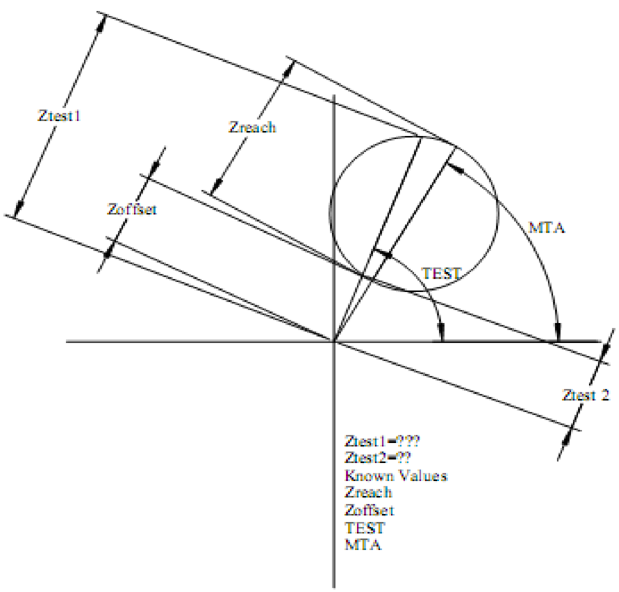
Optimierung Consulting

TITLE: Calculate any point on a circle with Excel

Description:

In electrical engineering, an impedance circle is drawn touching the origin in a graph then rotated by an angle. The formula "Line = Diameter * cos (Angle-LineAngle)" calculates the length of a line drawn from the origin to the edge of the circle at a known angle.
It is possible for the circle to be offset from the origin. From my research, the formula becomes more complex. (line - offset)(line-reach)cos(Angle-LineAngle)=0 or line^2-line(reach+offset)cos(Angle-LineAngle)+reach*offset..
See the attached pdf for more details.
I need a formula that will solve for line at any given angle from 0-360.
I also would like the formula applied to the attached spreadsheet.

Example figure:



Input variables:

$$MTA = \alpha$$
$$TEST = \beta$$
$$Z_{reach} = 2r, r > 0$$
$$Z_{offset} = \sqrt{x_0^2 + y_0^2} - r = d$$

Constraints:

$$x_0 = (d + r)\cos(\alpha) = \left(Z_{offset} + \frac{Z_{reach}}{2}\right)\cos(MTA)$$
$$y_0 = (d + r)\sin(\alpha) = \left(Z_{offset} + \frac{Z_{reach}}{2}\right)\sin(MTA)$$

Solving:

From circumference equation and Ztest line equation we obtain

$$\left.\begin{array}{l} Circumference: (x-x_0)^2+(y-y_0)^2=r^2, |x-x_0|\le r, |y-y_0|\le r \\ Ztestline: y=\tan(\beta)x \end{array}\right\} \Rightarrow \begin{cases} (x-x_0)^2+(\tan(\beta)x-y_0)^2=r^2, |x-x_0|\le r \\ y=\tan(\beta)x, |y-y_0|\le r \end{cases}$$

Rewriten x-equation using input variables we obtain

$$\begin{cases} \left(X_{test}-\left(Z_{offset}+\frac{Z_{reach}}{2}\right)\cos(MTA)\right)^2+\left(\tan(TEST)X_{test}-\left(Z_{offset}+\frac{Z_{reach}}{2}\right)\sin(MTA)\right)^2=\left(\frac{Z_{reach}}{2}\right)^2 \\ \left|X_{test}-\left(Z_{offset}+\frac{Z_{reach}}{2}\right)\cos(MTA)\right|\le\frac{Z_{reach}}{2} \end{cases}$$

$$X_{test}^2\cdot\sec^2(TEST)-2\cdot X_{test}\cdot\left(Z_{offset}+\frac{Z_{reach}}{2}\right)\cdot(\cos(MTA)+\sin(MTA)\tan(TEST))+\left(Z_{offset}^2+Z_{offset}Z_{reach}\right)=0$$

$$X_{test}^2-2\cdot X_{test}\cdot\left(Z_{offset}+\frac{Z_{reach}}{2}\right)\cdot\cos(TEST)\cdot\cos(MTA-TEST)+\left(Z_{offset}^2+Z_{offset}Z_{reach}\right)\cdot\cos^2(TEST)=0$$

Solving x-equation and applying this result in y-equation we obtain

$$X_{test1,2}=\cos(TEST)\cdot\left(\left(Z_{offset}+\frac{Z_{reach}}{2}\right)\cdot\cos(MTA-TEST)\pm\sqrt{\left(\left(Z_{offset}+\frac{Z_{reach}}{2}\right)\cdot\cos(MTA-TEST)\right)^2-\left(Z_{offset}^2+Z_{offset}Z_{reach}\right)}\right)$$

$$Y_{test1,2}=\sin(TEST)\cdot\left(\left(Z_{offset}+\frac{Z_{reach}}{2}\right)\cdot\cos(MTA-TEST)\pm\sqrt{\left(\left(Z_{offset}+\frac{Z_{reach}}{2}\right)\cdot\cos(MTA-TEST)\right)^2-\left(Z_{offset}^2+Z_{offset}Z_{reach}\right)}\right)$$

Solution:

Then, if the results above are real numbers, it will be possible to obtain *Ztest* from

$$Z_{test1,2}=\left|\left(Z_{offset}+\frac{Z_{reach}}{2}\right)\cdot\cos(MTA-TEST)\pm\sqrt{\left(\left(Z_{offset}+\frac{Z_{reach}}{2}\right)\cdot\cos(MTA-TEST)\right)^2-\left(Z_{offset}^2+Z_{offset}Z_{reach}\right)}\right|$$

Ztest values only will be a solution if Xtest and Ytest values are on circumference line, which can be checked by

$$\sqrt{\left(a\cdot X_{test1,2}-\left(Z_{offset}+\frac{Z_{reach}}{2}\right)\cos(MTA)\right)^2+\left(a\cdot Y_{test1,2}-\left(Z_{offset}+\frac{Z_{reach}}{2}\right)\sin(MTA)\right)^2}=\frac{Z_{reach}}{2}$$

$$a=\begin{cases} -1, \text{for only } X_{test,2} \text{ and } Y_{test,2} \text{ and } \left|Z_{offset}-\frac{Z_{reach}}{2}\right|\le\left|Z_{offset}-\frac{Z_{reach}}{2}\right| \\ 1, otherwise \end{cases}$$

The auxiliary a-values comes from that if $(0,0)$-point is inside the circumference then $Ztest\,1$ and $Ztest\,2$ will be in opposite phases, otherwise they will be in phase.

Verification:

Input variables:

Zreach:	3.0	unit
MTA:	60	deg
Zoffset:	-1.0	unit
Test:	150	deg

Solution:

Ztest1 =	1.4	unit
Ztest2 =	1.4	unit

Graph's points

Zoffset		Ztest1	1.41
x	y	x	y
0.0	0.0	0.0	0.0
-0.5	-0.9	-1.2	0.7
Zreach		Ztest2	1.41
x	y	x	y
-0.5	-0.9	0.0	0.0
1.0	1.7	1.2	-0.7
x0 =		0.3 unit	
y0 =		0.4 unit	

Graph