Some automated test software support modeling the various forms of residual compensation setting and the resistive blinder so that the reactance line can be modeled in per-phase. If the test software does not support this the best way to test the relay is to model the entire polygon characteristic in the loop impedance plane and include the effect of residual compensation.

11. Advanced test systems include a library of relay models where one enters the relay settings or imports them from files exported from relay software. The test software automatically creates the characteristic models for all the zones and fault loops.

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9. F6TesT Visual Relay Testing software, Doble Engineering Company.

Biographical Sketch

Quintin Verzosa, Jr. "Jun", received his BSEE degree from Mapua Institute of Technology (Manila, Philippines) in 1976. He joined National Power Corporation (Philippines) in 1978 as a Relay Engineer and was later promoted to Principal Engineer and then Manager of Power System Analysis & Protection. He became Manager of Protection, Control & Communications Engineering Design. In 1993 Jun joined GEC ALSTHOM where he worked on protection and control applications and testing, and later became Protection Systems Engineering Design Manager. Jun joined Doble Engineering Company in 1998 as a Senior Protection Engineer and is currently Manager of Protection & Automation Engineering and Services. He is involved in protection test systems marketing, training and support, research and development of protection models and testing algorithms, and the application of new technology. He is a member of IEEE and is actively involved in the Power Systems Relaying Committee (PSRC) working groups.

Appendix A. Derivation of ground-return compensation factors

Residual Compensation Factor, KN

Consider a system shown in Figure a1, with a ground distance relay at bus R protecting a transmission line, where a phase A-ground fault occurs at point F at a distance n from the relaying point. The distance n is in per unit of the relay setting **Z1** and **Z0**.

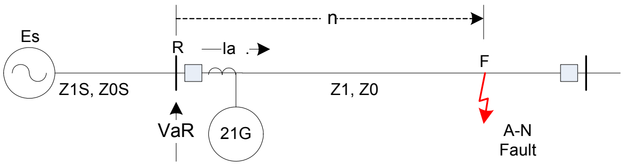
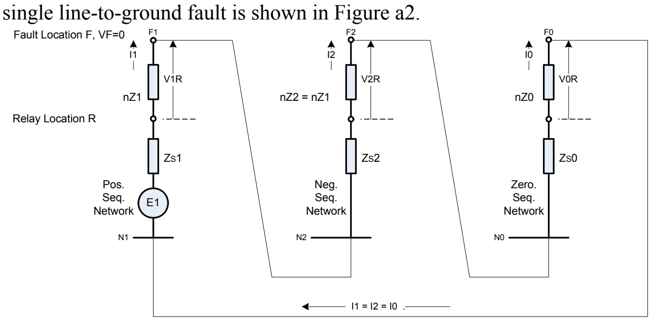


Figure a1

The symmetrical component sequence diagram for the system above for a single line-to-ground fault is shown in Figure a2.



Symmetrical Component Network for SLG fault at F

Figure a2. Symmetrical component sequence diagram

The voltage at the fault point F is zero, assuming that the fault is bolted, and the sequence voltages are:

$$\mathbf{V1R} = \mathbf{I1} \bullet n \bullet \mathbf{Z1} \quad \mathbf{V2R} = \mathbf{I2} \bullet n \bullet \mathbf{Z1} \quad \mathbf{V0R} = \mathbf{I0} \bullet n \bullet \mathbf{Z0} \qquad \text{(a1)}$$

The phase A to neutral voltage **VaR** at the relay point is then the sum of the sequence voltages

$$\begin{aligned} \mathbf{VaR} &= \mathbf{V1R} + \mathbf{V2R} + \mathbf{V0R} \\ &= \mathbf{I1} \bullet n \bullet \mathbf{Z1} + \mathbf{I2} \bullet n \bullet \mathbf{Z1} + \mathbf{I0} \bullet n \bullet \mathbf{Z0} \qquad \text{(a2)} \end{aligned}$$

The phase A current **Ia** at the relaying point is then

$$\mathbf{Ia} = \mathbf{I1} + \mathbf{I2} + \mathbf{I0} \qquad \text{(a3)}$$

and, since **I1** = **I2** = **I0**, the residual (neutral) current is

$$\begin{aligned} \mathbf{In} &= \mathbf{Ia} + \mathbf{Ib} + \mathbf{Ic} = 3\mathbf{I0} \qquad \text{(a4)} \\ \mathbf{I0} &= \mathbf{In} / 3 = \mathbf{Ia} / 3 \end{aligned}$$

Adding and subtracting **I0•n•Z1** in equation (a2), factoring out **n•Z1** and **I0**, and substituting equations (a3) and (a4)

$$\begin{aligned} \mathbf{VaR} &= \mathbf{I1} \bullet n \bullet \mathbf{Z1} + \mathbf{I2} \bullet n \bullet \mathbf{Z1} + \mathbf{I0} \bullet n \bullet \mathbf{Z1} - \mathbf{I0} \bullet n \bullet \mathbf{Z1} + \mathbf{I0} \bullet n \bullet \mathbf{Z0} \\ &= (\mathbf{I1} + \mathbf{I2} + \mathbf{I0}) \bullet n \bullet \mathbf{Z1} - \mathbf{I0} \bullet n \bullet \mathbf{Z1} + \mathbf{I0} \bullet n \bullet \mathbf{Z0} \\ &= \mathbf{Ia} \bullet n \bullet \mathbf{Z1} + \mathbf{I0} \bullet (\mathbf{Z0} - \mathbf{Z1}) \bullet n \qquad \text{(a5)} \\ &= \mathbf{Ia} \bullet n \bullet \mathbf{Z1} + (\mathbf{Ia}/3) \bullet (\mathbf{Z0} - \mathbf{Z1}) \bullet n \qquad \text{(a6)} \end{aligned}$$

If we use the voltage **VaR** and the current **Ia** directly for measurement the apparent impedance that is measured is

$$\mathbf{ZRapparent} = \mathbf{VaR} / \mathbf{Ia} = n \bullet \mathbf{Z1} + (\mathbf{Z0} - \mathbf{Z1}) \bullet n/3$$

The extra second term makes the result not very usable. To make the relay easier to use, the objective in the design of most ground distance relays is to make the relay measure only the first term, **n•Z1**. If we substitute **In**/3 for **I0** in equation (a5) and multiply the second term by **Z1**/**Z1** and simplify the equation to express the impedances as a factor of **Z1**, we obtain

$$\begin{aligned} \mathbf{VaR} &= \mathbf{Ia} \bullet n \bullet \mathbf{Z1} + (\mathbf{In}/3) \bullet (\mathbf{Z0} - \mathbf{Z1}) \bullet n \bullet \mathbf{Z1}/\mathbf{Z1} \\ &= [\mathbf{Ia} + \mathbf{In} \bullet (\mathbf{Z0} - \mathbf{Z1})/(3\mathbf{Z1})] \bullet n \bullet \mathbf{Z1} \qquad \text{(a7)} \end{aligned}$$

If we define a constant **KN**= (**Z0** – **Z1**)/(3**Z1**) which is valid for a given line configuration and ground resistivity, **VaR** simplifies to

$$\mathbf{VaR} = (\mathbf{Ia} + \mathbf{KN} \bullet \mathbf{In}) \bullet n \bullet \mathbf{Z1} \qquad \text{(a8)}$$

We can now use a modified current (**Ia** + **KN•In**), in which a portion **KN** of the residual current **In** is added to the phase current, to measure the impedance.

$$\mathbf{Zrelay} = \mathbf{VaR} / (\mathbf{Ia} + \mathbf{KN} \bullet \mathbf{In}) = n \bullet \mathbf{Z1} \qquad \text{(a9)}$$

This technique of measurement to express the fault impedance in terms of the positive-sequence impedance, by adding to the phase current a portion of the residual current, is known as *residual compensation*. The constant **KN** is called the *residual compensation factor*, which is defined as

$$\begin{aligned} \mathbf{KN} &= (\mathbf{Z0} - \mathbf{Z1}) / 3\mathbf{Z1} \\ &= (\mathbf{Z0}/\mathbf{Z1} - 1) / 3 \qquad \text{(a10)} \end{aligned}$$

Ground-return Impedance, ZN

Considering equation (a7) and expressing the voltage drops in terms of **Z1**,

$$\mathbf{VaR} = \mathbf{Ia} \bullet n \bullet \mathbf{Z1} + \mathbf{In} \bullet n \bullet (\mathbf{Z0} - \mathbf{Z1})/3 \qquad \text{(a11)}$$

This is the loop voltage from the relay terminal to the fault point and back, through a ground-return impedance **n•ZN**=**n•(Z0-Z1)**/3, to the neutral of the relay location. Hence, we can model the network as shown below

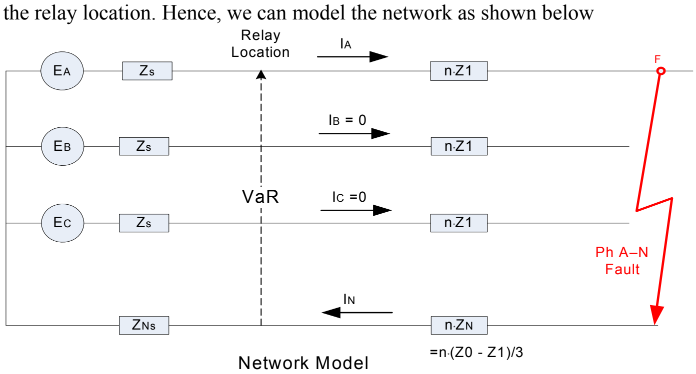


Figure a3. Simplified equivalent circuit

The impedance **ZN** is called the ground-return (or residual) impedance and is defined as

$$\mathbf{ZN} = (\mathbf{Z0} - \mathbf{Z1})/3 \qquad \text{(a12)}$$

Note also the relationship

$$\mathbf{ZN} = \mathbf{KN} \bullet \mathbf{Z1} \quad \text{or} \quad \mathbf{KN} = \mathbf{ZN} / \mathbf{Z1} \qquad \text{(a13)}$$

Zero-sequence current Compensation Factor, K0

In equation (a7), if we replace **In** by 3•**I0** we get

$$\begin{aligned} \mathbf{VaR} &= [\mathbf{Ia} + 3 \bullet \mathbf{I0} \bullet (\mathbf{Z0} - \mathbf{Z1})/(3\mathbf{Z1})] \bullet n \bullet \mathbf{Z1} \\ &= [\mathbf{Ia} + \mathbf{I0} \bullet (\mathbf{Z0} - \mathbf{Z1})/(\mathbf{Z1})] \bullet n \bullet \mathbf{Z1} \end{aligned}$$

We now define a constant **K0** = (**Z0** – **Z1**)/(**Z1**) and simplify the equation to

$$\mathbf{VaR} = (\mathbf{Ia} + \mathbf{K0} \bullet \mathbf{I0}) \bullet n \bullet \mathbf{Z1} \qquad \text{(a14)}$$

Using the relay voltage **VaR** and the modified current (**Ia** + **K0•I0**) yields an impedance measurement of

$$\mathbf{Zrelay} = \mathbf{VaR} / (\mathbf{Ia} + \mathbf{K0} \bullet \mathbf{I0}) = n \bullet \mathbf{Z1} \qquad \text{(a15)}$$

This technique of measurement to express the fault impedance in terms of the positive-sequence impedance, by adding to the phase current a portion of the zero-sequence current, is known as *zero-sequence compensation*. The constant **K0** is called the *zero-sequence current compensation factor*, which is defined as

$$\begin{aligned} \mathbf{K0} &= (\mathbf{Z0} - \mathbf{Z1}) / \mathbf{Z1} \\ &= \mathbf{Z0}/\mathbf{Z1} - 1 \qquad \text{(a16)} \end{aligned}$$

Residual Compensation Factors, RE/RL and XE/XL

Some relays compensate separately for the resistive part and for the reactive part. The loop impedance for a phase to ground fault consisting of **Z1** and **ZN** is shown in vector form in Figure a3.

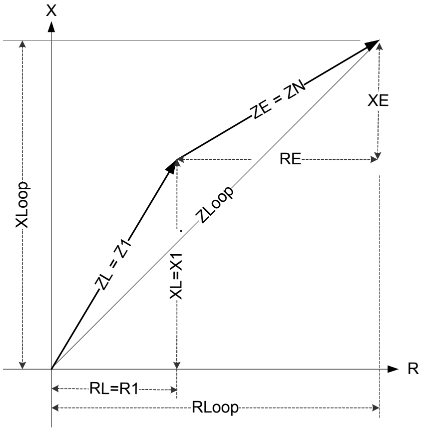


Figure a4. Loop Impedance vector diagram

To make it consistent with nomenclature of relays that use this type of compensation, the subscripts "L" and "E" are used in place of "1" and "N".
Hence, the loop impedance is

ZLoop = **ZL** + **ZE**

We can express ZLoop into its resistive and reactive components

ZLoop = RLoop + j XLoop

And we know that **ZL** = **Z1** = R1 + j X1 = RL + jXL

If we expand equation (a12) into its resistive and reactive components,

ZE = **ZN** = (**Z0** – **Z1**) / 3
= (R0 – R1)/3 + j (X0 – X1)/3
= RE + j XE

So RE = (R0 – R1)/3 and
XE = (X0 – X1)/3 (a17)

From Figure (a4) we see that the loop resistance is RLoop = RL + RE. If we express RLoop in terms of RL and introduce a constant RE/RL we get

RLoop = RL + RE
= RL (1 + RE/RL)

If we now take the constant RE/RL substitute their respective formulas from equation (a17)

RE/RL = [(R0 – R1)/3] / R1

$$\frac{RE}{RL} = \frac{1}{3}\left(\frac{R0}{R1} - 1\right) \qquad \text{(a18)}$$

Similarly, we can derive the reactive compensation factor XE/XL. Hence,

$$\frac{XE}{XL} = \frac{1}{3}\left(\frac{X0}{X1} - 1\right) \qquad \text{(a19)}$$

Note the general inequality:

$$\boldsymbol{KN} = \frac{\boldsymbol{ZN}}{\boldsymbol{Z}1} = \frac{\boldsymbol{ZE}}{\boldsymbol{ZL}} = \frac{RE + jXE}{RL + jXL} \neq \frac{RE}{RL} + j\frac{XE}{XL}$$

However, for any point along the loop impedance line the inequality does not exist.

Appendix B. Worked Examples in Modeling Some Polygon Characteristics in the Loop Impedance Plane

When the test software does not support testing, using the form of residual compensation and/or the resistive blinder is difficult to use for per-phase testing, it is necessary to model the ground distance polygon characteristic the loop impedance plane. This will simplify testing without bothering about these complexities in the calculation of test voltages and currents.

Example 1.

Given:
The relay setting and characteristic is shown below.
Impedance reach: Z1 = 5Ω @ 82°
Resistive reach: 10Ω, separate fault resistance, or ohms per loop
Reactance line tilt: -3º
Directional line angle: -8º
Residual compensation: KN = 0.774 @ -14.29º

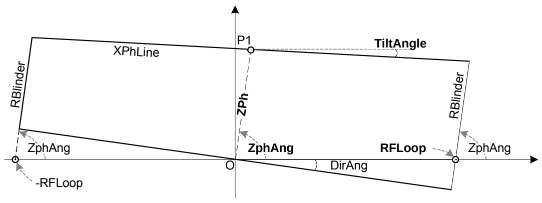


Figure b1. Per-phase impedance characteristic

Additional information:
The residual compensation factor formula is: KN = (Z0-Z1)/3Z1
The resistive reach blinder on the left-hand side the same as that of the right-hand blinder
The resistance blinder angle is influenced by the residual compensation
The reactance line tilt angle setting is that of the loop characteristic
The angle of the directional line is affected by the residual compensation in the same way as the resistive blinder.

Required:
Model the loop characteristic, defining the line segments with a point and angle. Draw the loop characteristic.

1. Calculate the loop impedance
   ZLoop = ZPh /ZphAng ∗ (1+KN)
   = 5 /82 ∗ (1 + 0.774 /-14.29)
   = 8.80Ω / 75.77º

2. The loop reactance line can be drawn. It passes through point P2 (which is the tip of the loop impedance) and has an angle –3º, which is the same as the original line.

3. The right-hand side resistive blinder passes through the point RFLoop (the resistive setting) and has an angle equal to the ZLoop impedance of ZloopAng = 75.77º.

4. The left-hand side resistive blinder passes through the point –RFLoop, also with an angle equal to ZloopAng = 75.77º.

5. The directional line is affected be the residual compensation by the same angle as that of the blinders. The new directional line angle is equal to the original set angle plus the difference of ZloopAng and ZPhAng.

   DirLoopAng = (-8) + (75.77-82) = -14.23º

6. The directional line is drawn passing through the origin and has an angle of –14.23º.

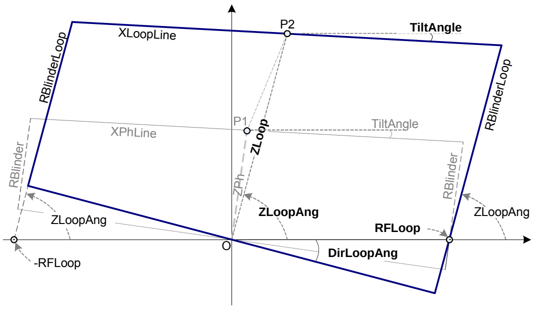


Figure b2. Loop impedance characteristic after applying KN compensation

Example 2.

Given:
The relay setting and characteristic is shown below.
Impedance reach: Z1 = 5Ω @ 82º
Resistive reach: 3.107Ω per phase
Reactance line tilt angle: -3º
Directional line angle (right): -22º
Directional line angle (left): 120º
Residual compensation factors: RE/RL = 2.109, XE/XL = 0.723

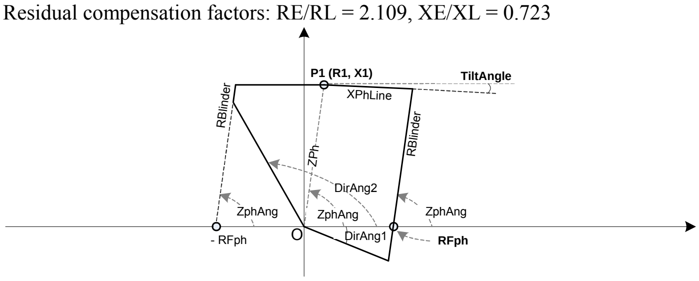


Figure b3. Per-phase impedance characteristic with compensation RE/RL & XE/XL

Additional information:
The reach reactance line is made up of a horizontal line to the left of the impedance reach point P1 and a tilted line to the right of the reach point. The resistive reach blinder on the left-hand side the same as that of the right-hand blinder.
It is also known that the relay directional lines are affected in the by the compensation factors in the same way as the reactance line and the blinders.

Required:
Model the loop characteristic, defining the line segments with a point and angle. Draw the loop characteristic.

1. Consider two per phase points – P1 (R1, X1) and P2 (R2, X2) on the R-X diagram.

The line connecting these two points has a slope of

$$Tan\,\theta = \frac{X2 - X1}{R2 - R1}$$

(b1)

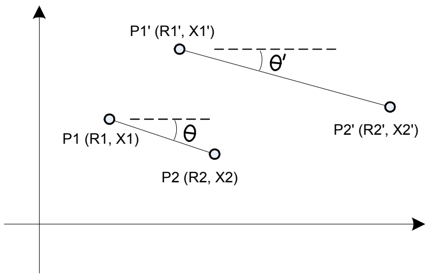


Figure b4

2. One can calculate the loop points P1' and P2' using the compensation constants
R1' = R1 (1+RE/RL)
X1' = X1 (1+XE/XL)
R2' = R2 (1+RE/RL)
X2' = X2 (1+XE/XL)

3. The new slope of the per-loop line that connect these two points is

$$Tan\,\theta' = \frac{X2' - X1'}{R2' - R1'} = \frac{X2(1+XE/XL) - X1(1+XE/XL)}{R2(1+RE/RL) - R1((1+RE/RL)} = \frac{(X2 - X1)}{(R2 - R1)} \cdot \frac{(1+XE/XL)}{(1+RE/RL)}$$

Substituting equation (b1),

$$Tan\,\theta' = Tan\,\theta \cdot \frac{1+XE/XL}{1+RE/RL}$$

The new line angle is

$$\theta' = ArcTan\left(Tan\,\theta \cdot \frac{1+XE/XL}{1+RE/RL}\right) \qquad (b2)$$

This formula will be used in calculating the new angles of all per loop line segments.

4. All reactance lines will pass through point P2, which is the ZLoop reach. The ZLoop reach components are:

RLoop = ZPh∗Cos (ZPhAng) ∗ (1+RE/RL)
= 5∗Cos(82) ∗ (1 + 2.109) = 2.163Ω

XLoop = ZPh∗Sin (ZPhAng) ∗ (1+XE/XL)
= 5∗Sin(82) ∗ (1 + 0.723) = 8.531Ω

ZLoopMag = Sqrt(RLoop^2 + XLoop^2) = 8.801Ω

ZLoopAng = ArcTan(XLoop/RLoop) = 75.77º

5. The loop line segment to the left of point P2 has an angle of 0 and the loop reactance line to the right of P2 has a new angle of

TiltAngleLoop = ArcTan{ Tan (-3) ∗ (1 + 0.723) / (1 + 2.109)}
= -1.664º

6. The resistive blinders have a reach of
RLoop=RPh∗ (1+RE/RL) = 3.107∗ (1 + 2.109) = 10Ω

7. The resistive blinders now have a new inclination angle of

RBlinderAngle = ArcTan { Tan (80) ∗ (1 + 0.723) / (1 + 2.109) }
= 75.77º
It can be proven that this angle is actually equal to ZLoopAng.

8. The loop directional line angles are

DirLoopAng1 = ArcTan { Tan (-22) ∗ (1 + 0.723)/ (1 + 2.109)}
= -12.621º
DirLoopAng2 = ArcTan { Tan (120) ∗ (1 + 0.723)/(1 + 2.109)}
= 136.172º

The loop impedance model is drawn below, based on these calculated points and angles.

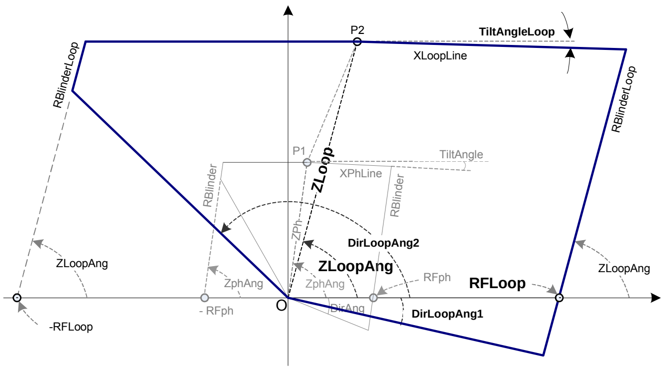


Figure b5. Loop impedance characteristic after compensating for RE/RL and XE/XL