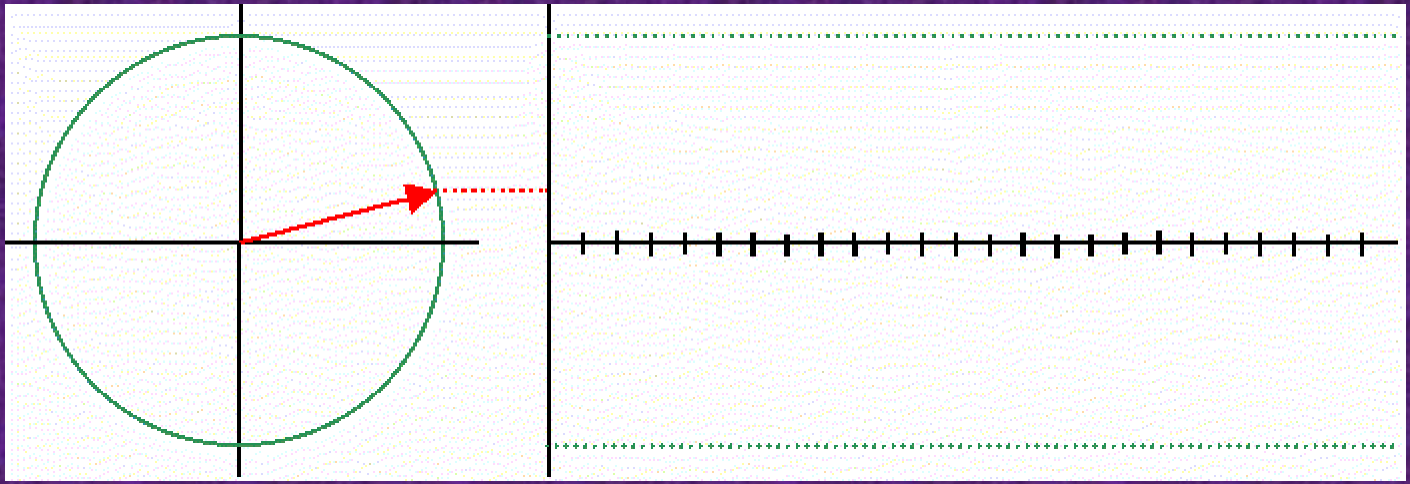


BASIC MATH FOR RELAY TECHNICIANS

BONNEVILLE POWER ADMINISTRATION

for

WSU HANDS-ON RELAY SCHOOL

INTRODUCTION - Math I

This lecture will cover aspects of DC vs AC, why AC is so different, what is a sine wave and how it relates to phasors. We will also demonstrate how to use calculations involving right triangles to determine power factor, and to plot phasors for protective relay work.

In Service Lab

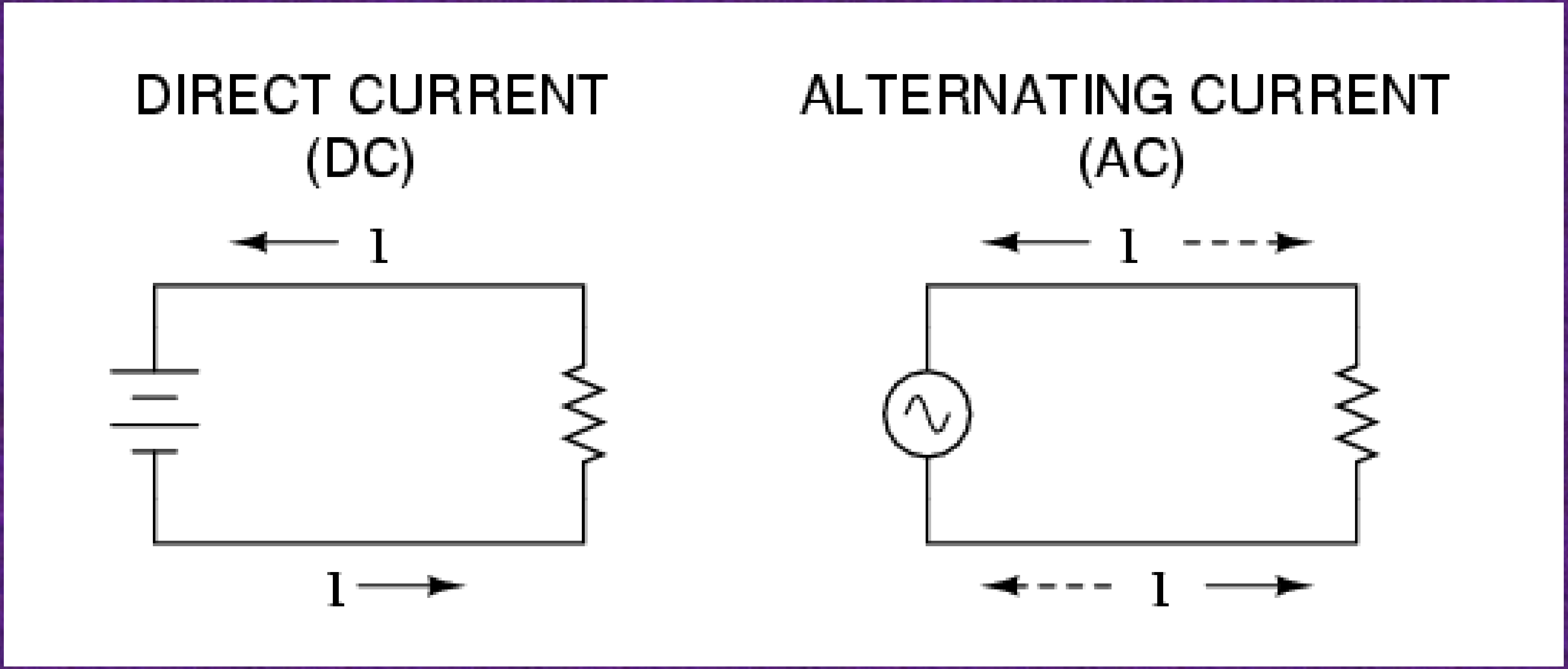
- Thursday afternoon there will be an opportunity to explore the application of phasors as used in determining whether sensing devices are wired correctly, and reporting accurate information to relays.
- The following information will help understand what is being done.

Mathematical Relationship of R, I, V, and P in DC

- **All you need to Remember is: V=IR (Ohms law), and P=VI (Joules law)…**
  The rest just simple algebraic manipulation and substitution
- **[A] V=IR $\Rightarrow$ [1] I= V/R and [2] R = V/I**
- **[B] P=VI $\Rightarrow$ [3] I= P/V and [4] V = P/I**
- **P using [B] and [A] $\Rightarrow$ P = (IR)I = I^2R**
- **P using [B] and [1] $\Rightarrow$ P = V(V/R) = V^2/R**

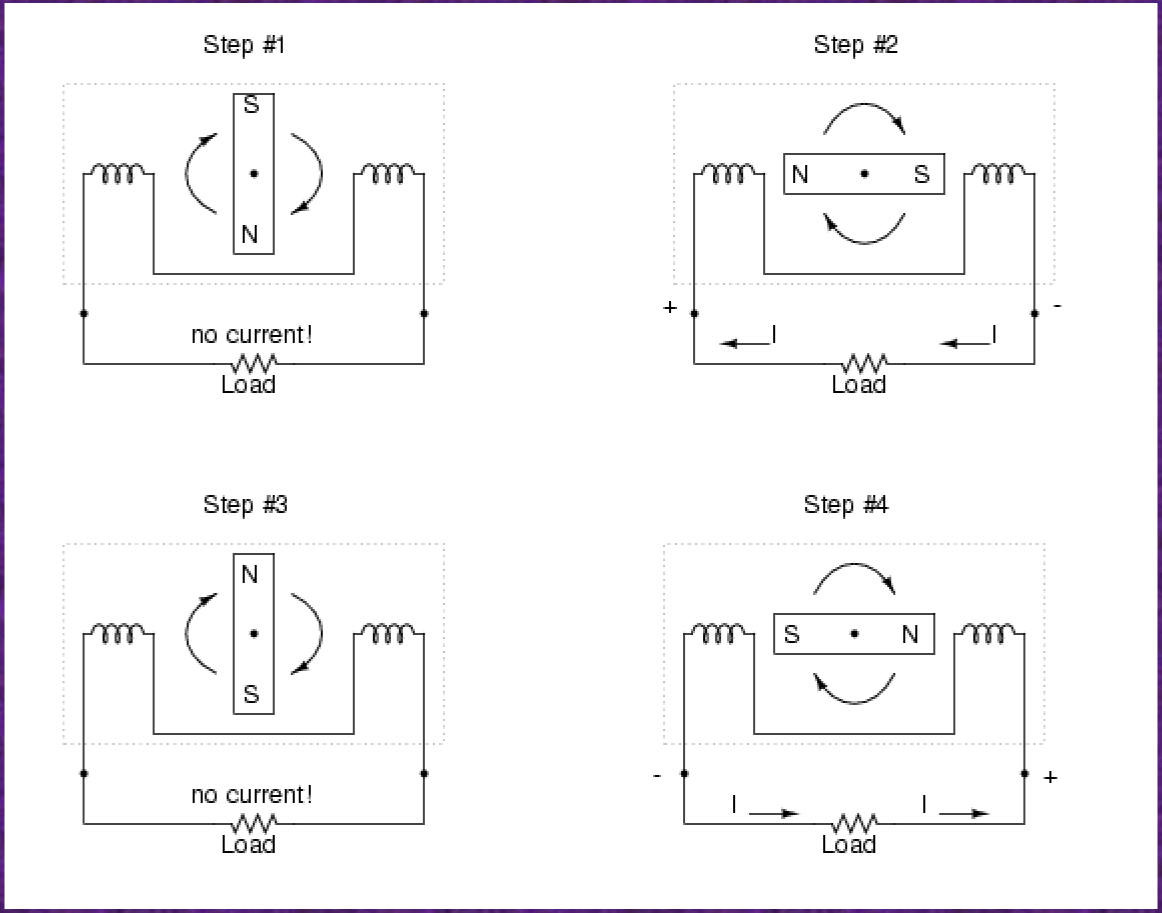
AC vs DC

Due to the nature of AC, not only do quantities such as voltage and current oscillate, but we have influences (inductance, capacitance) that can effect both a measured value and displacement in time.



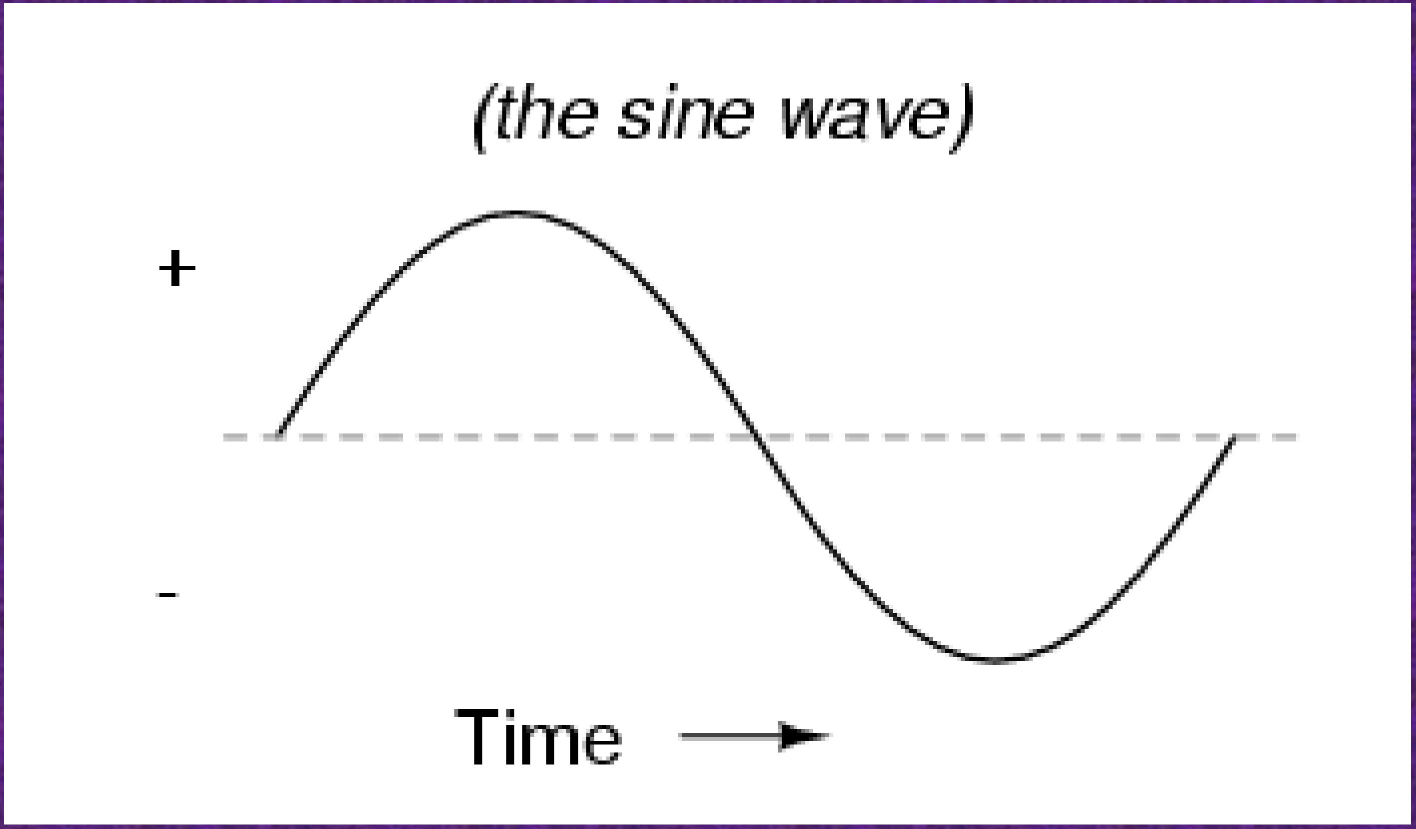
AC generation

as the magnet turns…

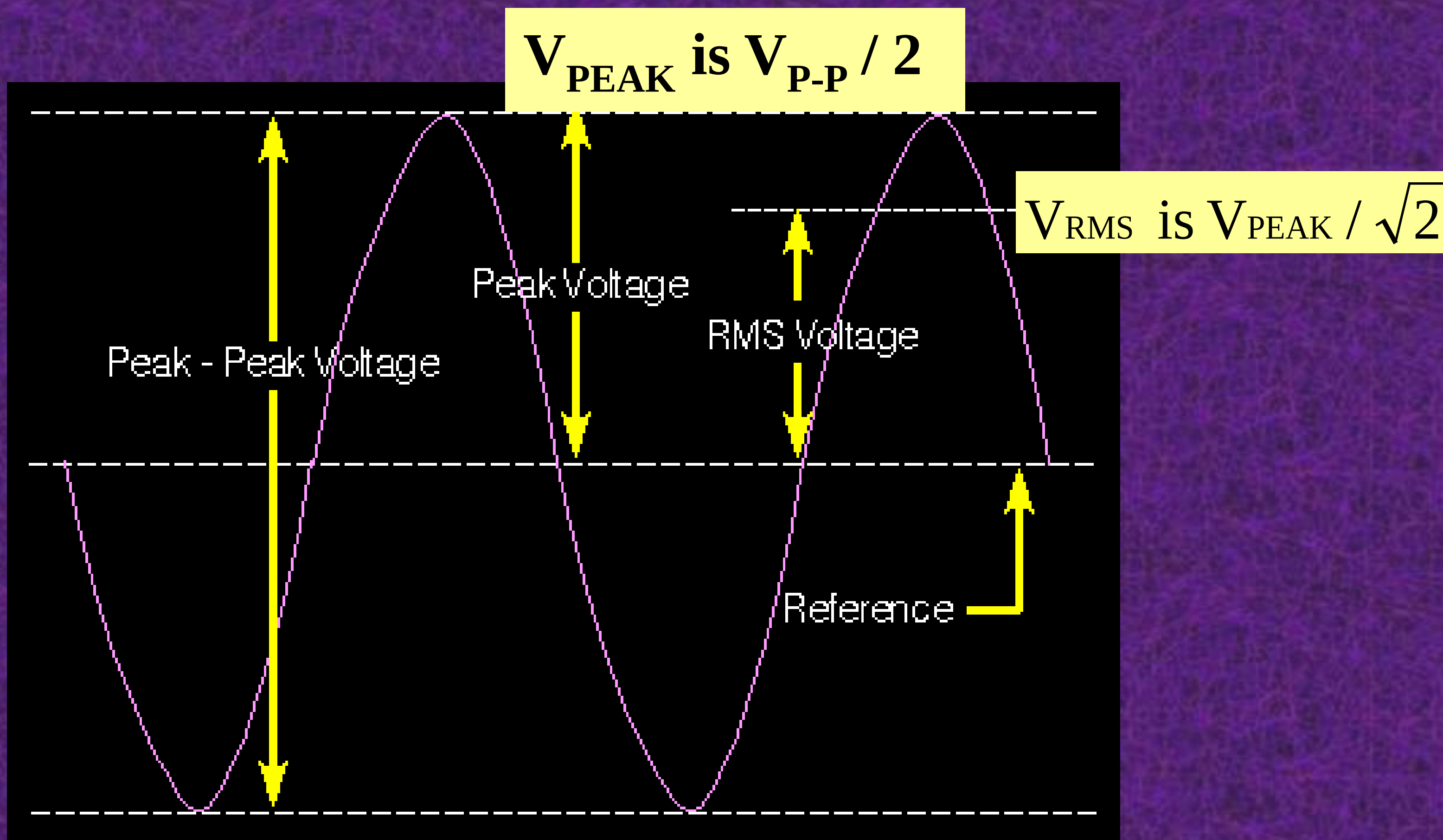


This way of creating energy that builds and collapses in different directions creates a sine wave…

The way AC is generated creates a ‘sine wave.’

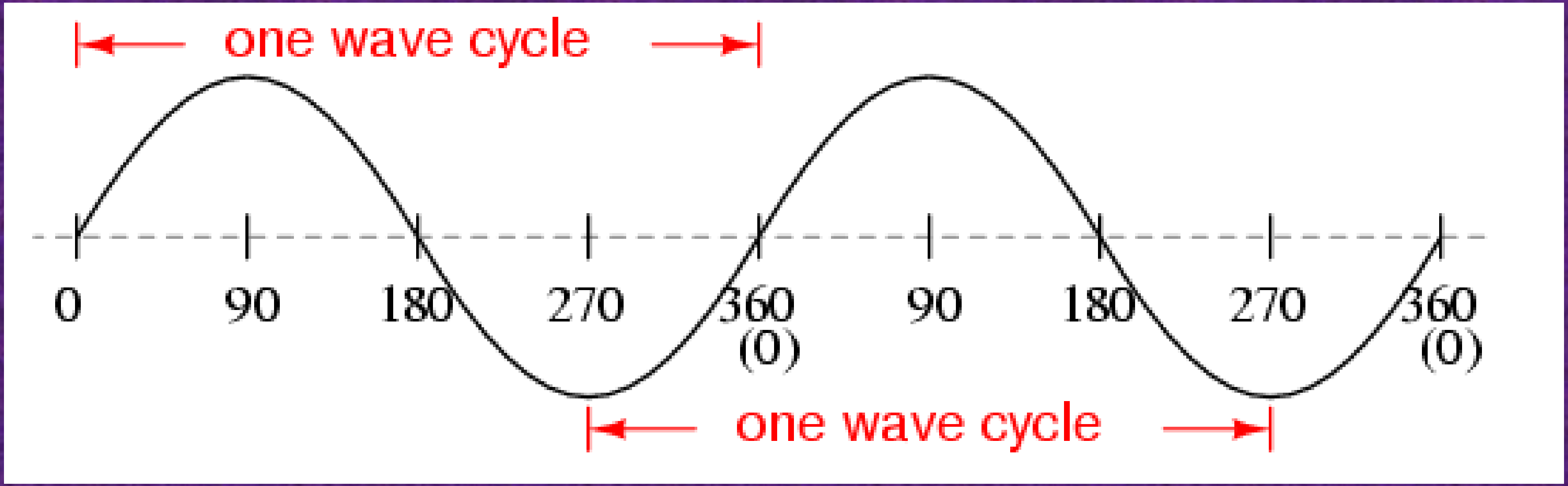


AC Sinusoidal Wave Magnitude Relationships



In the power world, we refer to voltages at RMS value, not peak.

Example of two wave cycles. For relay work, this sinusoidal representation can be plotted on a coordinate system, a common language in the relay world for both engineers and craftspeople.

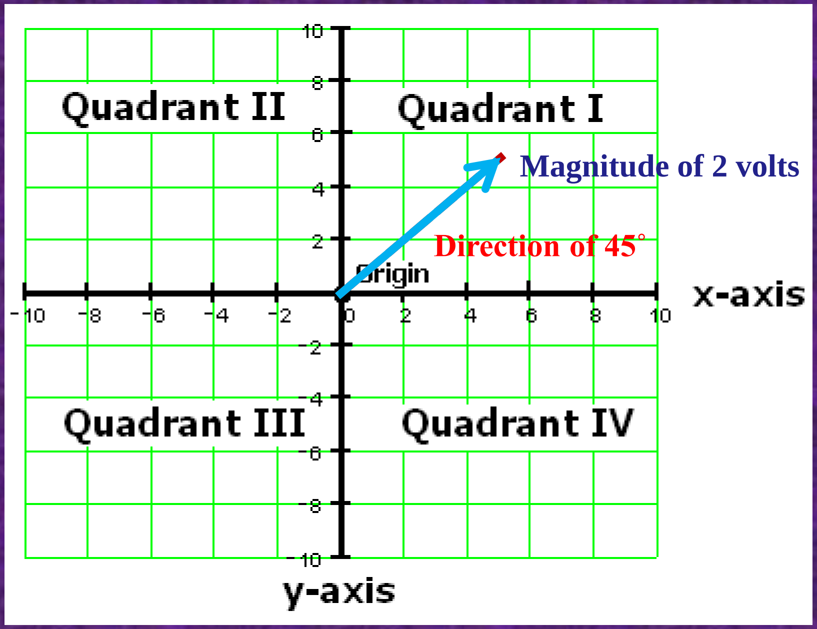


Remember: one cycle = one 360° rotation. It can be measure from any point on the sine wave, for example, from 270° to 270°.

One cycle of the same sine wave we just viewed can be graphically represented on a Cartesian coordinate system similar to this one, depending on where in time you take your measurements. One critical point, you must draw it with reference to it's place in time (direction) and at it's full RMS value.

90˚ (or -270)

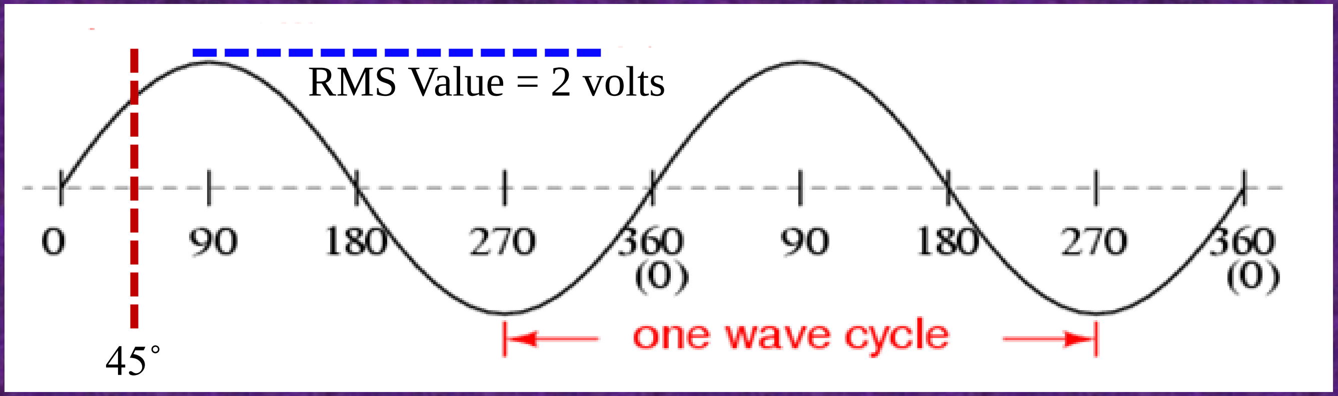
180˚

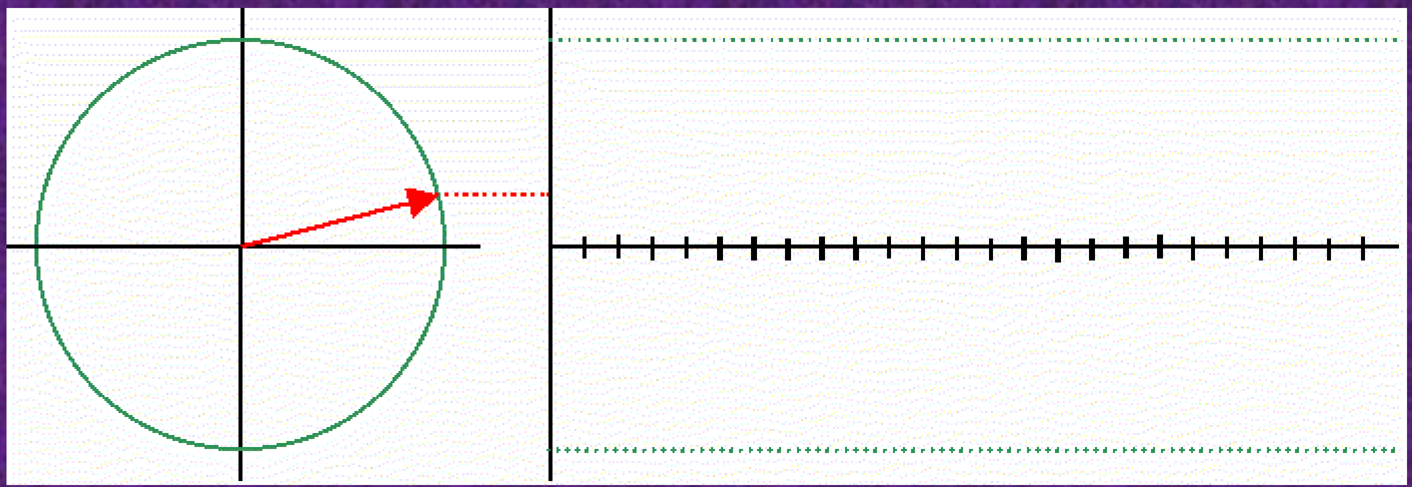


0, or 360˚

Plotted with Lead angles

270˚ (or -90)





Formulas for
Z, I, V, P, Q, VA, PF, and RF in AC

Basic relationships

V=IZ,

Volt-Amps=VI

Z = Impedance in ohms

Z=V/I*

(division reverses the polarity of the angle)

In addition:

Power Factor, or P.F. = COS θ

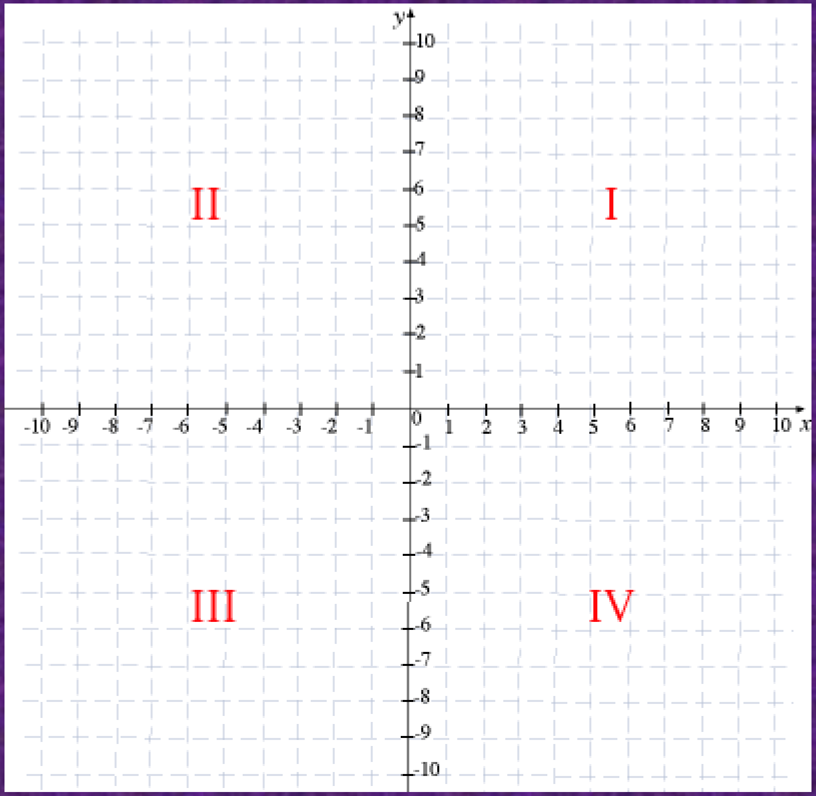
Reactive Factor, or R.F. = SIN θ

P = Watts = VA x P.F. = VI•COS θ

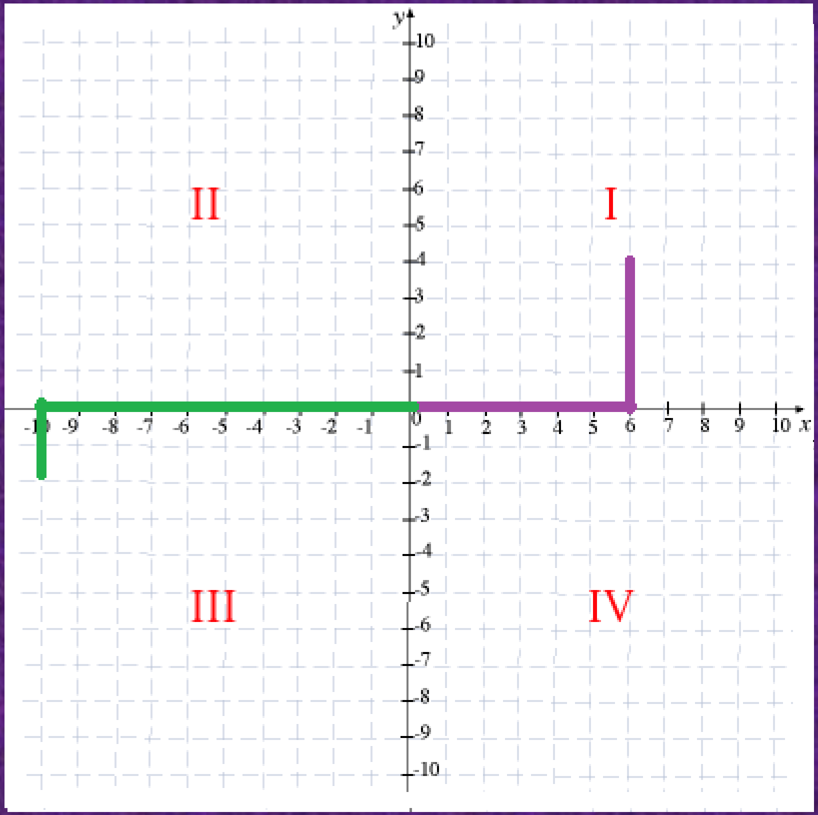
Q = Vars = VA x R.F. = VI•SIN θ

Cartisian Coordinate System

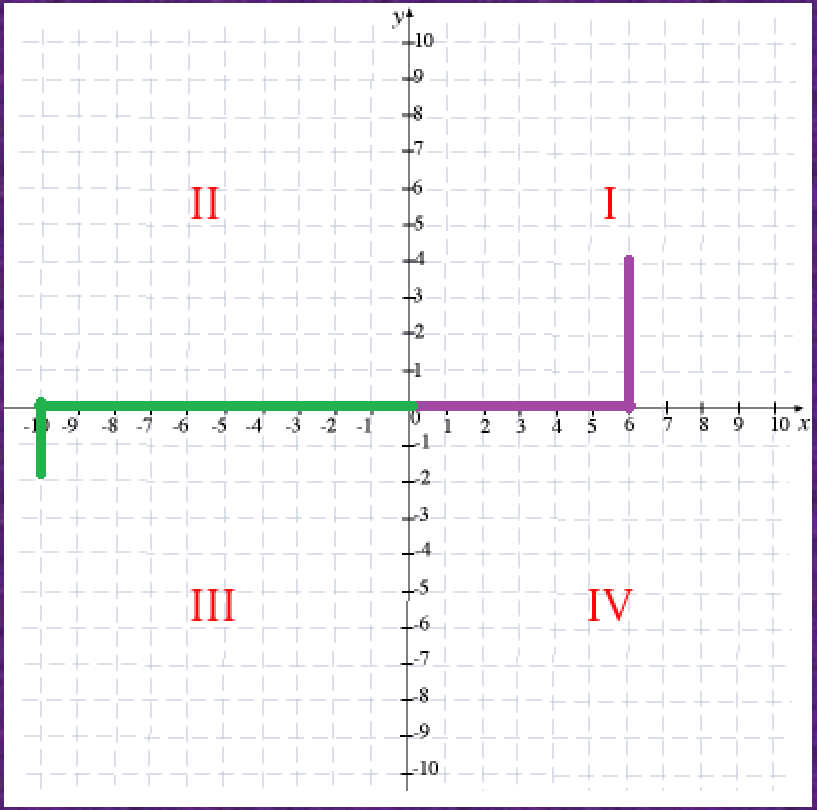
(used for plotting phasors)



We used this to plot points in school. For example: **X = 6, Y = 3 (purple)**; or perhaps **X = -10, Y = -2 (green)** as seen below, to define a point on the coordinate system.



As an example, values as below can be representative of True Power (watts) on the horizontal line, and Reactive Power (vars) on the vertical line. By using those two quantities, we can find the 'hypotenuse' or VA.

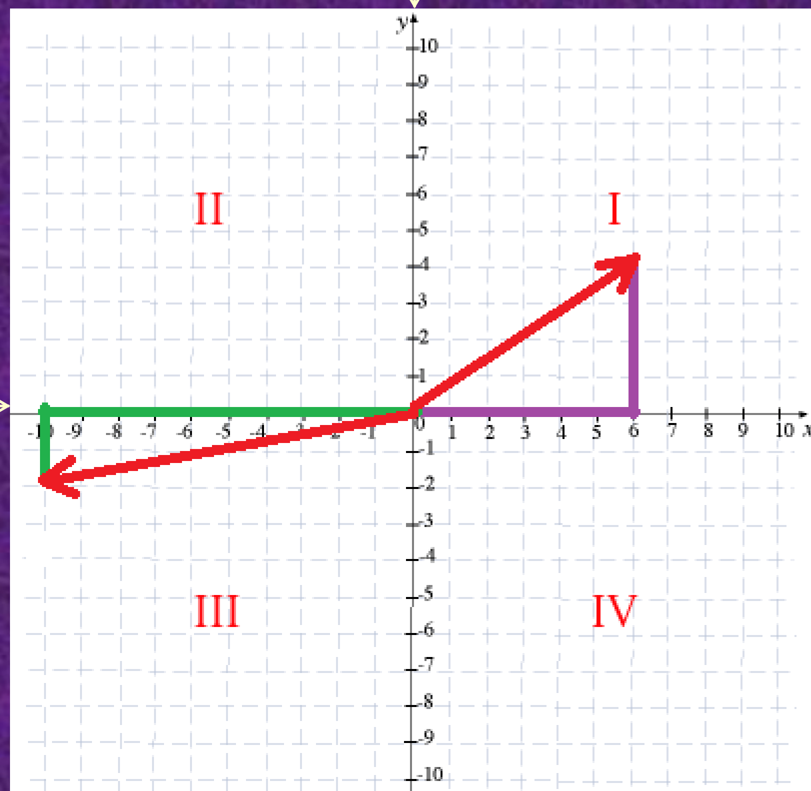


Cartesian Coordinate System

The summation of these quantities is proven through some basic trigonometric math. After we know the resultants magnitude and direction, we can then draw their resultant as a phasor.

y, or j, is 90˚

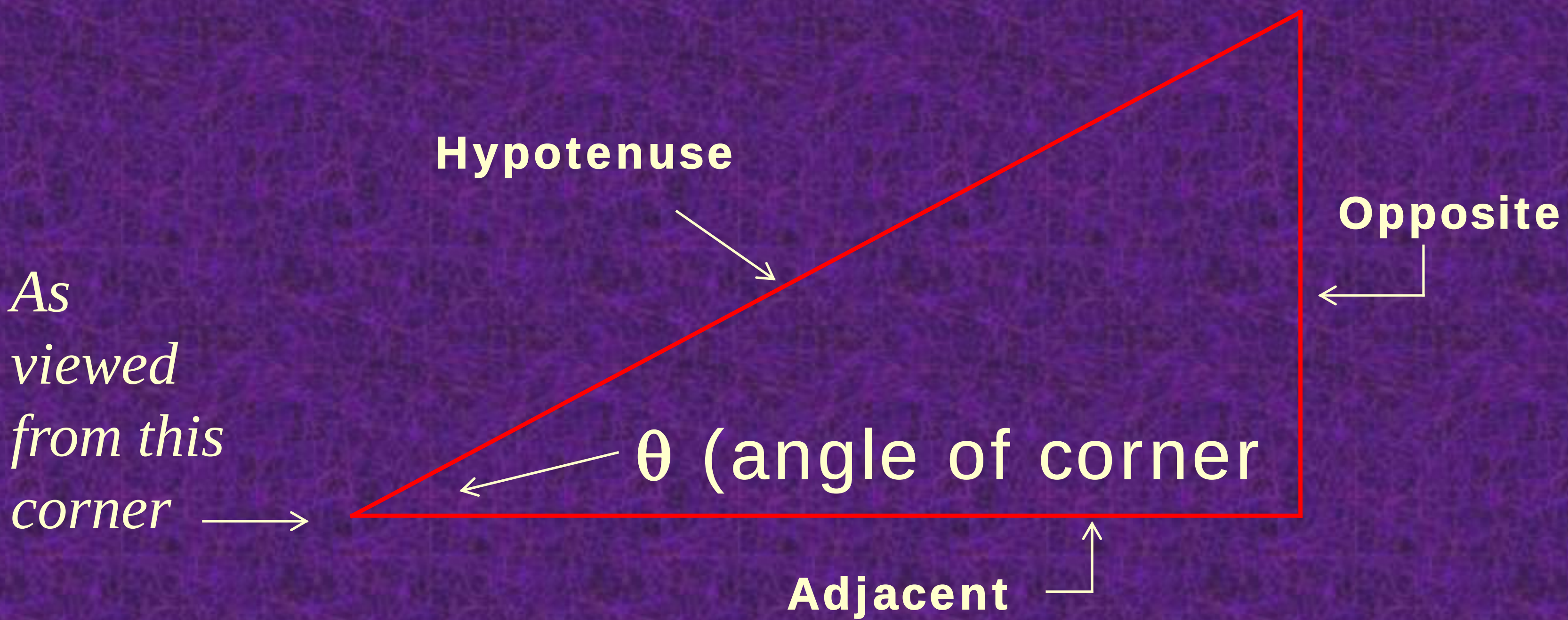
-x, or –r is 180˚



x, or r, is 0˚, (sometimes referred to as 360 ˚)

-y, or –j, is 270˚

Trigonometric Functions of a "Right Triangle"



Functions

sine(θ) = opp / hyp

cos(θ) = adj / hyp

tan(θ) = opp / adj

Inverse Functions

arcsine(opp / hyp) = SIN^{-1} = θ

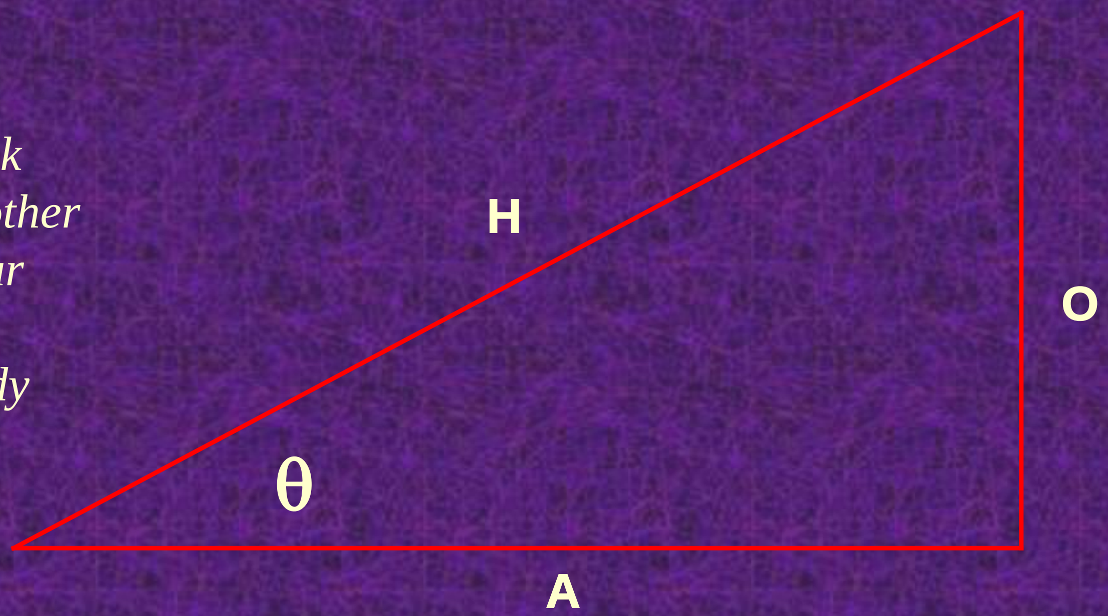
arccosine(adj / hyp) = COS^{-1} = θ

arctangent(opp / adj) = TAN^{-1} = θ

It would be nice to have an easy way to remember this!

Trigonometric Functions of a "Right Triangle"

Sin =	$\frac{O}{H}$	*oh* *heck*
Cos =	$\frac{A}{H}$	*another* *hour*
Tan =	$\frac{O}{A}$	*of* *Andy*



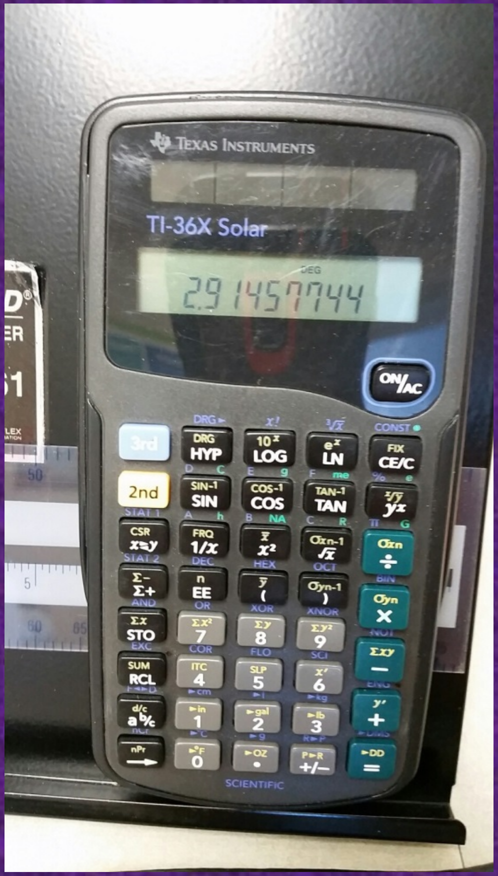
Another way to remember is to use the mnemonic "Oh Heck Another Hour Of Andy!"

What you are doing is finding the ratio, or difference in size, between the numerator (top number) and the denominator (bottom number). This ratio will be given as a percentage. That percentage can then be used to find the angle of the triangle, or θ.

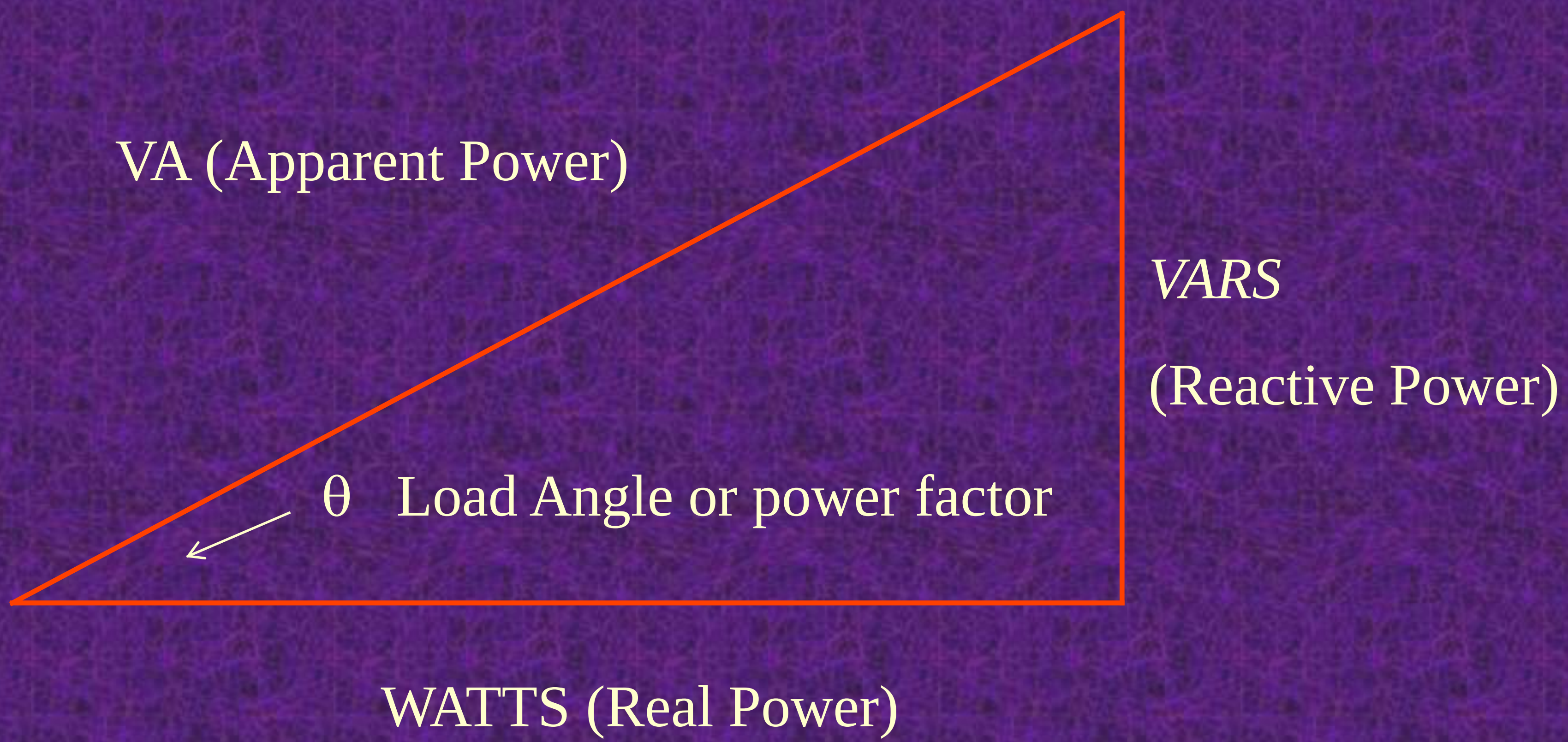
Example if H=10, O=5……

Sine = O/H = 5/10 = 0.5 (ratio), thereby SIN^{-1} of 0.5 = 30 degrees.

YOU DON'T NEED A FANCY CALCULATOR. JUST SOMETHING SIMPLE SUCH AS THIS $14 TI-36X. YOU WILL NEED TRIGONOMETRIC FUNCTIONS (sine, cosine, tangent) AS WELL AS AN R»P / R«P FUNCTION (rectangular, polar).

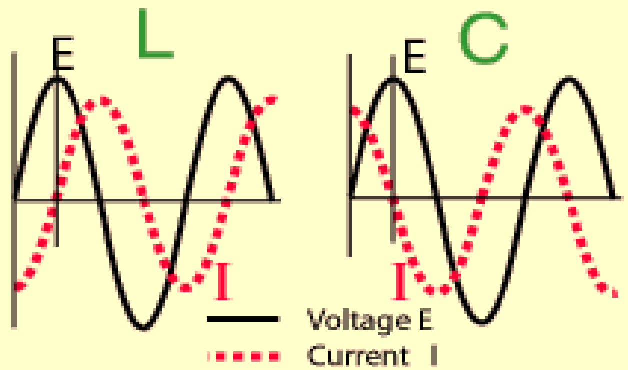


Understanding the AC System using the Power Triangle
Q: Why can't AC power just be resistive?
Q: What are Vars?



A mnemonic for the phase relationships of current and voltage.

When a voltage is applied to an inductor, it resists the change in current. The current builds up more slowly than the voltage, lagging it in time and phase.



Since the voltage on a capacitor is directly proportional to the charge on it, the current must lead the voltage in time and phase to conduct charge to the capacitor plates and raise the voltage.

Voltage leads Current

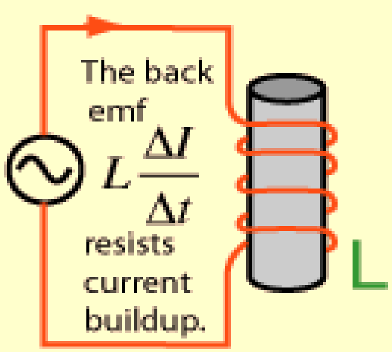
E L I the I C E man

in an inductor

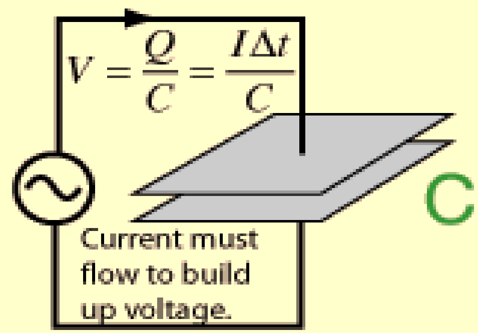
Current leads Voltage

in a capacitor

Inductance L



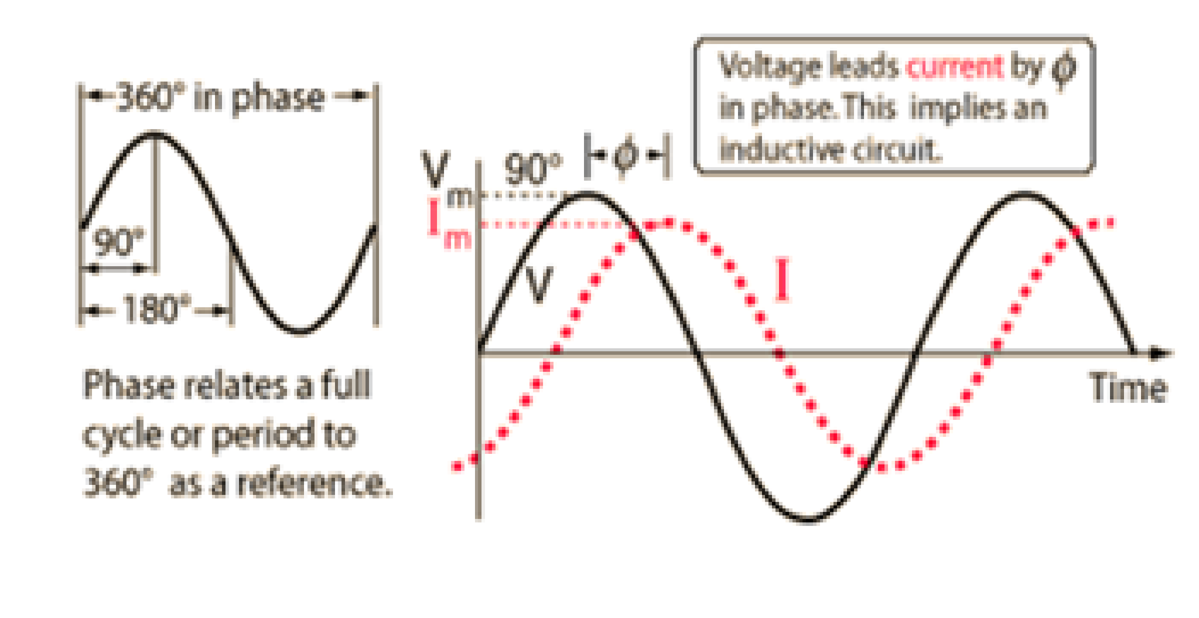
Capacitance C



When inductors or capacitors are used in an AC circuit, the circuit is no longer purely resistive with voltage and current in phase. Rather, current and voltage do not cross the zero reference at the same time (shown below).

An inductor alone, without any circuit resistance, would cause current to lag by 90 degrees from its relative voltage. If resistance is added, that lag would decrease from 90 degrees depending on the amount of resistance.

A capacitor alone, without any circuit resistance, would cause current to lead its voltage by 90 degrees. If resistance is added, that lead would decrease from 90 degrees depending on the amount of resistance.

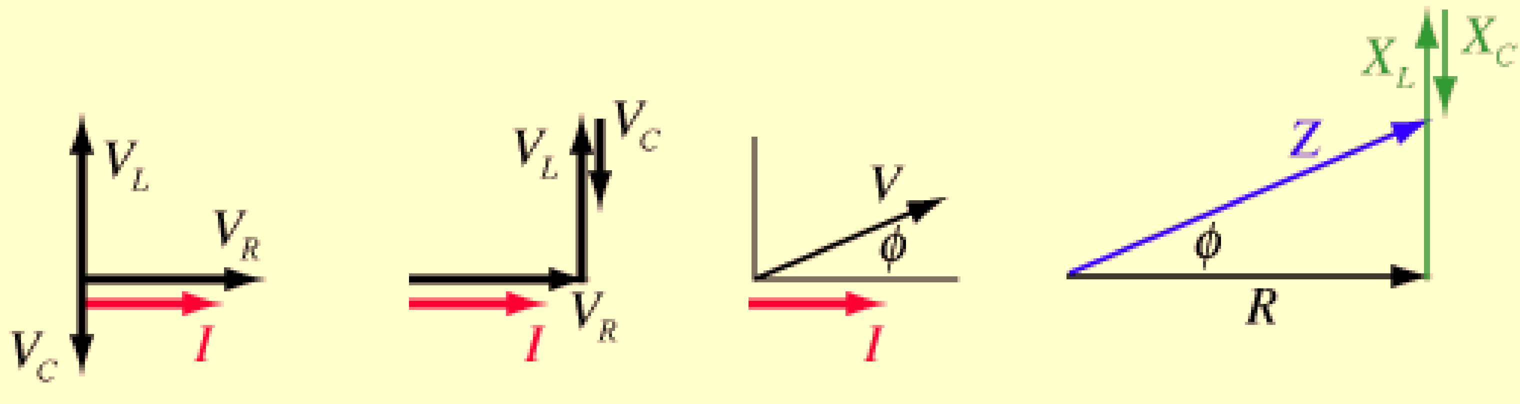


ELI the ICEman

Phasor Diagrams

The usual reference for zero phase is taken to be the positive x-axis and is associated with the resistor, or resistive part of an AC power system, since the voltage and current associated with the resistor are in phase.

The length of the phasor is proportional to the magnitude of the quantity represented, and its angle represents its phase relative to that of the current through the resistor. The phasor diagram for the RLC series circuit shows the main features.



$$V = \sqrt{V_R^2 + (V_L - V_C)^2} \qquad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

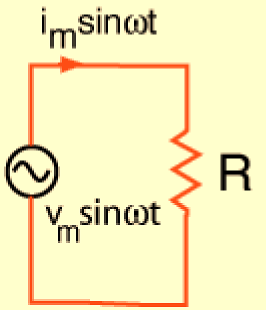
$$\phi = \tan^{-1}\frac{V_L - V_C}{V_R} \qquad \phi = \tan^{-1}\frac{X_L - X_C}{R}$$

Resistor AC Response

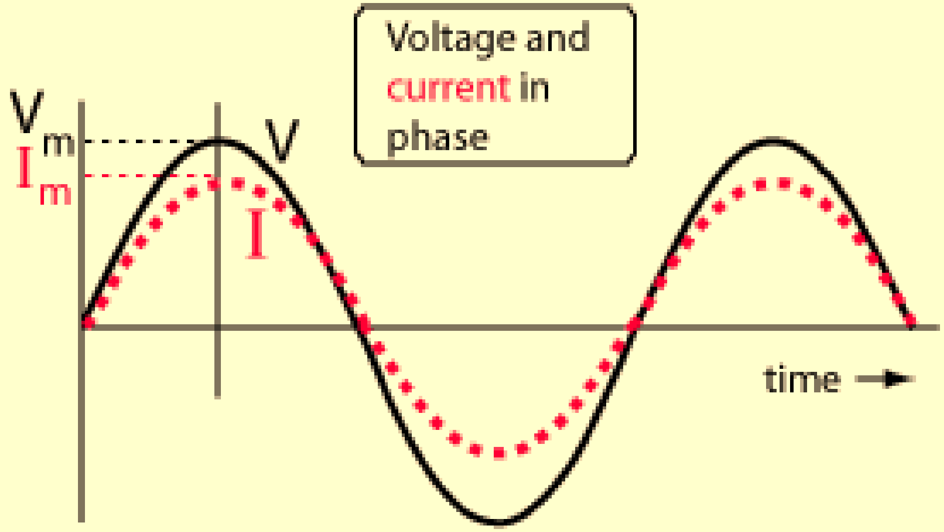
Impedance

$$I = \frac{V}{R}$$

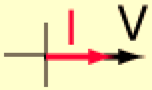
$$Z = R$$



$$I = \frac{I_m}{\sqrt{2}},\ V = \frac{V_m}{\sqrt{2}}$$



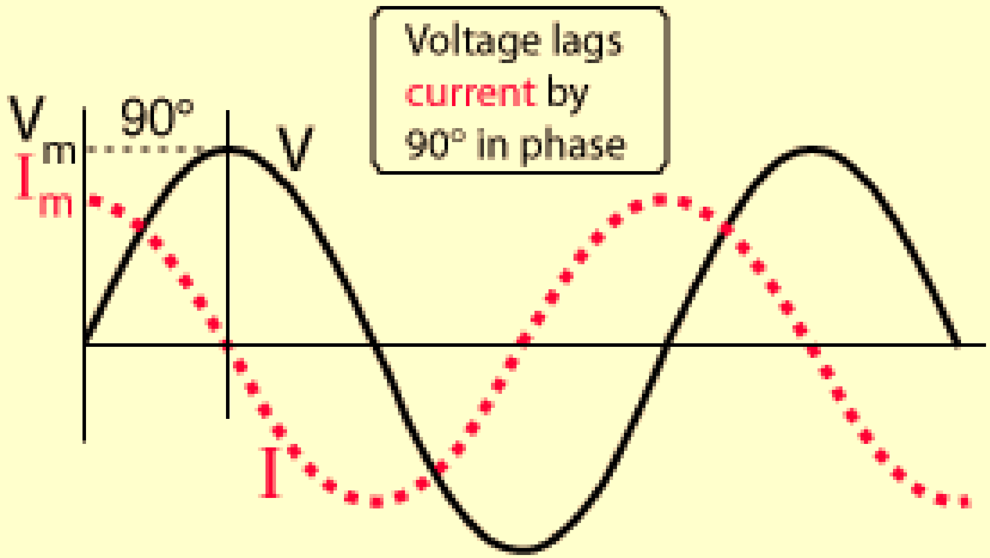
Phasor diagram

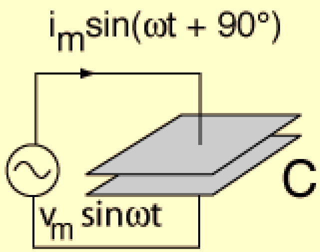


Impedance

$$I = \frac{V}{X_C}$$

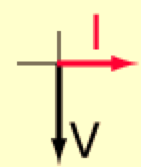
$$X_C = \frac{1}{\omega C}$$





$$I = \frac{I_m}{\sqrt{2}}, \quad V = \frac{V_m}{\sqrt{2}}$$

Phasor diagram

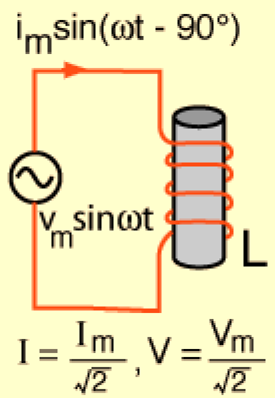


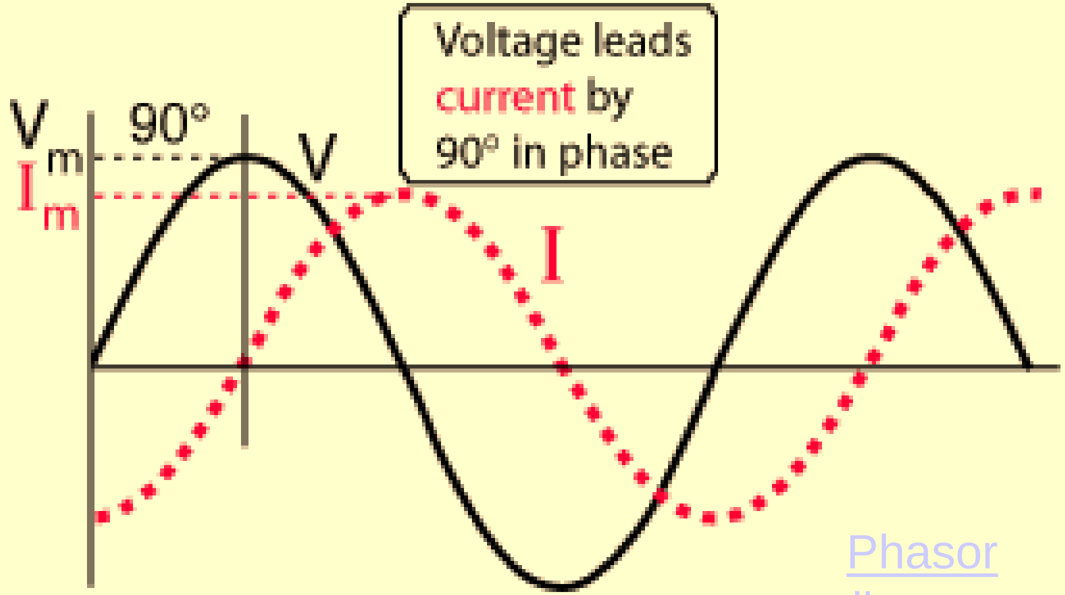
You know that the voltage across a capacitor lags the current (ICE) because the current must flow to build up the charge, and the voltage is proportional to that charge which is built up on the capacitor plates.

Impedance

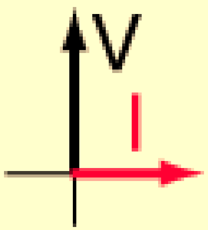
$$I = \frac{V}{X_L}$$

$$X_L = \omega L$$





Phasor diagram



You know that the voltage across an inductor leads the current (ELI) because of Lenz' law. The inductor resists the buildup of the current, and it takes time for an imposed voltage to force the buildup of current to its maximum.

Now that we understand the influence Vars have, let's talk some more about triangles. Particularly, a power triangle.

Apparent Power

$$S = VI\text{, units VA}$$

Real Power

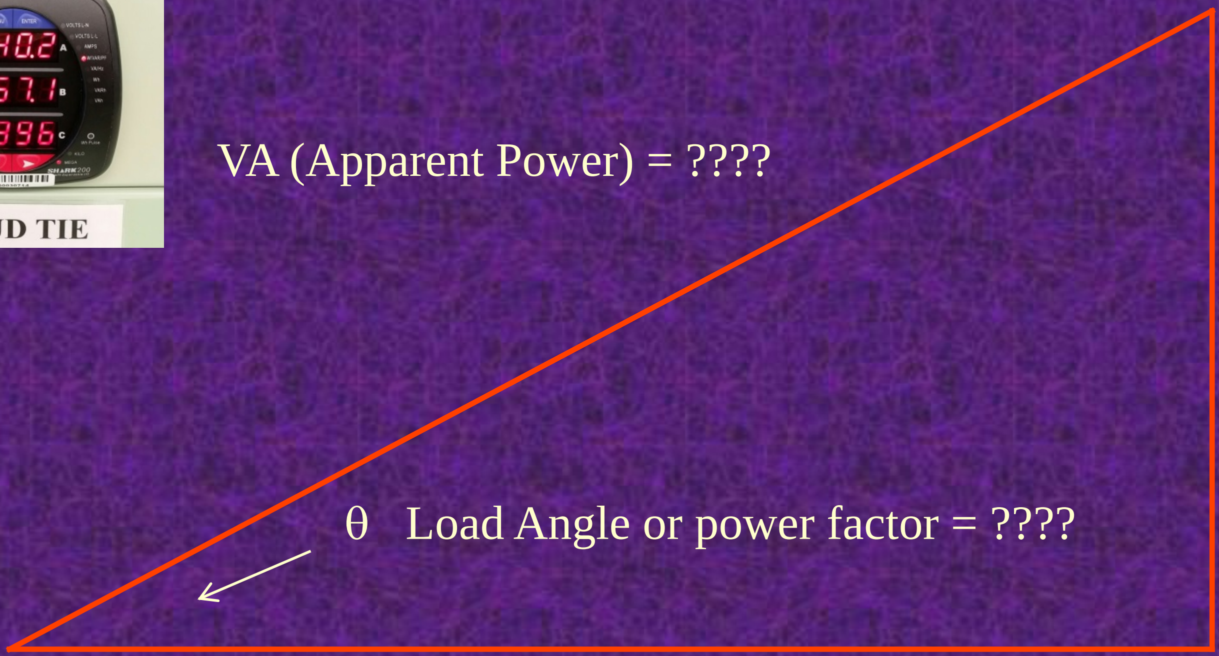
$$P = VI \cos \theta\text{, units Watts}$$

Reactive Power

$$Q = VI \sin \theta\text{, units Var (VA reactive)}$$

- Lagging load (positive vars)
- Leading load (negative vars)

Let's use what we have learned so far to find VA of a circuit using watts and vars from board meters.



VA (Apparent Power) = ????

MVARS = 57.1 out

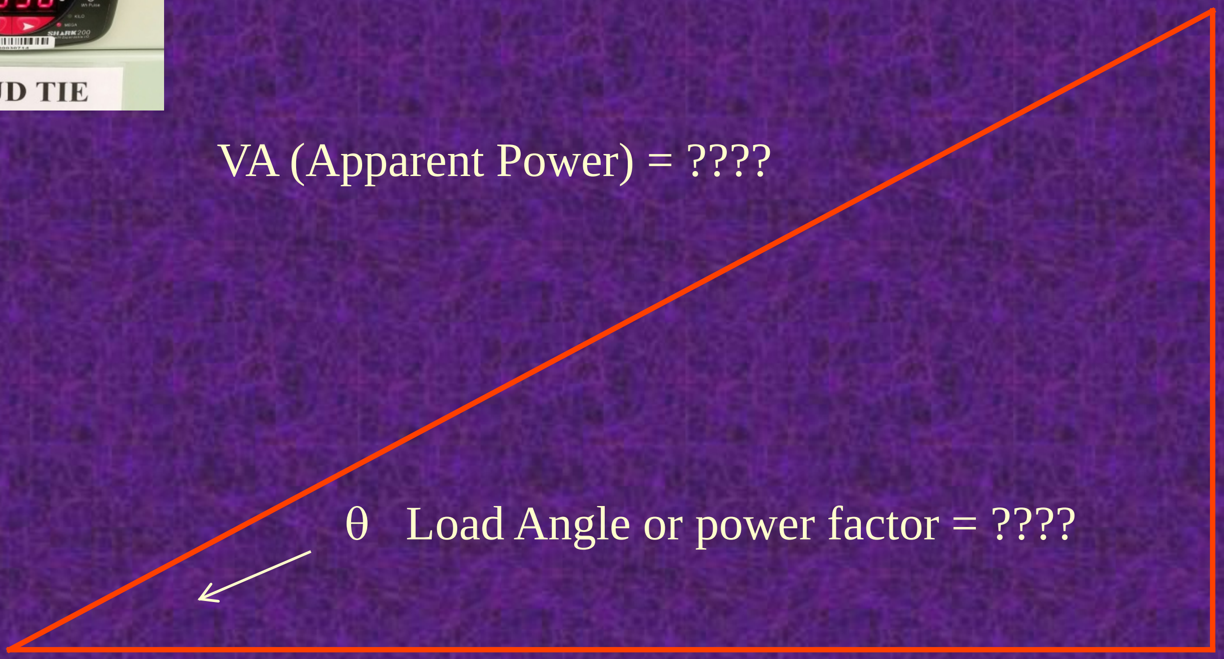
(Reactive Power)

θ Load Angle or power factor = ????

WATTS = 640.2 in
(Real Power)

Sin =	<u>O</u>	*oh*
	H	*heck*
Cos =	<u>A</u>	*another*
	H	*hour*
Tan =	<u>O</u>	*of*
	A	*Andy*

VA (Apparent Power) = ????



MVARS = 57.1 out

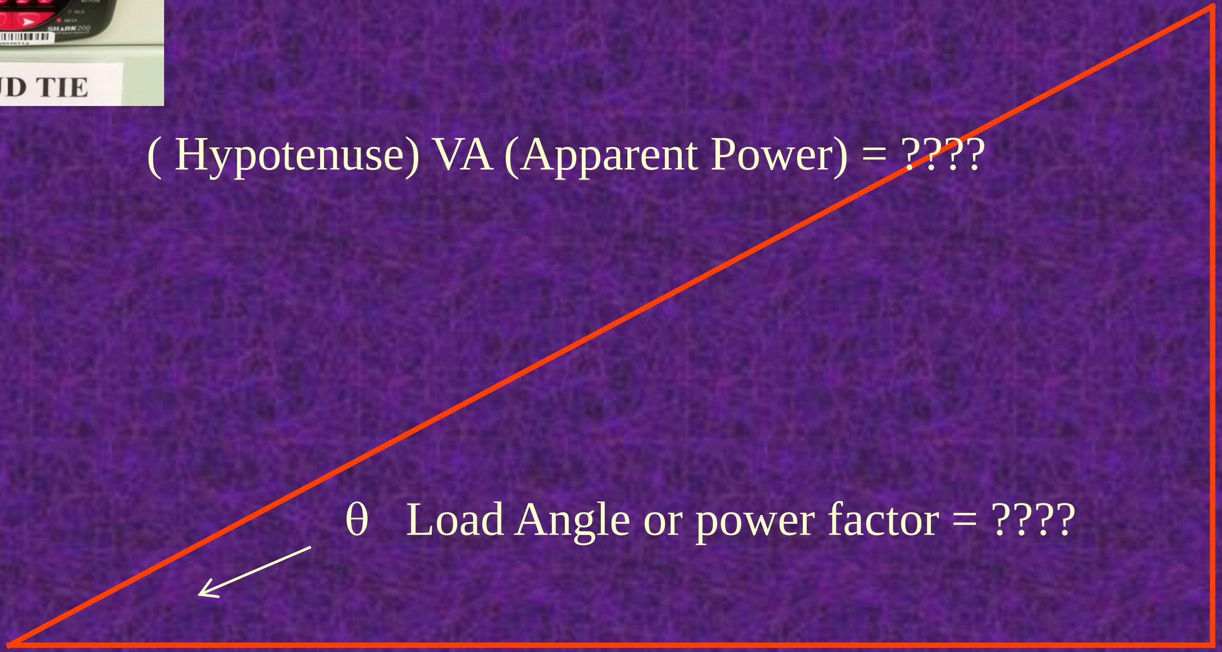
(Reactive Power)

θ Load Angle or power factor = ????

WATTS = 640.2 in
(Real Power)

Which values do we know as given from the board meters?

Sin =	O	*oh*
	H	*heck*
Cos =	A	*another*
	H	*hour*
Tan =	O	*of*
	A	*Andy*



(Hypotenuse) VA (Apparent Power) = ????

(Opposite) MVARS = 57.1 out

(Reactive Power)

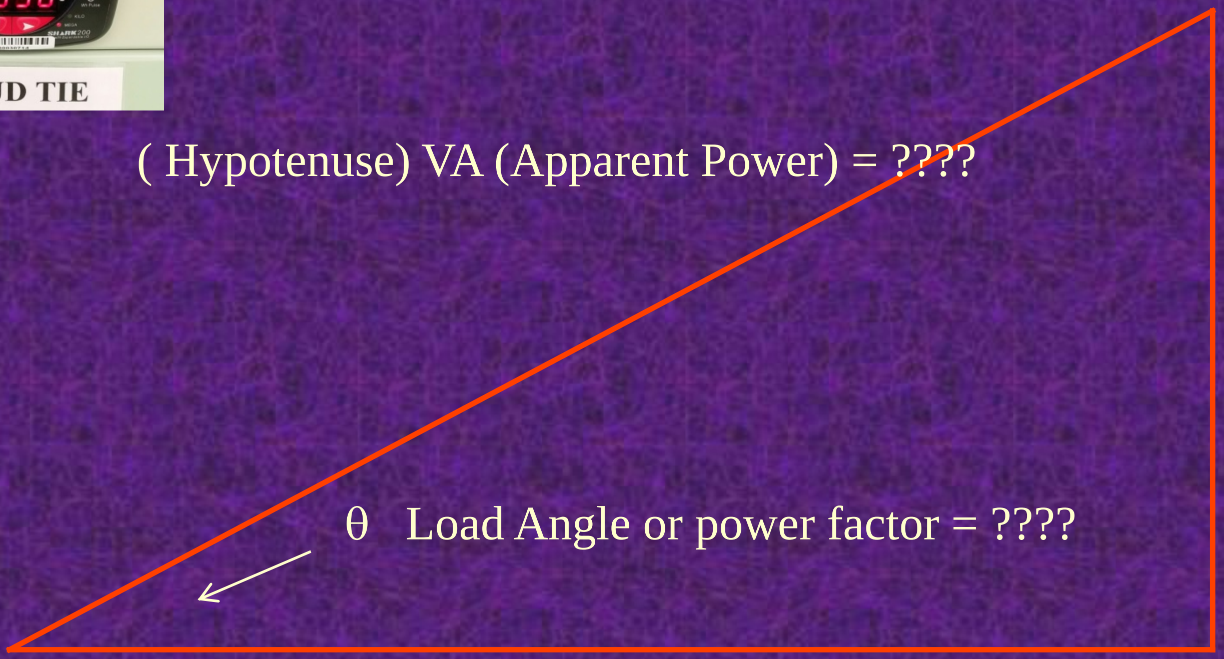
θ Load Angle or power factor = ????

(Adjacent) WATTS = 640.2 in (Real Power)

So, which math function above would I use?

Sin =	<u>O</u>	*oh*
	H	*heck*
Cos =	<u>A</u>	*another*
	H	*hour*
Tan =	<u>O</u>	*of*
	A	*Andy*

(Hypotenuse) VA (Apparent Power) = ????



(Opposite) MVARS = 57.1 out

(Reactive Power)

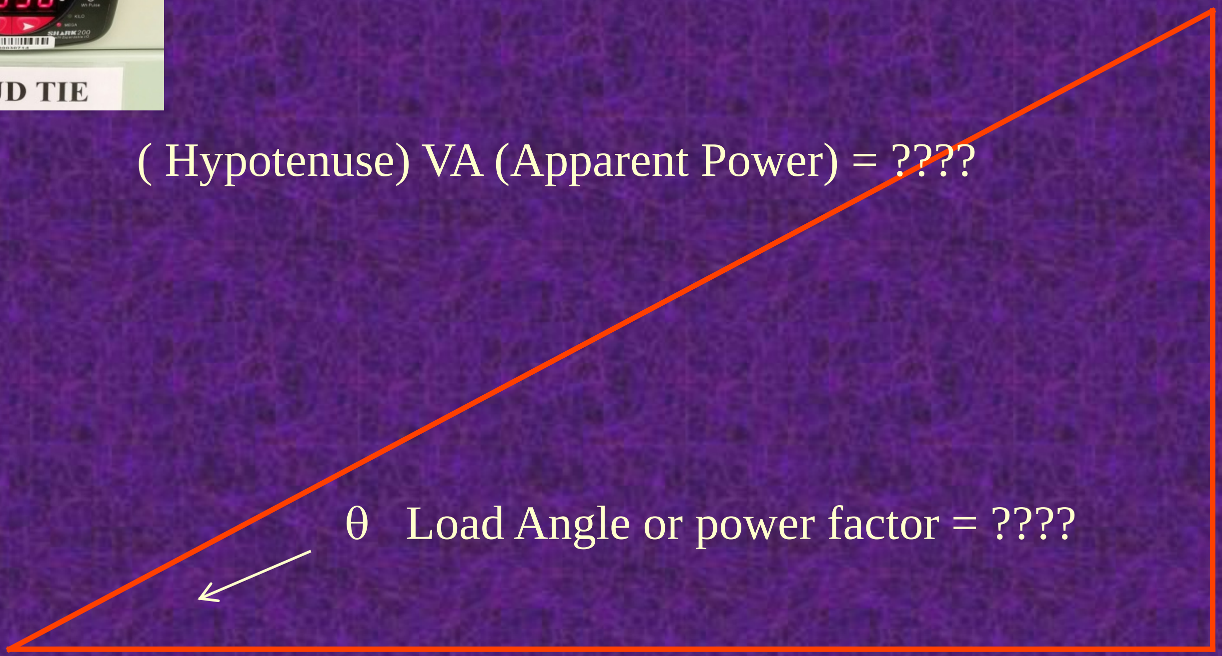
θ Load Angle or power factor = ????

(Adjacent) WATTS = 640.2 in (Real Power)

Since I have 'adjacent' & 'opposite' values, then use Tangent!

Sin =	<u>O</u>	*oh*
	H	*heck*
Cos =	<u>A</u>	*another*
	H	*hour*
Tan =	<u>O</u>	*of*
	A	*Andy*

(Hypotenuse) VA (Apparent Power) = ????



(Opposite) MVARS = 57.1 out

(Reactive Power)

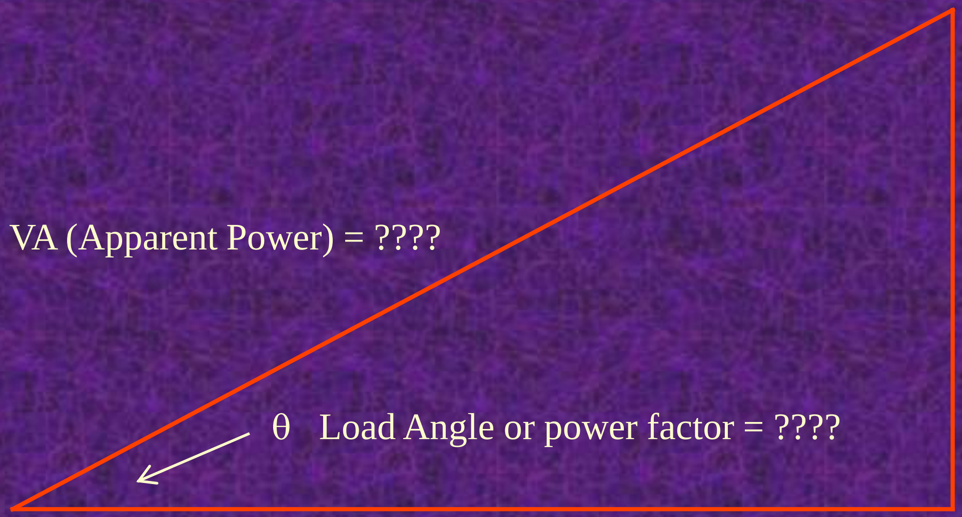
θ Load Angle or power factor = ????

(Adjacent) WATTS = 640.2 in (Real Power)

Answer: Tan = O/A; Tan = 57.1/640.2; = 0.08919; ArcTan of 0.08919 = 5.0968°
Our angle is now known.

Now, let's use the known angle of **5.0968** ° to see what VA, or the length of the hypotenuse line, is?

Sin =	<u>O</u>	*oh*
	H	*heck*
Cos =	<u>A</u>	*another*
	H	*hour*
Tan =	<u>O</u>	*of*
	A	*Andy*



(Hypotenuse) VA (Apparent Power) = ????

(Opposite) MVARS = 57.1 out

(Reactive Power)

θ Load Angle or power factor = ????

(Adjacent) WATTS = 640.2 in (Real Power)

For this, now knowing three quantities, you can use any of the equations.
Let's use Sin! Sin = O/H
Sin of angle 5.0968 ° = 57.1 / H; 0.088839 = 57.1 / H; H = 57.1/0.088839; H = 642.7 VA!

To prove this answer, we can use Pythagoreans theorem, which states $C^2 = A^2 + B^2$, or $C = \sqrt{(A^2 + B^2)}$
(SEE NEXT SLIDE)

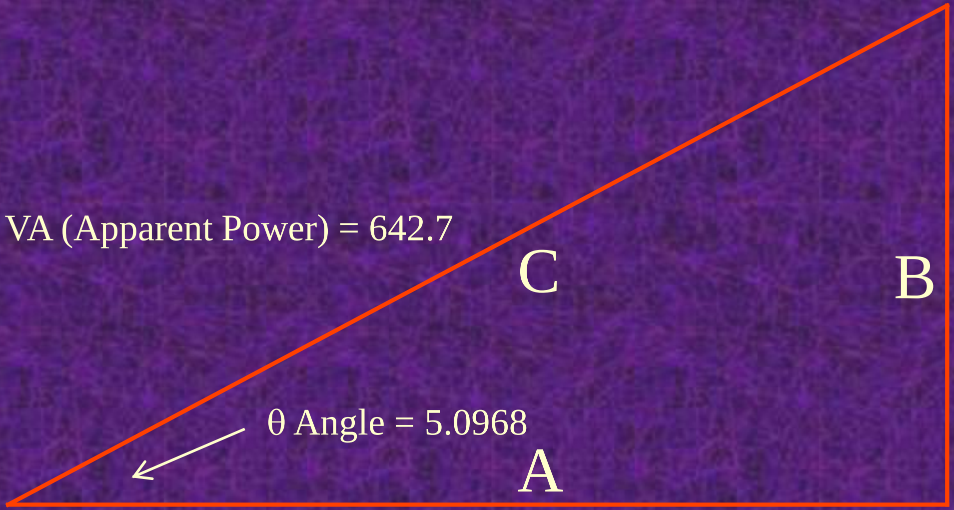
To prove this answer, we can use Pythagoreans theorem, which states $C^2 = A^2 + B^2$, or $C = \sqrt{(A^2 + B^2)}$
Therefore: $642.7 = \sqrt{(640.2^2 + 57.1^2)}$.
Did we prove our math???

Sin =	$\frac{O}{H}$	*oh* / *heck*
Cos =	$\frac{A}{H}$	*another* / *hour*
Tan =	$\frac{O}{A}$	*of* / *Andy*

(Hypotenuse) VA (Apparent Power) = 642.7

(Opposite) MVARS = 57.1 out

(Reactive Power)

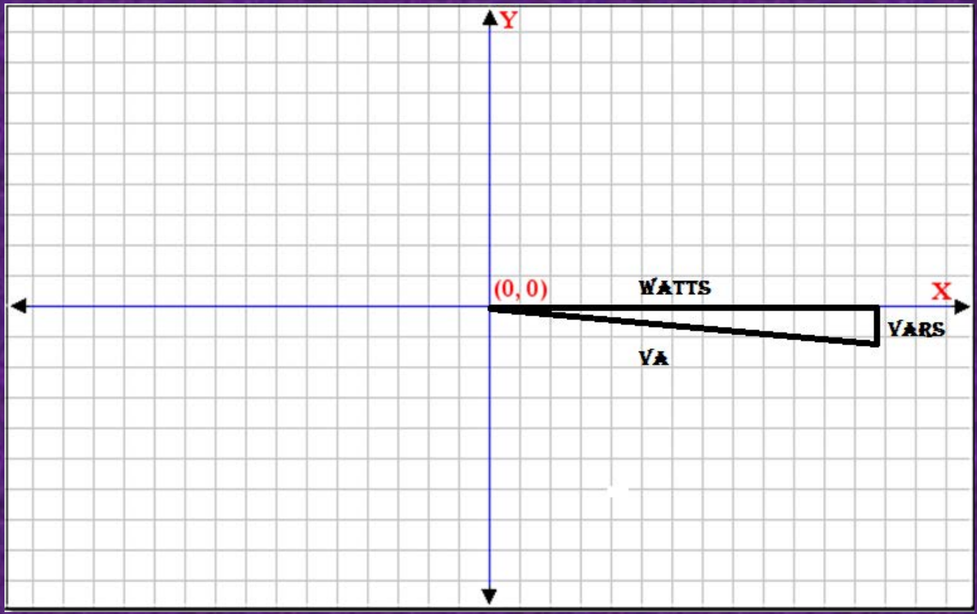


(Adjacent) WATTS = 640.2 in (Real Power)

Now, let's plot our known quantities on a Cartesian coordinate system.

We know that we have:

57.1 Vars out
640.2 watts in
An angle of 5.0968 degrees
VA = 642.7 (basically IE)

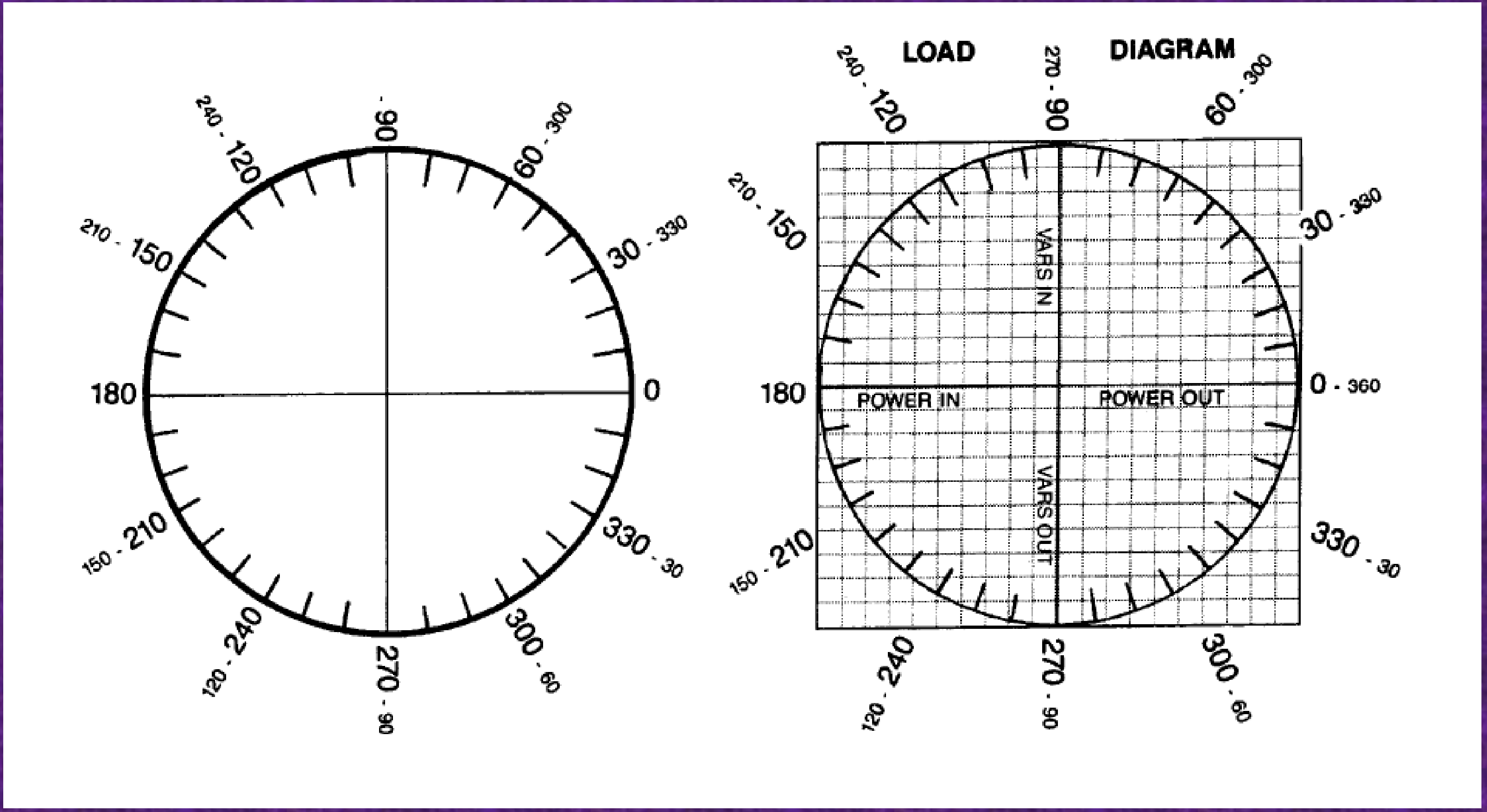


Q: why is the triangle upside down?

A: LEAD VS LAGGING

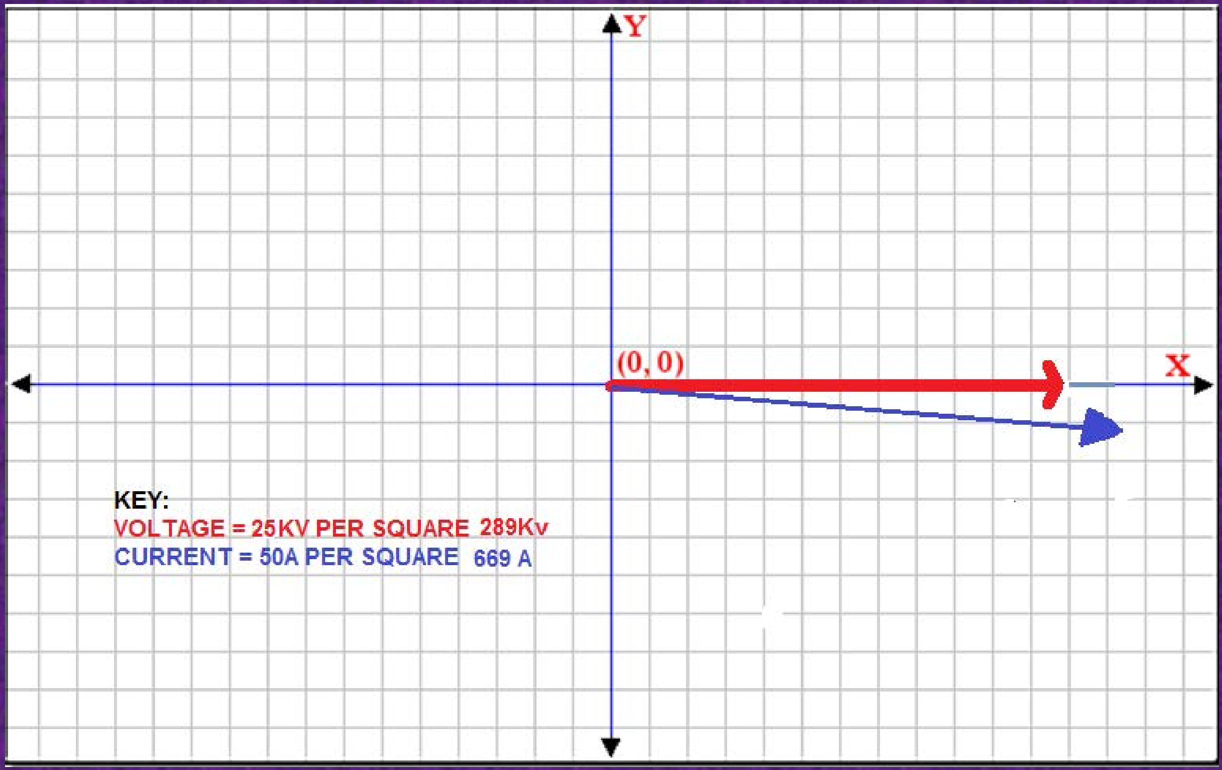
Additionally, we also know that VA = IE. If voltage is 289V, and we decide to plot that voltage at 0 degrees just like resistance/watts, that means current (I) would lag its respective voltage by the same angle used in finding VA.

The previous triangle was plotted upside down because we have 'vars out' according to our board. Which means our load on the end of the line looks like what? A capacitor or Inductor? What do you think?



Now, let's plot our known quantities on a Cartesian coordinate system.
We know that we have:

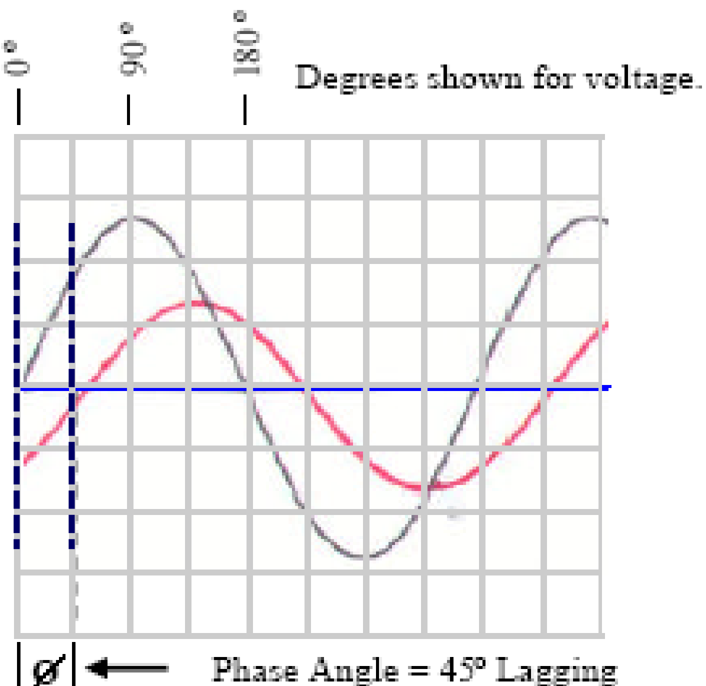
57.1 Vars out
640.2 watts in
An angle of 5.0968 degrees
VA = 642.7 (basically IE)   I = 669 amps
V = 500KV/$\sqrt{3}$ = 289KV



Additionally, we also know that VA = IE. If voltage is 289V, and we decide to plot that voltage at 0 degrees just like resistance/watts, that means current (I) would lag its respective voltage by the same angle used in finding VA. Congratulations! We just plotted phasors! ☺

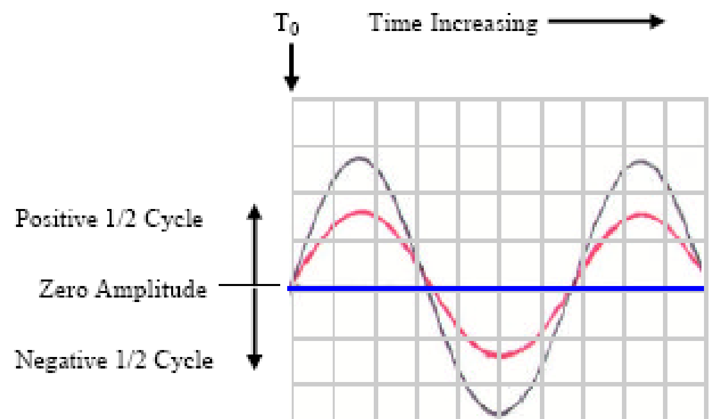
Sinusoidal “graphical” Comparison of V & I in AC

REAL AND REACTIVE POWER AT 70% P.F.



Cos^{-1} .70 (PF) ≈ 45°

Sine Wave Relationship
Red - Current
Black - Voltage



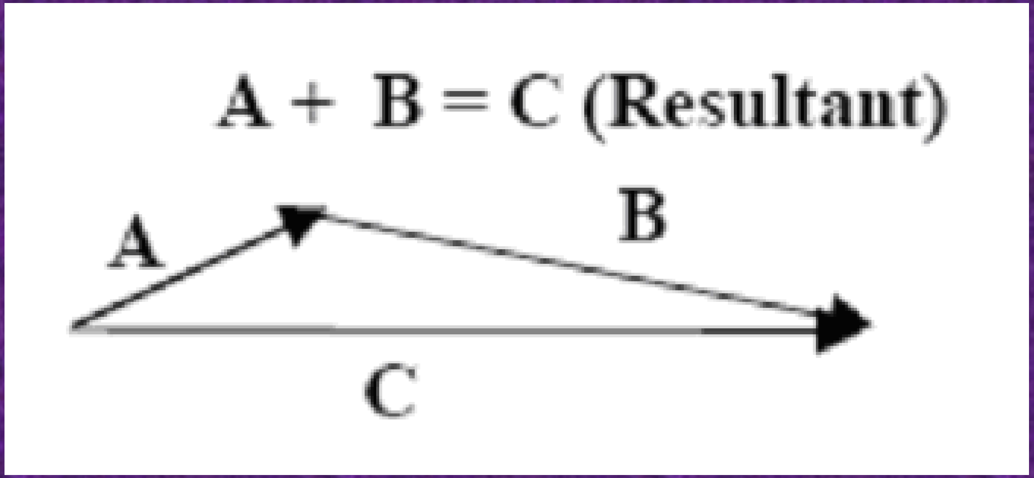
Cos of 0 = 1,
Therefore PF = 1

Sine Wave Relationship
Showing in-phase condition.
Red - Current
Black - Voltage

REAL POWER ONLY

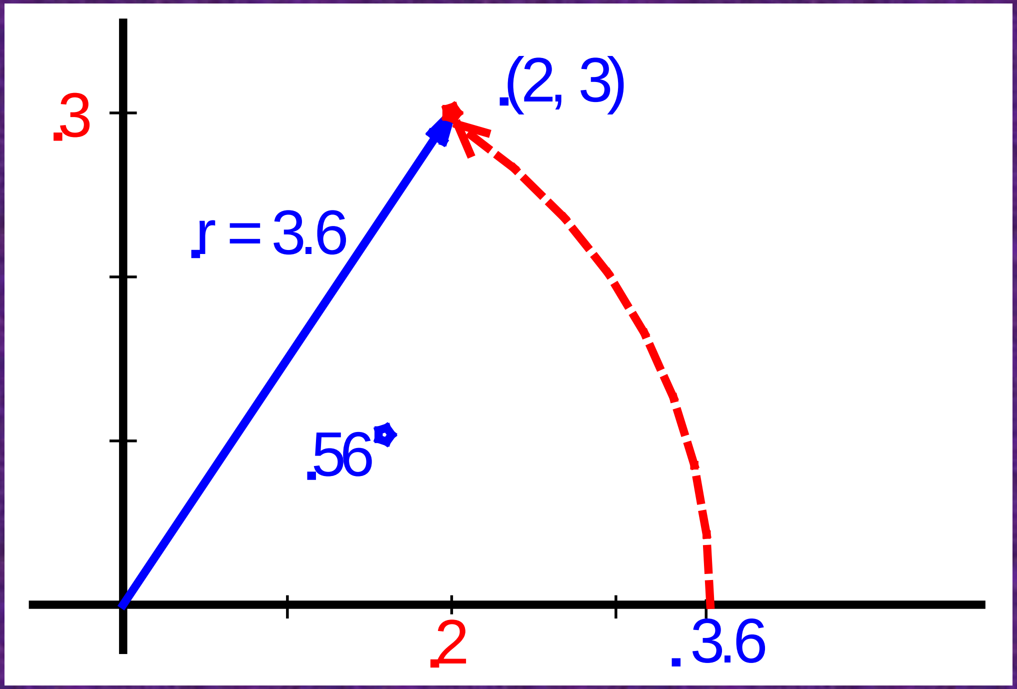
Graphic Vector Addition

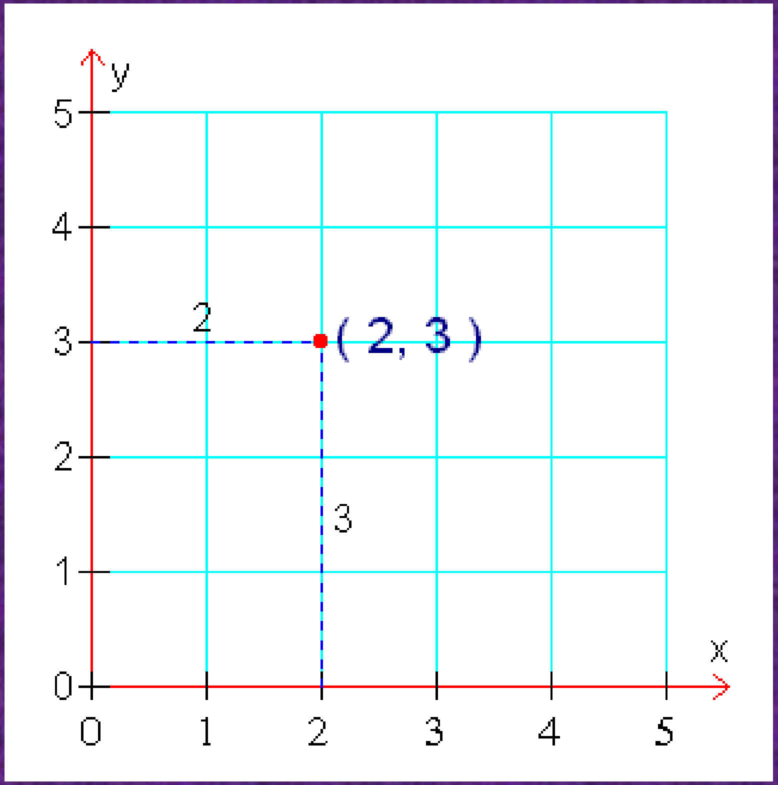
- **Adding and subtracting (graphically)**
- ADDING – TAIL to HEAD
- SUBTRACTING – still TAIL to HEAD (after subtracted vector is rotated 180°)



As we have seen, there are polar values, or rectangular values. Both show the same result, but polar is used when graphing phasors on a coordinate system.

Polar Coordinates





Rectangular Coordinates

Rectangular Coordinate Math

- **Adding & Subtracting (easy)**
- **Multiplication (hard)**
- **Division (hardest)**

$$\left(30+j20\right)+\left(20+\mathrm{j}10\right)=30+20+j20+j10=50+j30$$

$$\left(30+j20\right)-\left(20+\mathrm{j}10\right)=30-20+j20-j10=10+j10$$

Adding & Subtracting

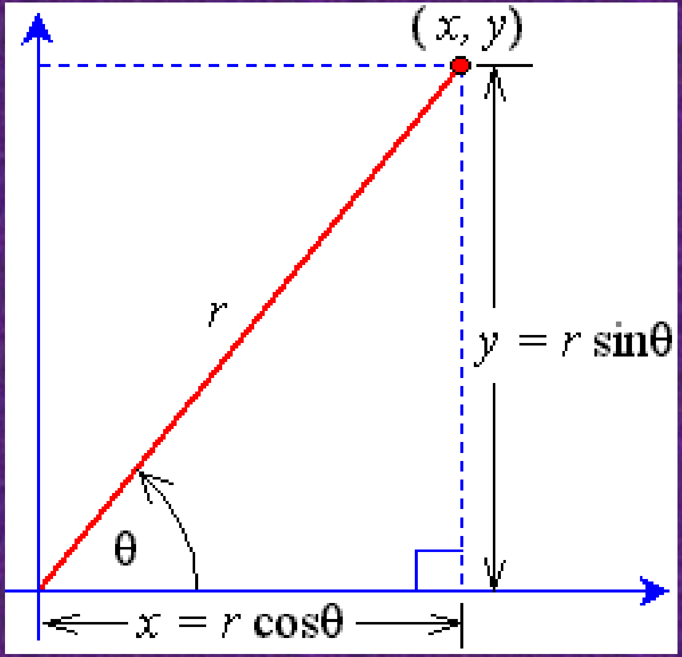
Polar Coordinate Math

- Multiplication (easy)
- Division (easy)

$$10\angle 20^\circ \text{ x } 20\angle 10^\circ = (10 \text{ x } 20)\ \angle(20^\circ + 10^\circ) = 200\angle 30^\circ$$

$$\frac{10\angle 20^\circ}{20\angle 10^\circ} = (10\,/\,20)\ \angle(20^\circ - 10^\circ) = 0.5\angle 10^\circ$$

Polar to Rectangular and Rectangular to Polar Conversions



P ⇒ R

$x = r \cos\theta$ $y = r \sin\theta$

R ⇒ P

$$r = \sqrt{x^2 + y^2}$$

$\theta = \tan^{-1}({}^{y}/_{x})$

Polar to Rectangular and Rectangular to Polar Conversions

Calculator that is capable of performing complex math

or

- Use P→R / R→P conversion calculator functions
- Adding and subtracting (using P→R conversion)
- Multiplication & Division (using R→P conversion)

Convert back and forth as need based on whether the operation is Addition / Subtraction or Multiplication / Division

Example

- Verify the line current based on the two ring bus breaker currents:
- $50a < -15^0$ and $70a < -30^0$

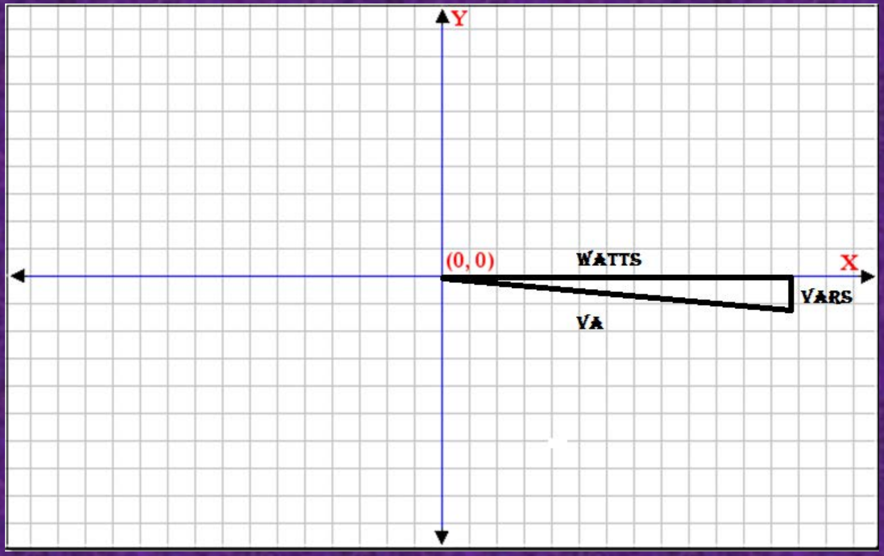
Example Answer

- Enter 50, x><y, -15
- 2nd button, P>R, jot down value, then x><y and jot down second value (48.3, -12.9)
- Do this again for 70<-30
- Add the X values and add the Y values
- Enter the resulting X, x><y, resulting Y, 3rd, jot down value, x><y, jot down second value
- 118a<-23.7^0

A Variety of Planes

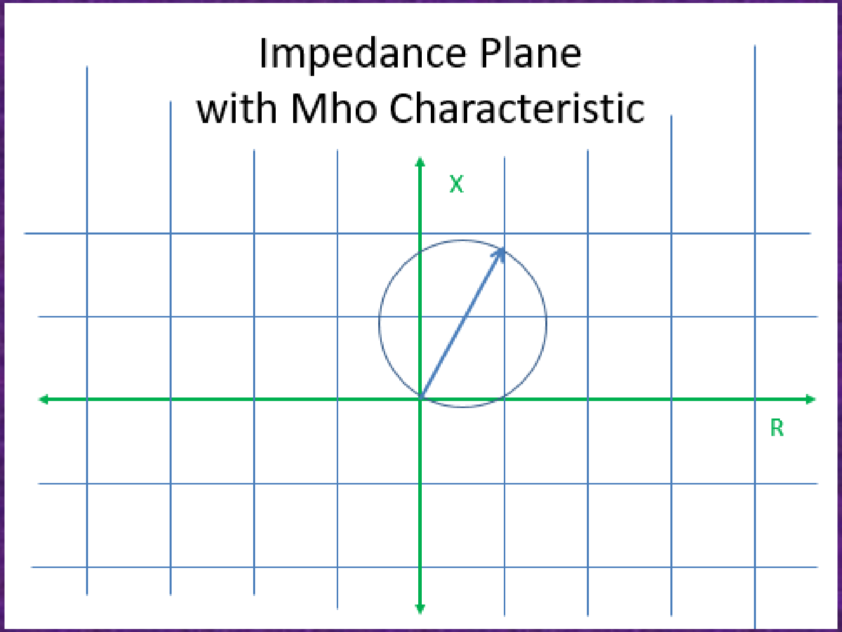
- Voltage and current phasors are represented on a plane that represents these magnitudes
- We use separate planes to represent other quantities
- P = VI so the power phasor has the same angle direction as the current phasor
- A bit confusing but

lagging current yields what are called “positive” Vars



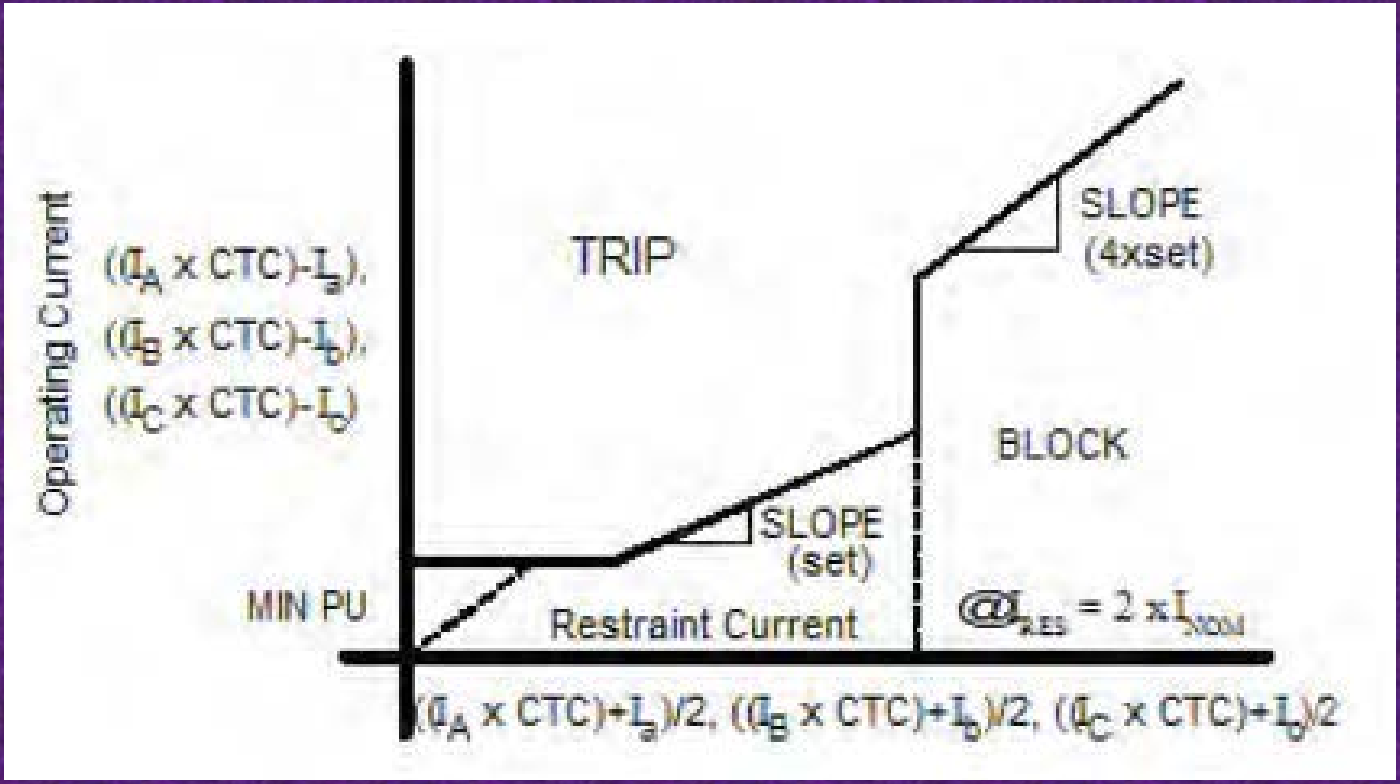
A Variety of Planes

- Z = V/I so the +/- sign of the angle changes between injection plane and impedance plane
- If the impedance is at 80^0, the current injection will be at -80^0
- The plane is defined by resistance and reactance



A Variety of Planes

- Transformer differential is represented on yet another plane with restraint and operate currents defining the plane



Ratios

- Ratios are used to compare two quantities
- They can be expressed as comparisons - 2:3 or as fractions - 2/3
- We see them in transformers – 115kV/12.47kV
- The full description of transformer parameters is:

  a (turns ration) = $N_1 / N_2 = V_1 / V_2 = I_2 / I_1$
- Can be simplified by dividing 115/12.47 = 9.2/1
- Also CTs 2000/5 = 400/1
- And PTs 115,000/115 = 1000/1

Per Unit

- Per unit means putting the base at one
- Like percentage without the factor of 100
- 50% is the same as 0.5 per unit
- A relay value called tap is a base – TOC

  2 x tap = 2 per unit
- Used in some relay settings – differential minimum op and unrestrained are often per unit settings, also directional power in some generator relays

$\frac{100kV}{115kV} = 0.87pu$

Per Unit Example

- 40MVA transformer, 115 kV / 12.47kV
- 40MVA = S_{base}, 115kV = V_{baseHS}, 12.47kV = $V_{base\ LS}$
- Rated current would be

  40,000,000va/115,000 v/ √3 = 602.4 a = I_{baseHS}
- If HS = 100 kV, this would be 0.87 V_{baseHS}

  (100kV/115kV = 0.87pu)

  if HS = 230 kV it would be 2V_{baseHS}

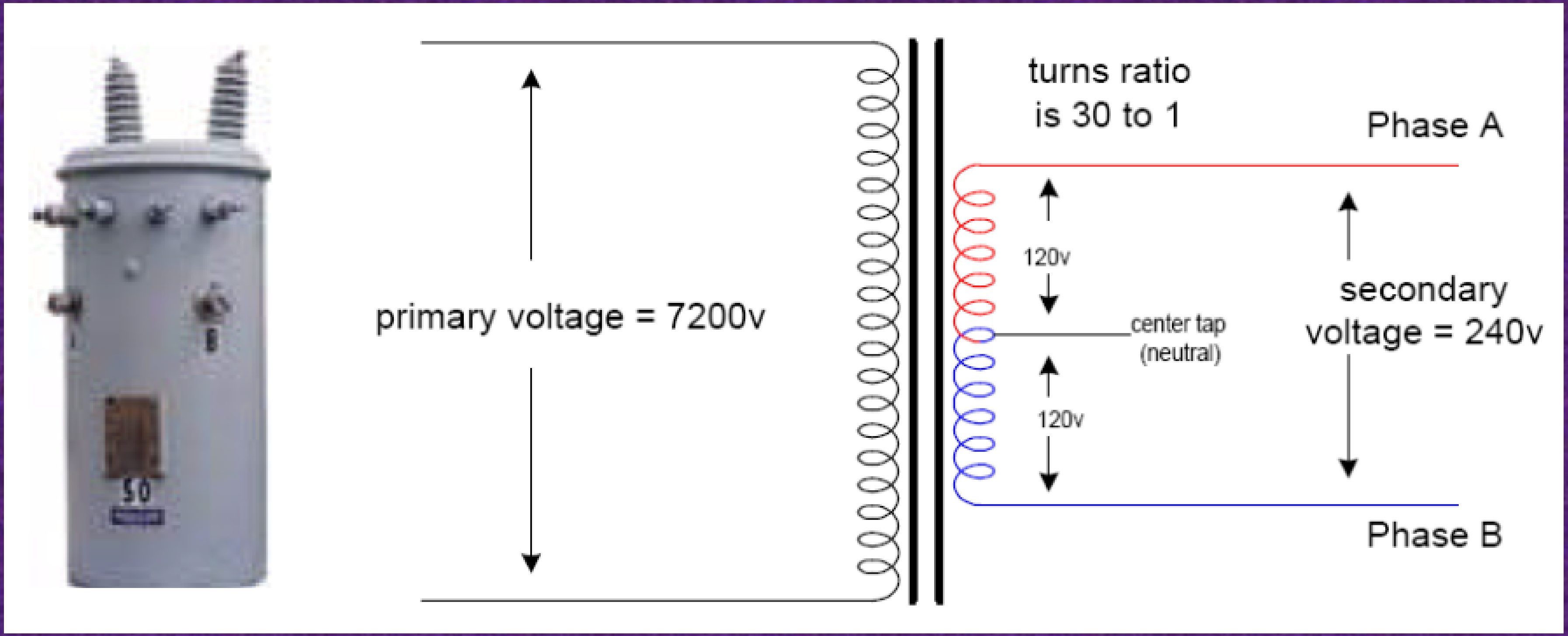
  (230kV/115kV = 2.0pu)
- If you measure 301.2 a, this would be 0.5 I_{baseHS}

  (301.2a/602.4a = 0.5pu)
- If you run it at 110 kV and 50 a, you would see 110 x 50 x √3 = 9.53 MVA or about 0.24 S_{base}

  (9.53MVA/40MVA = 0.24pu)

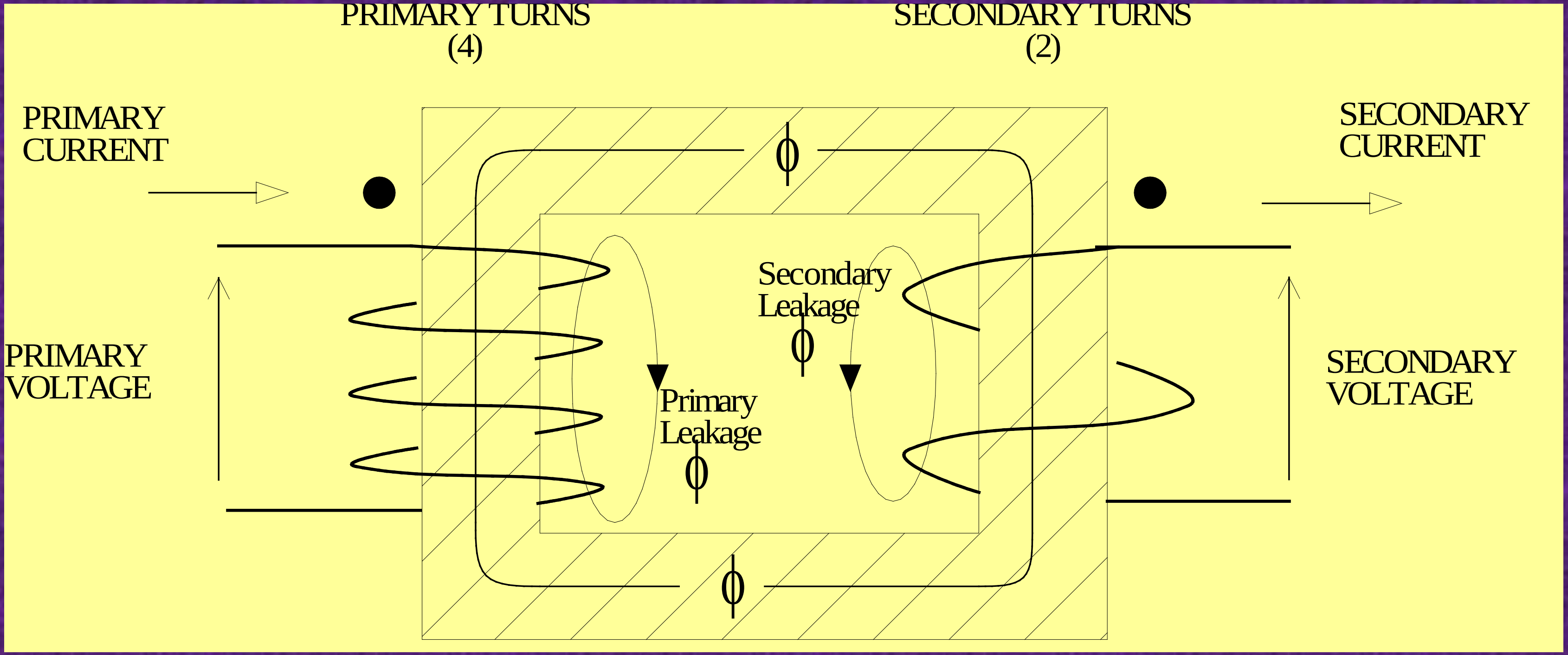
Thank you!!!

Transformer Ratio relationships by Turns, Voltage, and Current



- Turns ratio = Number of primary turns / Number of secondary turns
- V ratio = Turns ratio = $V_{PRIMARY}$ / $V_{SECONDARY}$
- I ratio = 1 / Turns Ratio = $V_{SECONDARY}$ / $V_{PRIMARY}$ = I_{PRI} / I_{SEC}

Instantaneous “phasor” Polarity of current and voltage on a transformer



- Voltage Aligns With Polarity on Both Windings
- Current In Polarity Terminal of One Winding Comes Out the Polarity Terminal of the Other Winding