

# Models of Perceptual Uncertainty and Decision Making

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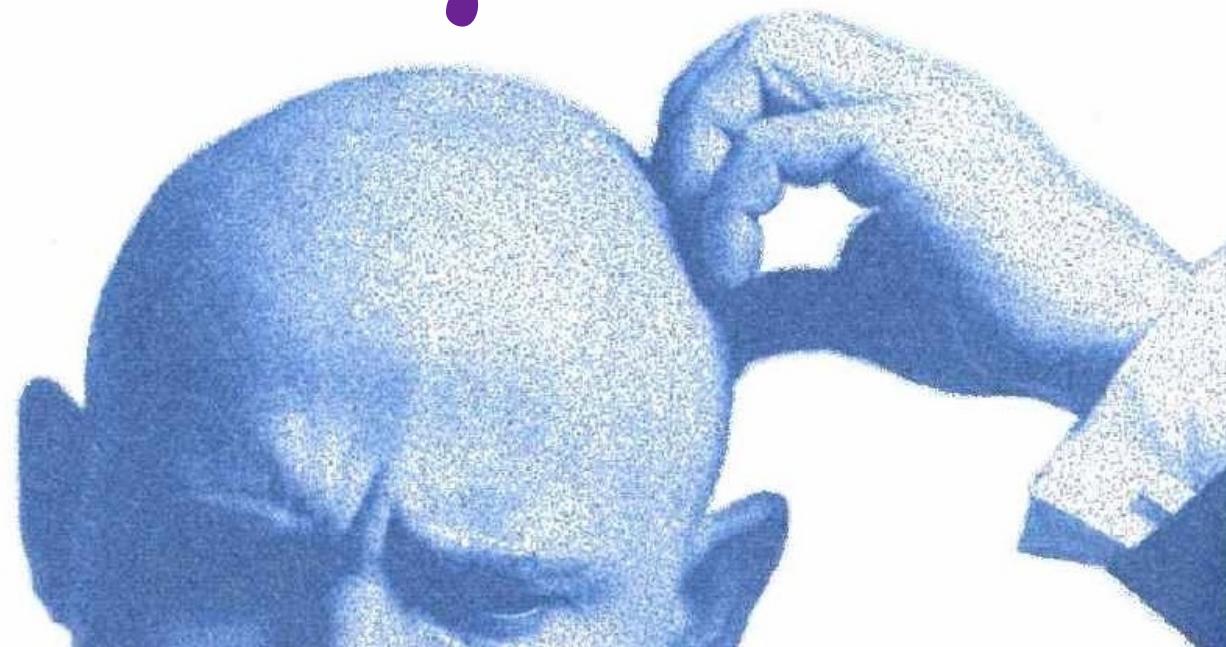
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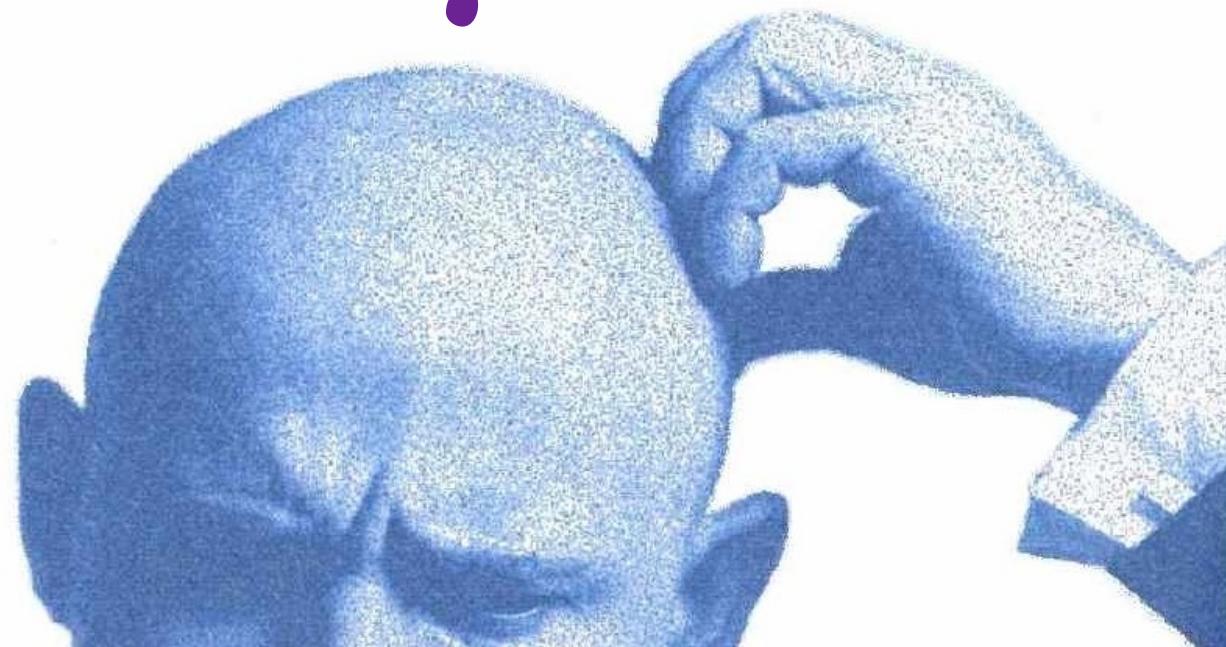
- How do we integrate sensory percepts?
- How do we weigh ambiguous evidence?

?



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- How do we weigh ambiguous evidence?

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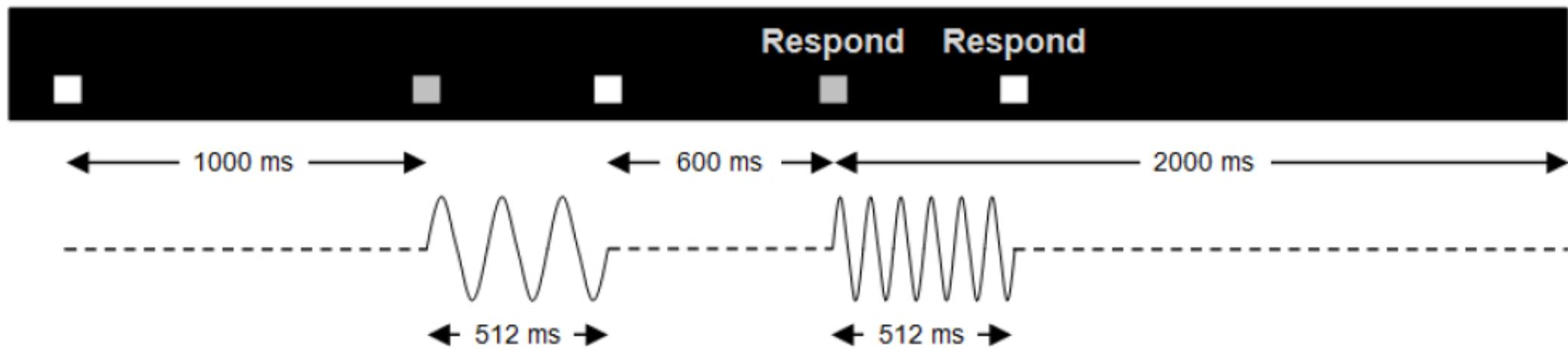


# Vibrotactile experiment: perceptual decision making



# Perceptual decision-making task

## Vibrotactile discrimination task



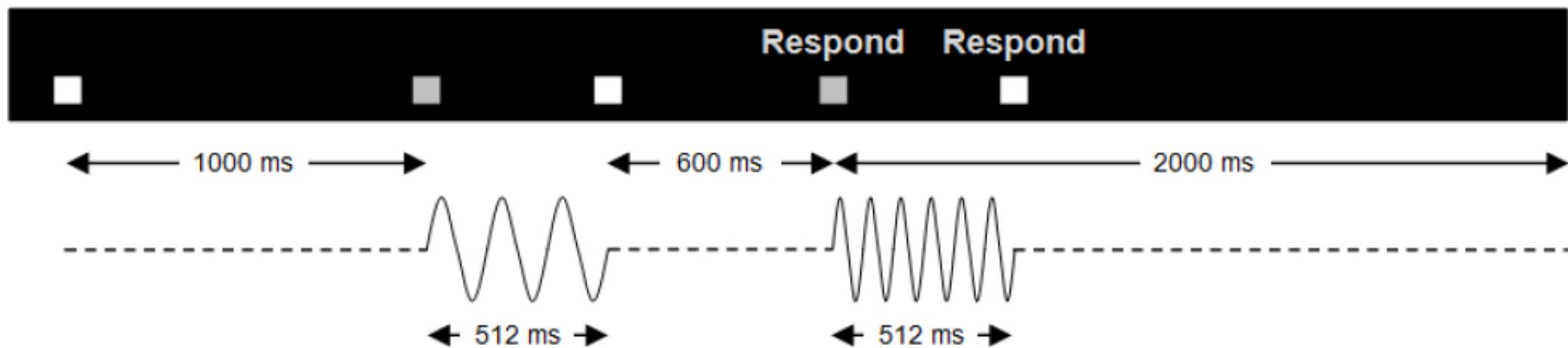
Two-alternative forced choice:

1: Same or Different? S D

2:  $f_2 > f_1$ ?

# Perceptual decision-making task

## Vibrotactile discrimination task



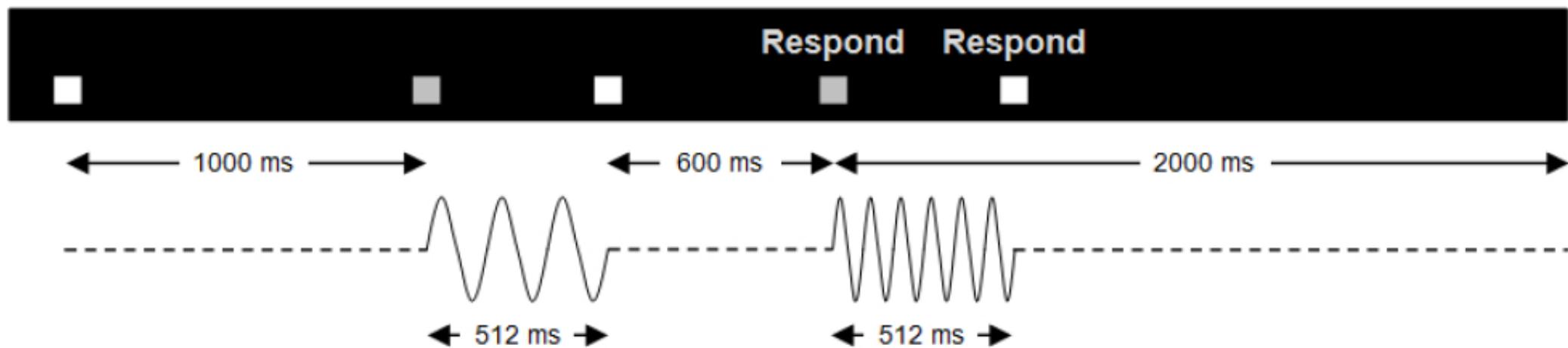
Two-alternative forced choice:

1: Same or Different?

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# Perceptual decision-making task

## Vibrotactile discrimination task



Two-alternative forced choice:

- 1: Same or Different? S D  
2: f<sub>2</sub>>f<sub>1</sub>? Y N

Subtraction

Subtraction weighted by precision of stimulus representation

# outline

- Same or Different? S D
  - Effective connectivity
  - Behavioral, fMRI, DCM
- $f_2 > f_1?$  Y N
  - Time-order effect
  - Behavioral, Bayesian approach

# Vibrotactile discrimination task

## Experiment 1: Same or Different?

Conditions:

1. Different or same frequencies – adaptation  
repetition suppression



# Vibrotactile discrimination task

Conditions:

1. Different or same frequencies – adaptation
2. Pure or noisy sinusoidal shape –

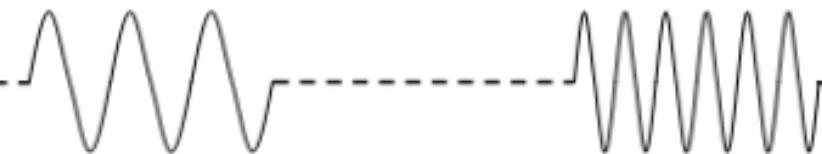
what is the role of noise in the input frequency?



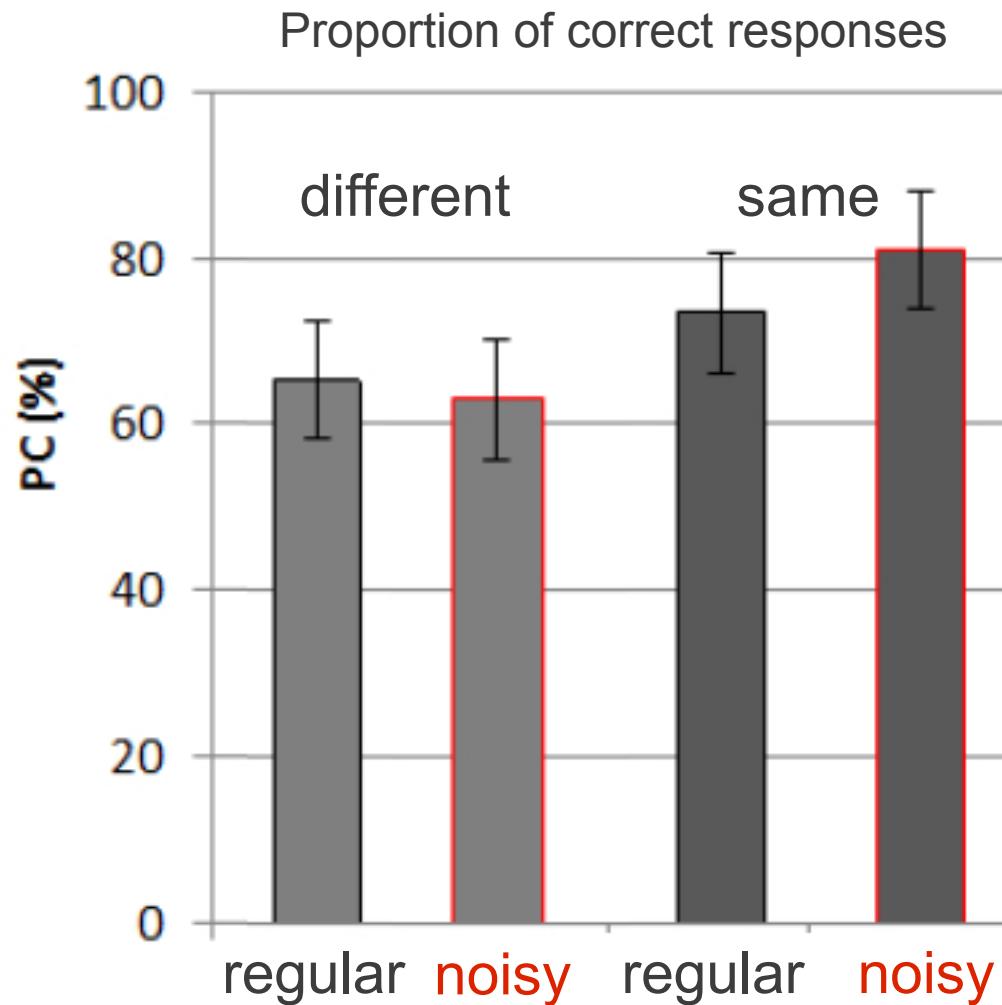
# Vibrotactile discrimination task

Conditions:

1. Different or same frequencies – adaptation
2. Pure or noisy sinusoidal shape –  
**what is the role of noise in the input frequency?**



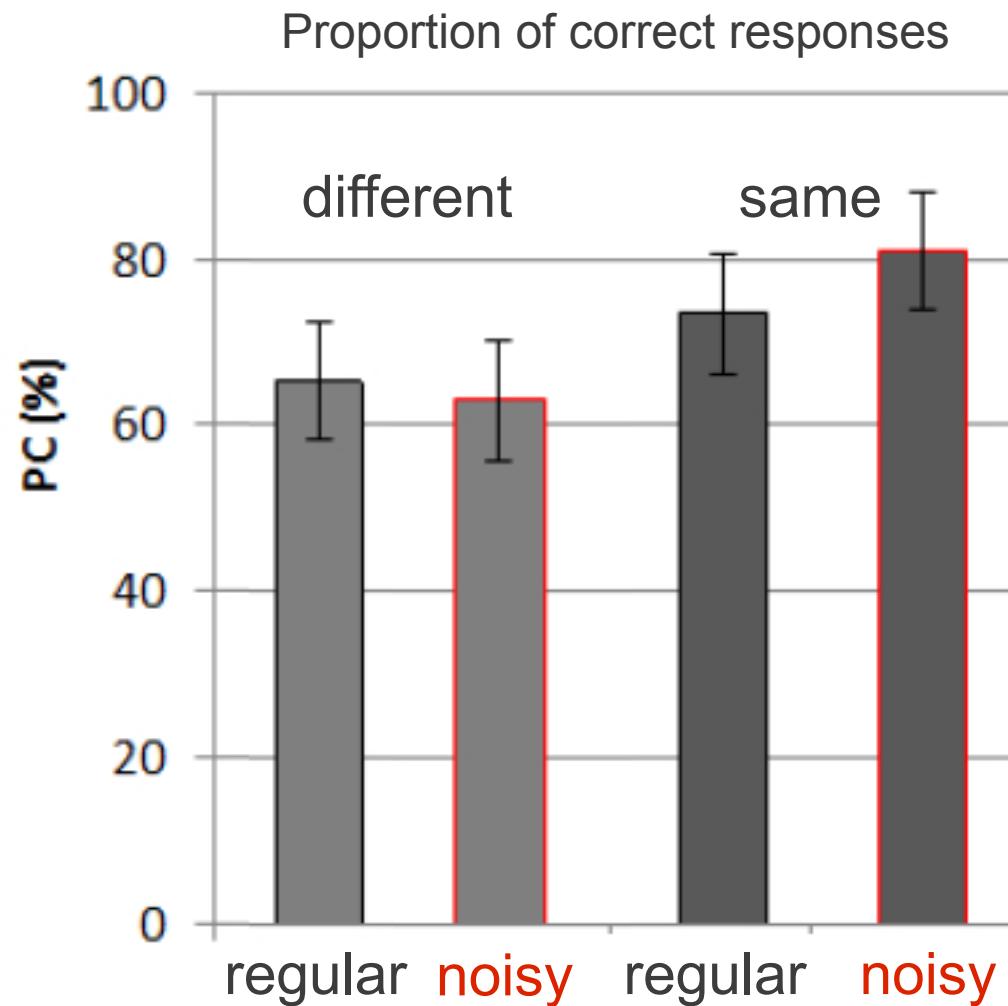
# Behavioral data



Noise reduces precision of  
stimulus representation

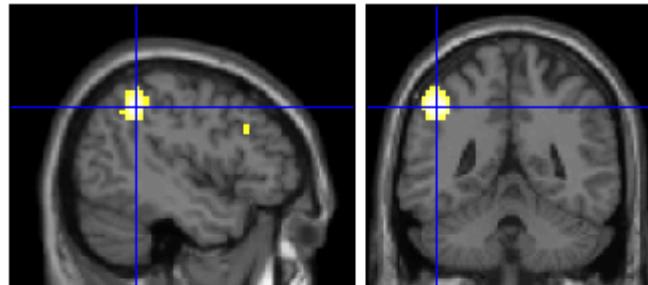
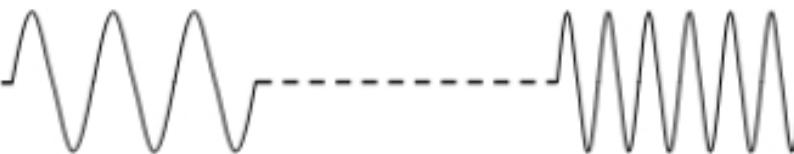
## Role of input noise

### Behavioral data

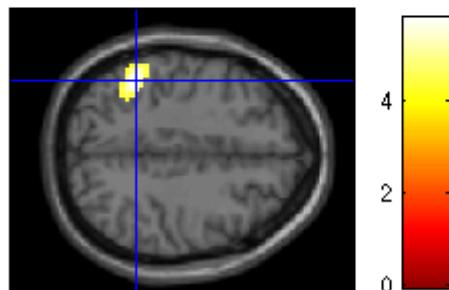


Noise degrades the perception of different trials and improves the perception of same trials (interaction:  $p=0.0007$ ,  $F_{1,15}=17.927$ )

# Perceptual decision-making task



Inferior parietal lobe

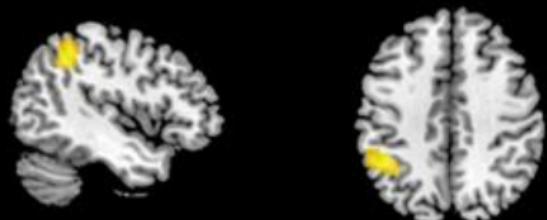


Greater activity for different  
than for same input frequencies

Different > Same

A. Left Inferior Parietal Lobe

IPL (x1)



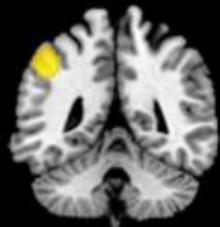
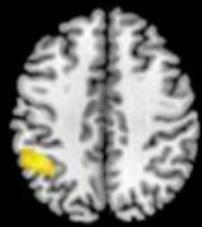
Inferior parietal lobe

Different > Same

Different > Same

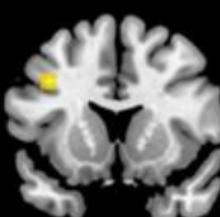
A. Left Inferior Parietal Lobe

IPL (x1)



B. Left Frontal Middle Gyrus

DLPFC (x2)

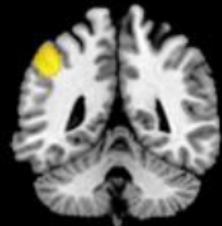
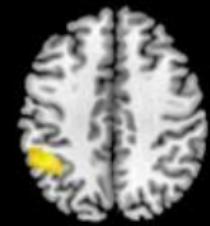


Dorsal lateral prefrontal cortex

Different > Same

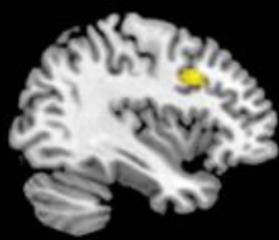
## Different > Same

A. Left Inferior Parietal Lobe

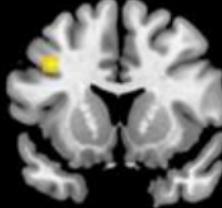


IPL (x1)

B. Left Frontal Middle Gyrus

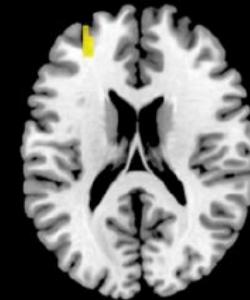
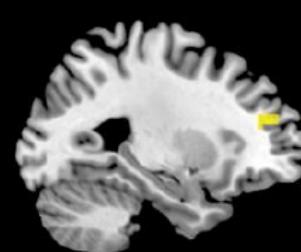


DLPFC (x2)



## Regular > Noisy

Left Frontal Middle Gyrus (rostral PFC)



rPFC (x4)

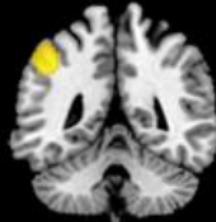
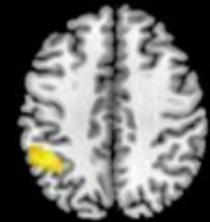


Rostral prefrontal cortex

Regular > Noisy

### Different > Same

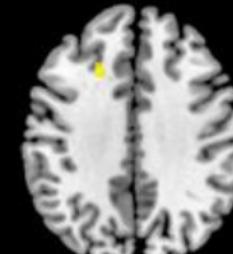
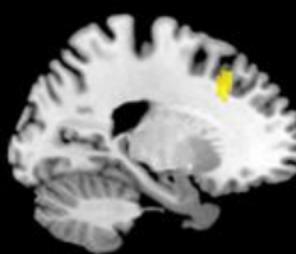
A. Left Inferior Parietal Lobe



IPL (x1)

### Noise × Difference Interaction

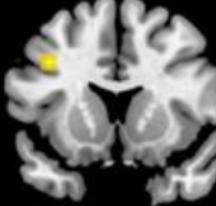
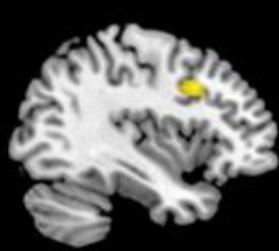
Left Superior Frontal Gyrus



SFG(x3)

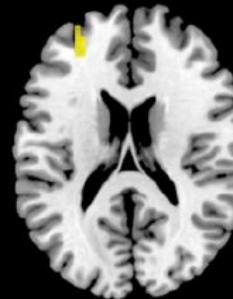
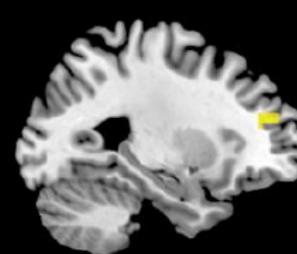
B. Left Frontal Middle Gyrus

DLPFC (x2)

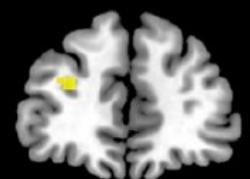


### Regular > Noisy

Left Frontal Middle Gyrus (rostral PFC)



rPFC (x4)



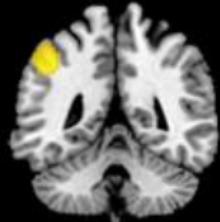
Superior frontal gyrus

Interaction effect

### Different > Same

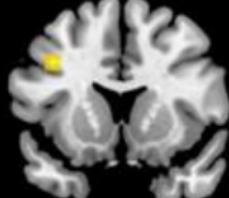
A. Left Inferior Parietal Lobe

IPL (x1)



B. Left Frontal Middle Gyrus

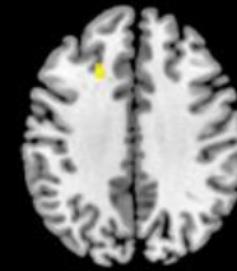
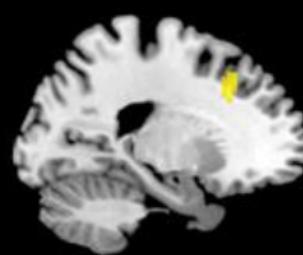
DLPFC (x2)



### Noise × Difference Interaction

Left Superior Frontal Gyrus

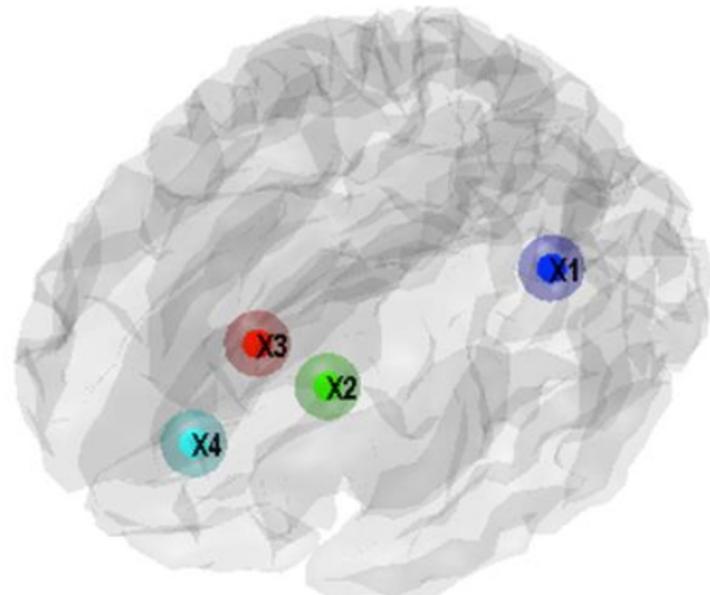
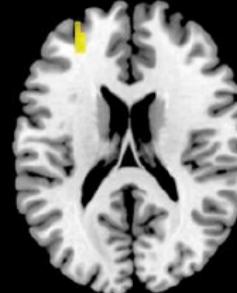
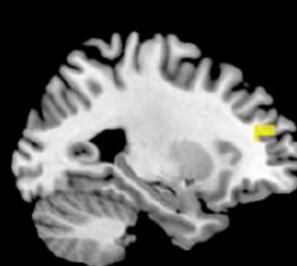
SFG(x3)



### Regular > Noisy

Left Frontal Middle Gyrus (rostral PFC)

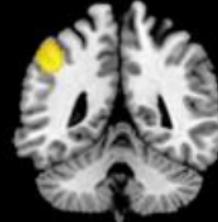
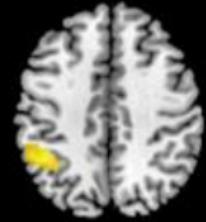
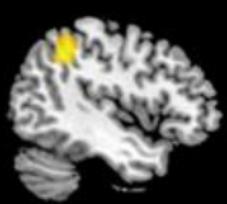
rPFC (x4)



### Different > Same

A. Left Inferior Parietal Lobe

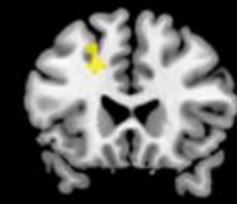
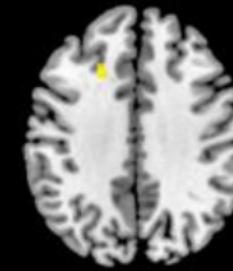
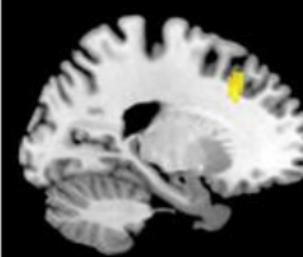
IPL (x1)



SFG(x3)

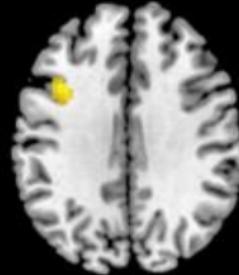
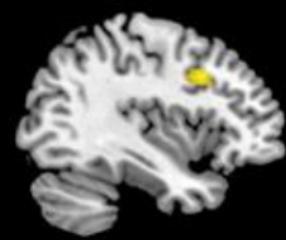
### Noise × Difference Interaction

Left Superior Frontal Gyrus



B. Left Frontal Middle Gyrus

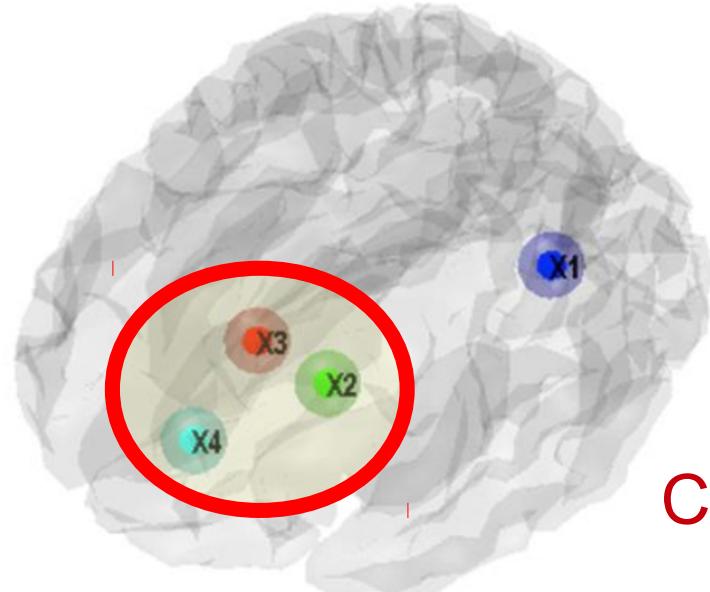
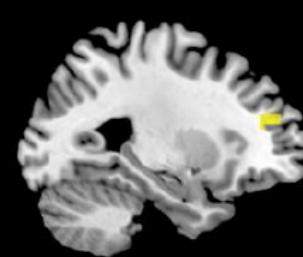
DLPFC (x2)



rPFC (x4)

### Regular > Noisy

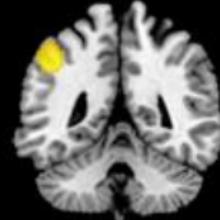
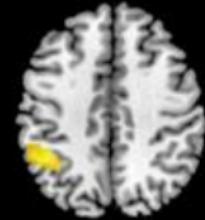
Left Frontal Middle Gyrus (rostral PFC)



Constellation of prefrontal regions

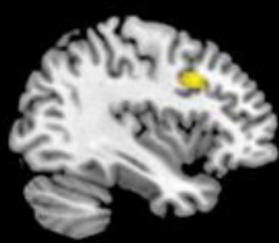
### Different > Same

A. Left Inferior Parietal Lobe

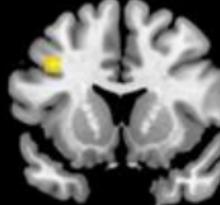


IPL (x1)

B. Left Frontal Middle Gyrus

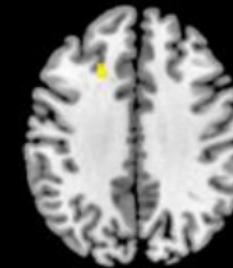
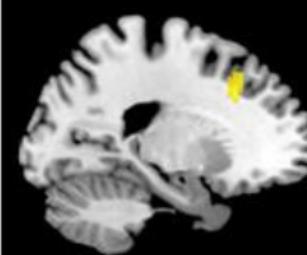


DLPFC (x2)



### Noise × Difference Interaction

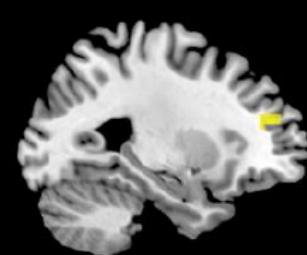
Left Superior Frontal Gyrus



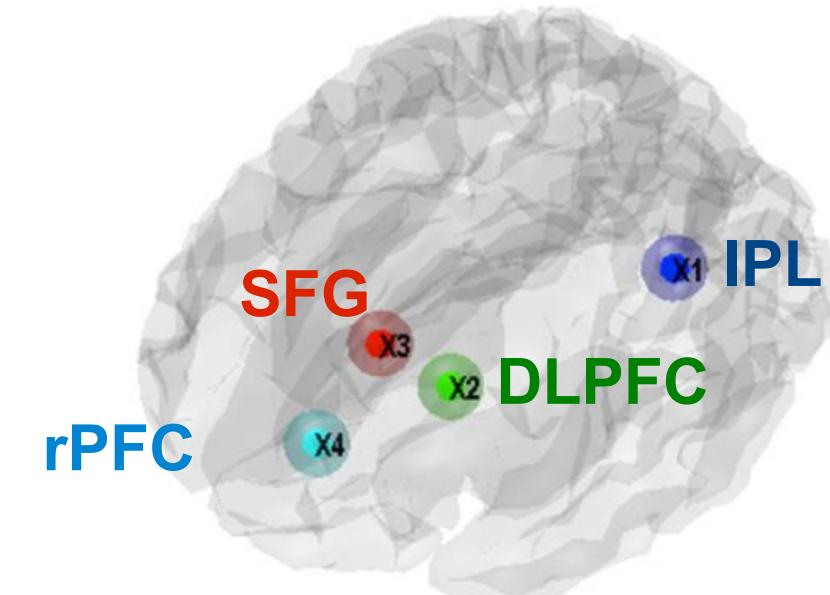
SFG(x3)

### Regular > Noisy

Left Frontal Middle Gyrus (rostral PFC)

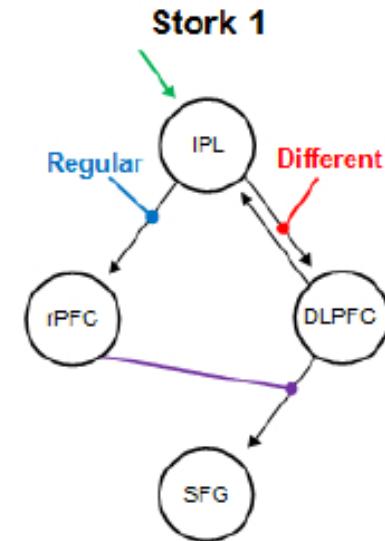
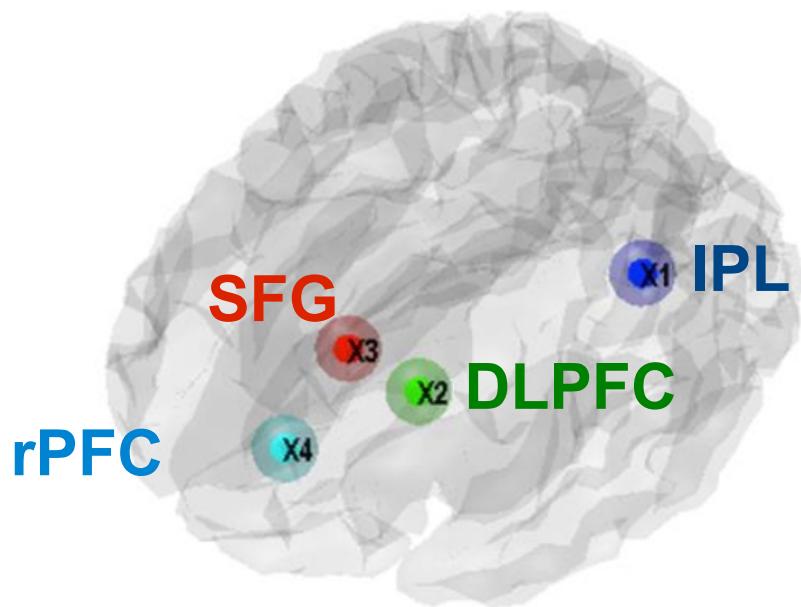


rPFC (x4)



How do these regions interact?

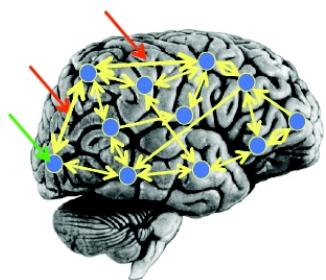
# Effective connectivity: Dynamic causal modelling



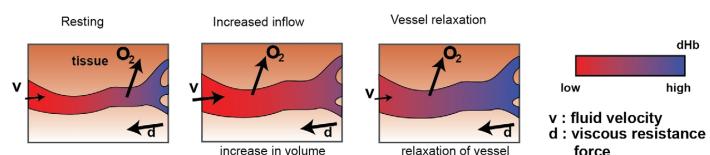
How do these regions interact?

# Dynamic causal modelling

Model



$$\dot{x} = F(x, u, \theta) \quad \text{neural state equation}$$



$$y = H(x, u, \theta) + \epsilon$$

hemodynamic state equations

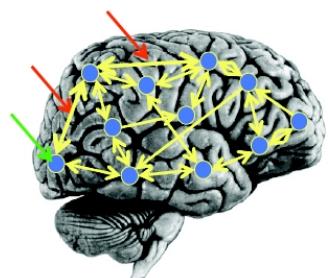
Prediction

estimated BOLD response

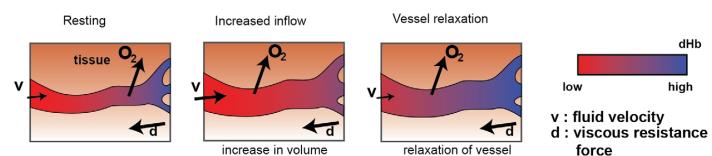
forward model

# Dynamic causal modelling

Model



$$\dot{x} = F(x, u, \theta)$$



$$y = H(x, u, \theta) + \epsilon$$

Prediction

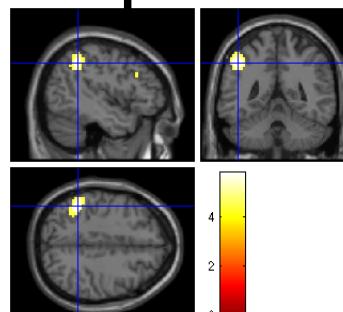
forward model

Model evidence

$$p(y|m) = \int p(y|\theta, m) p(\theta|m) d\theta$$

Probability of observing the data  $y$  given the model  $m$

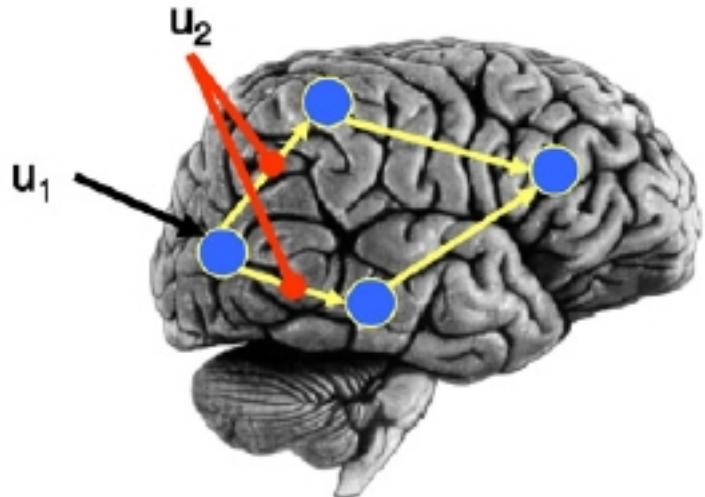
variational Bayes



Empirical effect

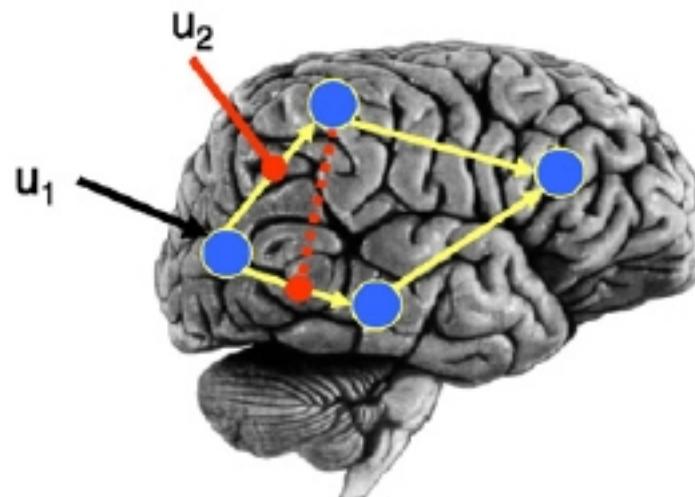
model inversion

## bilinear DCM



Bilinear state equation

## nonlinear DCM



Nonlinear state equation

$$\frac{dx}{dt} = \left( A + \sum_{i=1}^m u_i B^{(i)} \right) x + C u$$

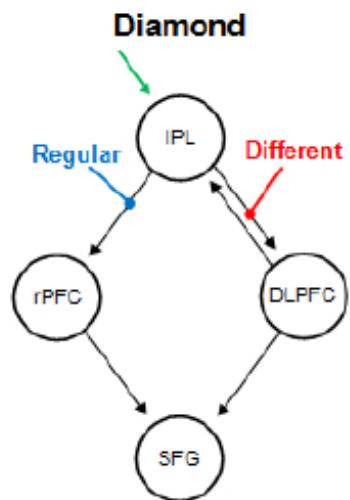
state changes  
modulation  
input  
effective connectivity

$$\frac{dx}{dt} = \left( A + \sum_{i=1}^m u_i B^{(i)} + \sum_{j=1}^n x_j D^{(j)} \right) x + C u$$

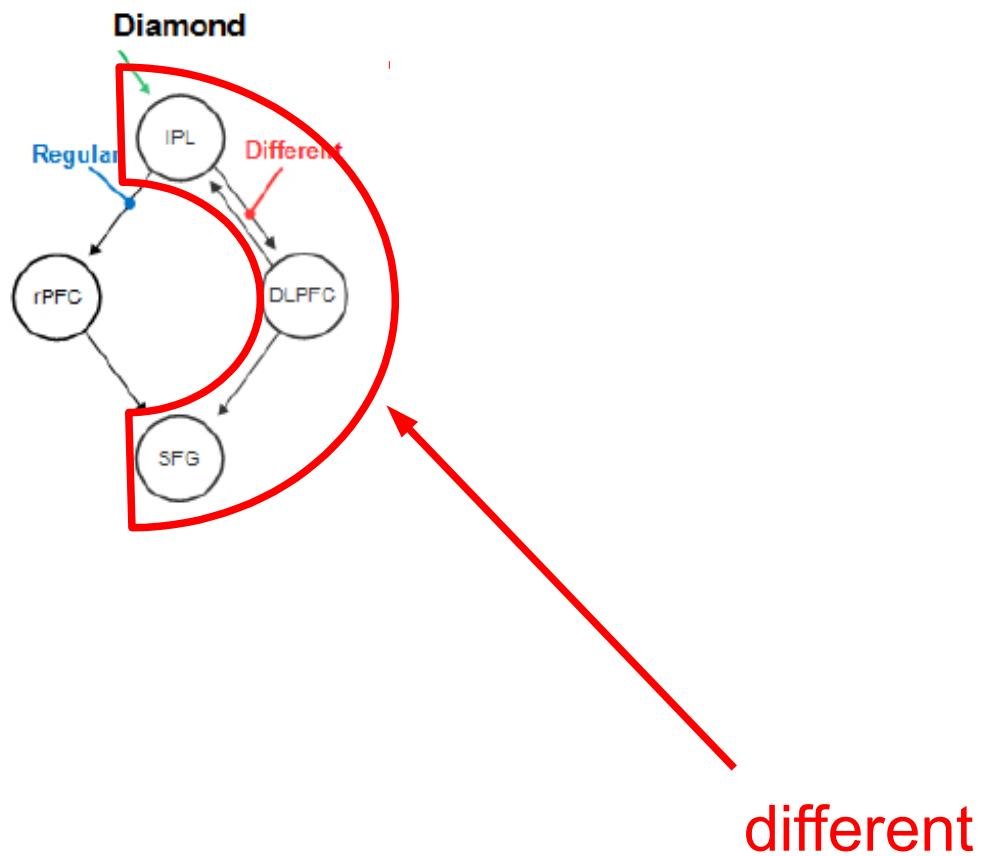
voltage dependent gating  
(via NMDA receptors)

Stephan et al. (2008) Neuroimage

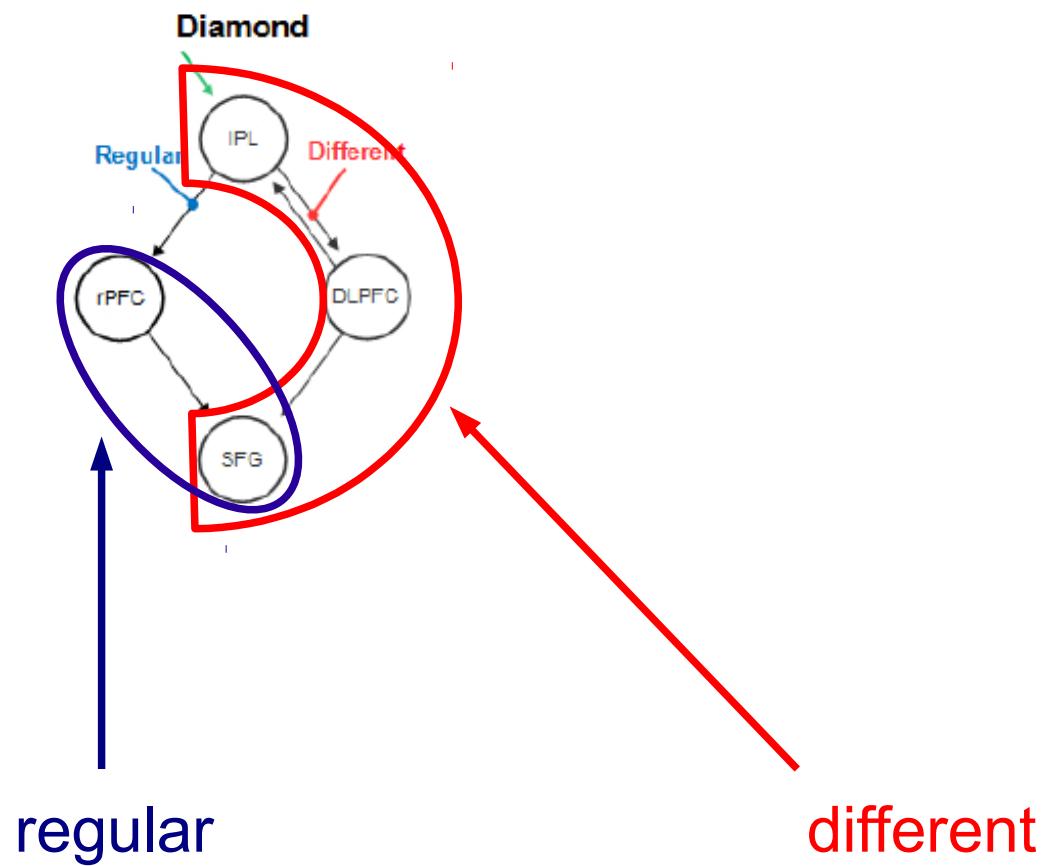
# Set of minimal models



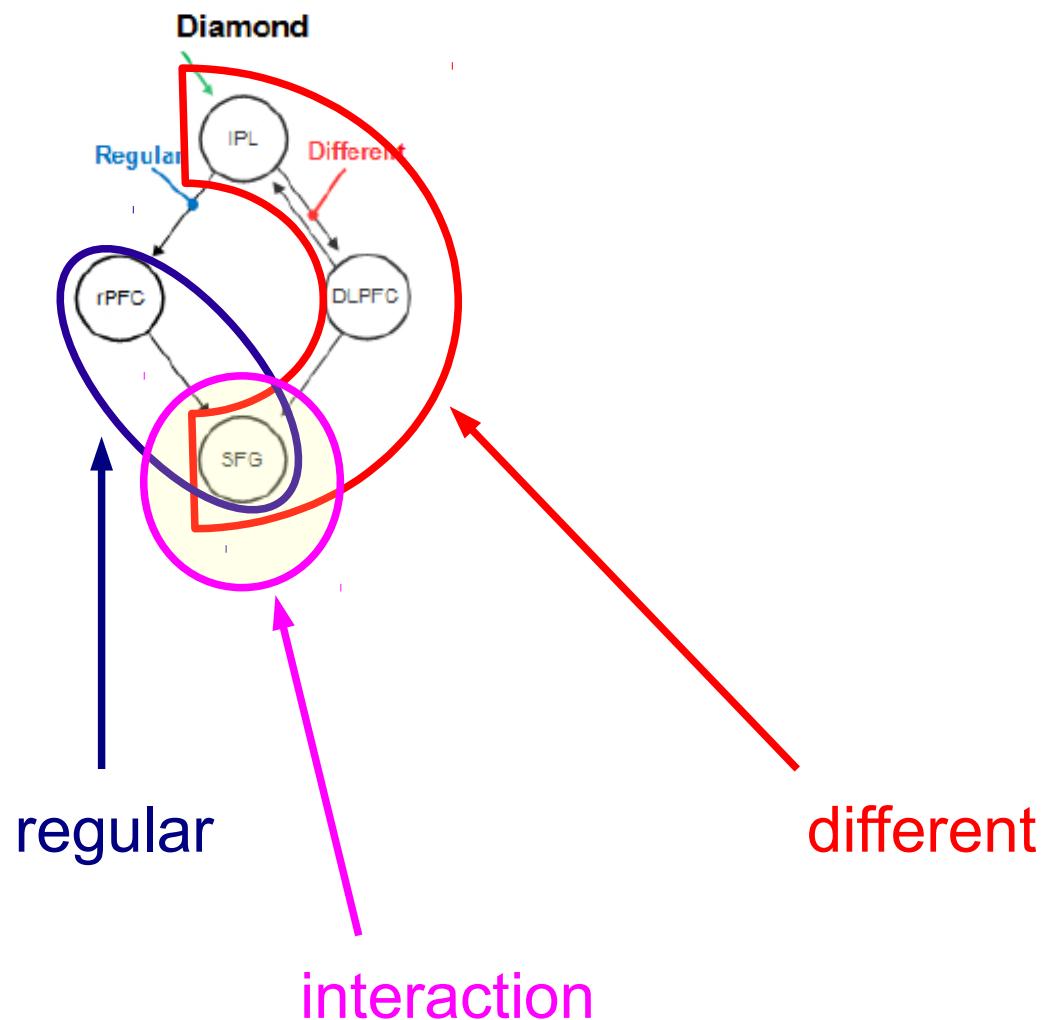
# Set of minimal models



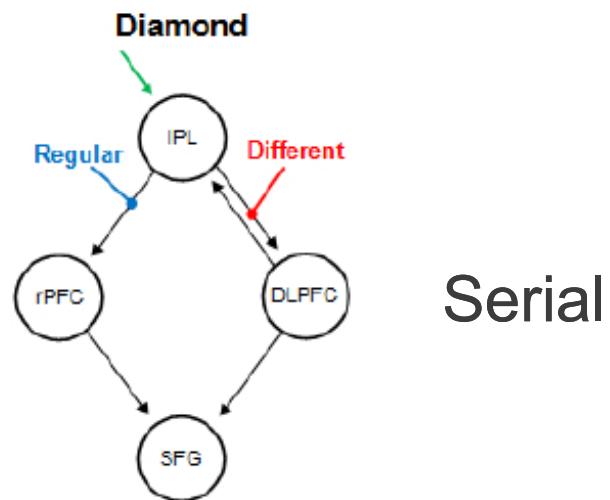
# Set of minimal models



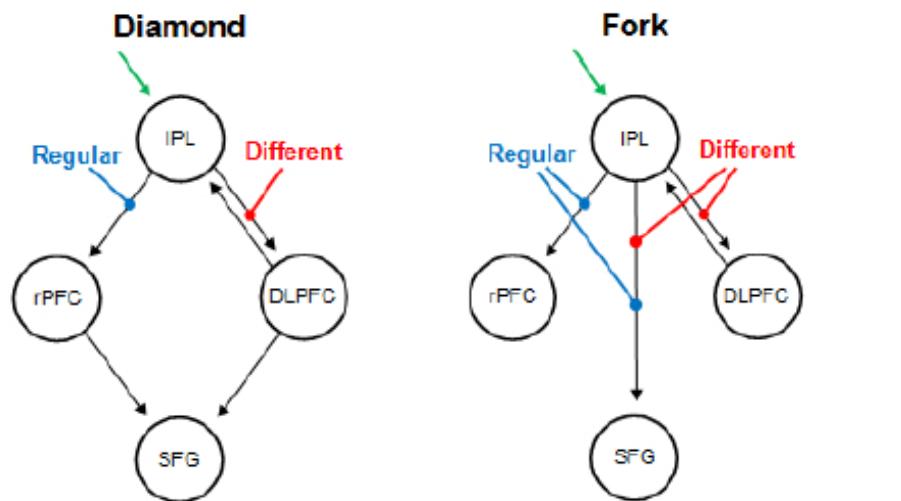
# Set of minimal models



# Set of minimal models

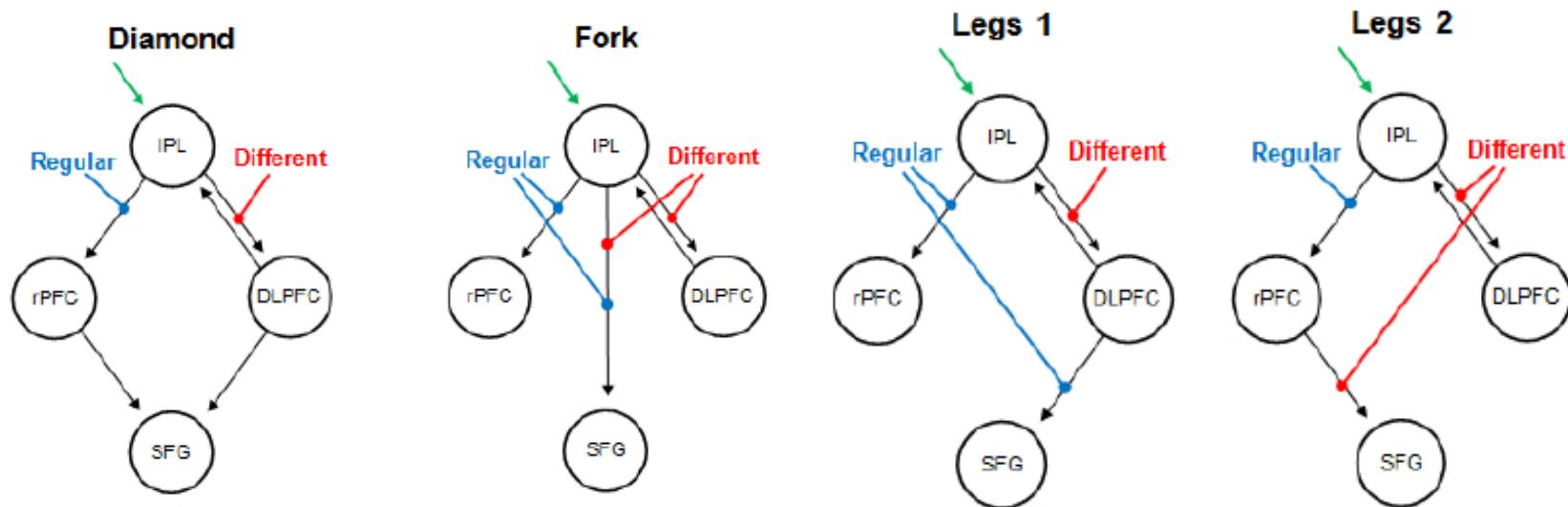


# Set of minimal models



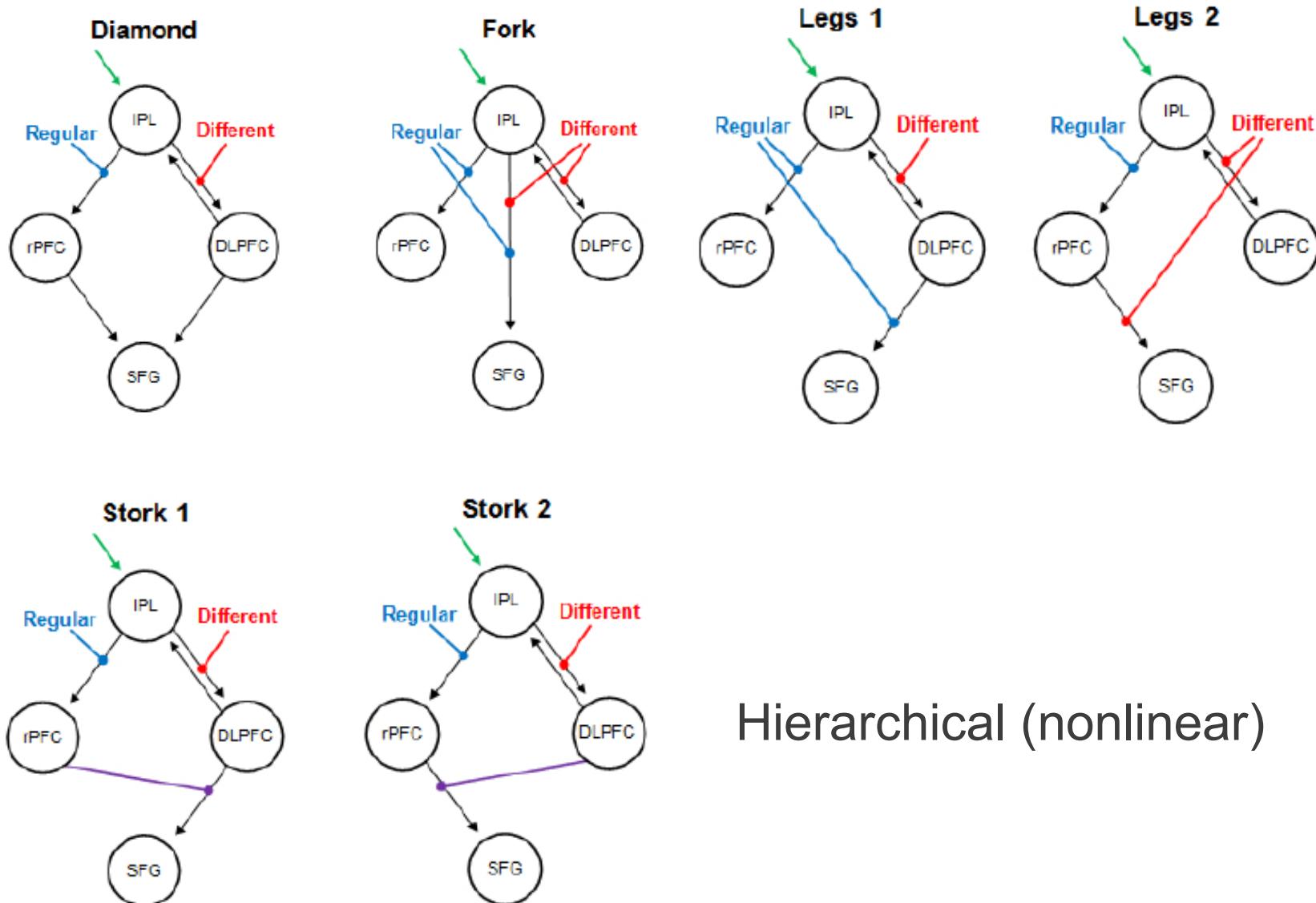
Parallel

# Set of minimal models



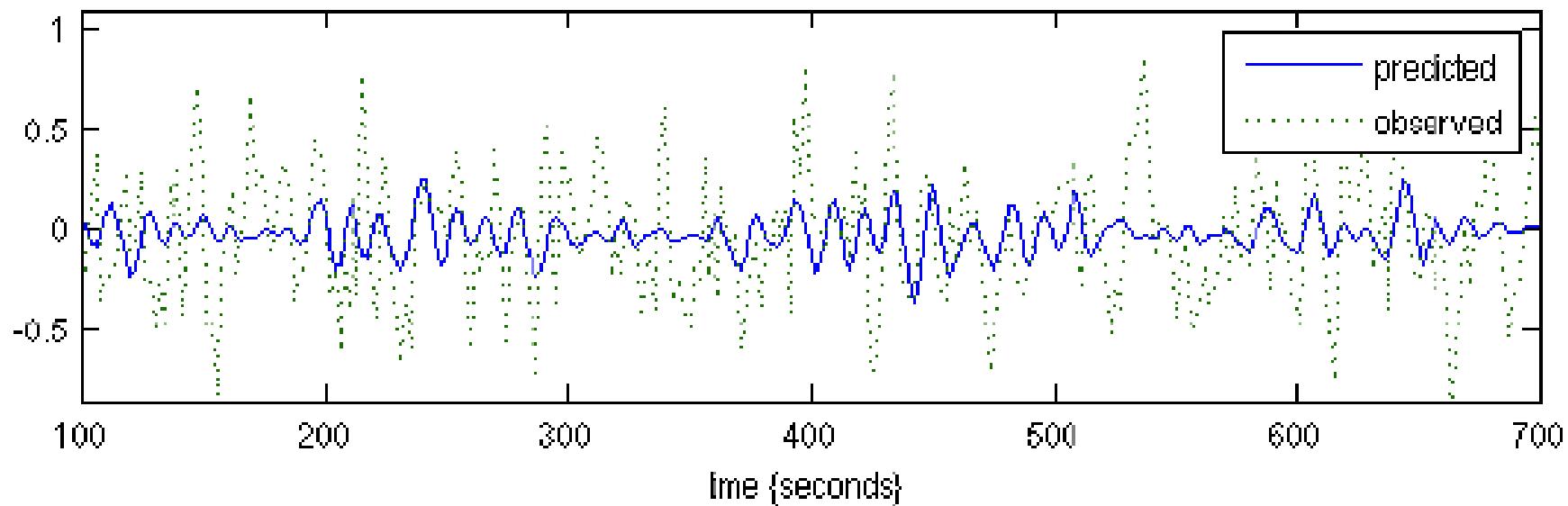
Hierarchical (bilinear)

# Set of minimal models



# Example time series

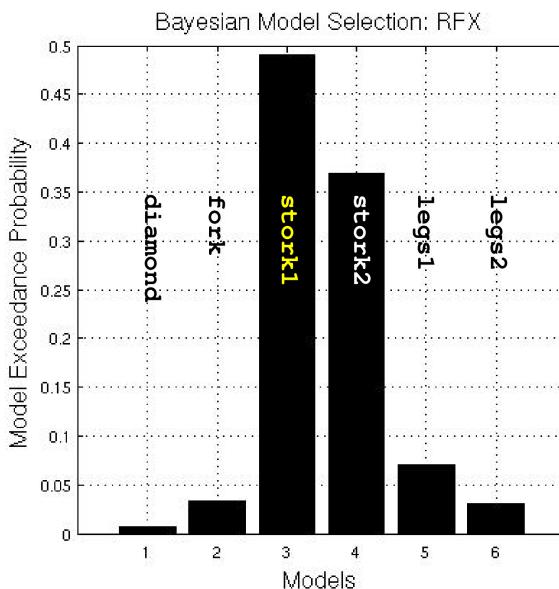
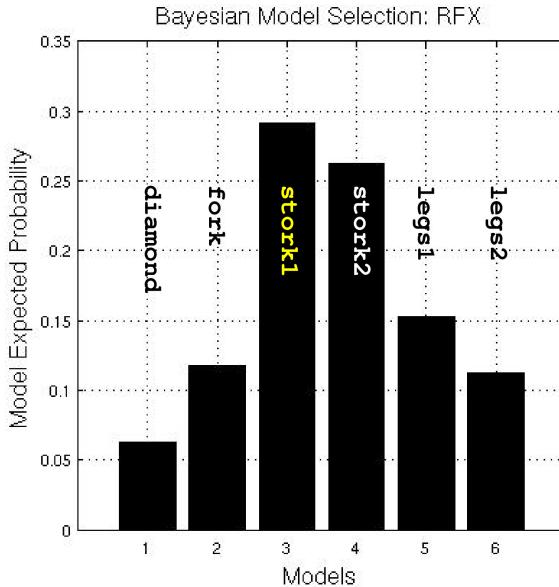
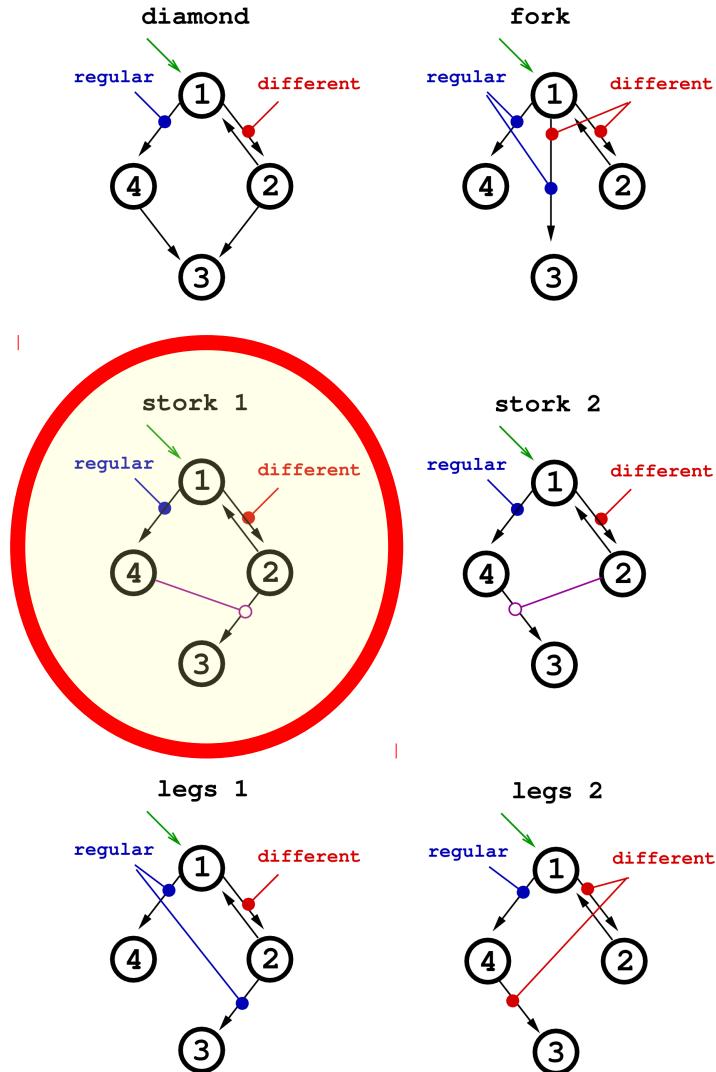
X4: responses and predictions



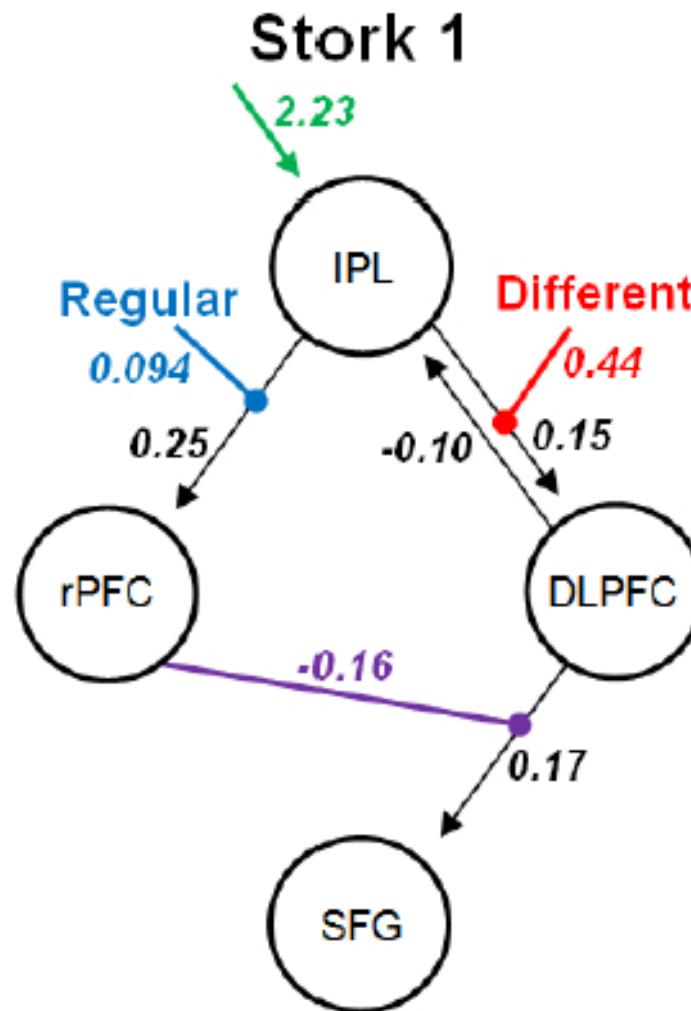
Expectation maximisation optimises the posterior likelihood  
of each model given the data

Stork 1

# Model comparison

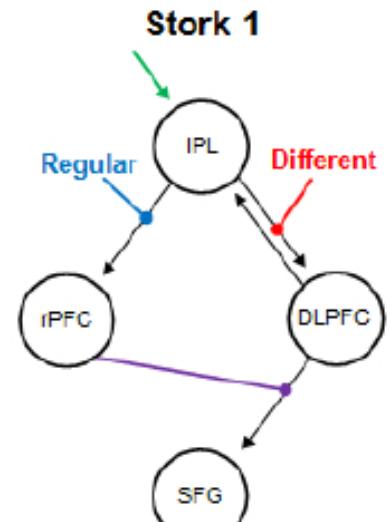
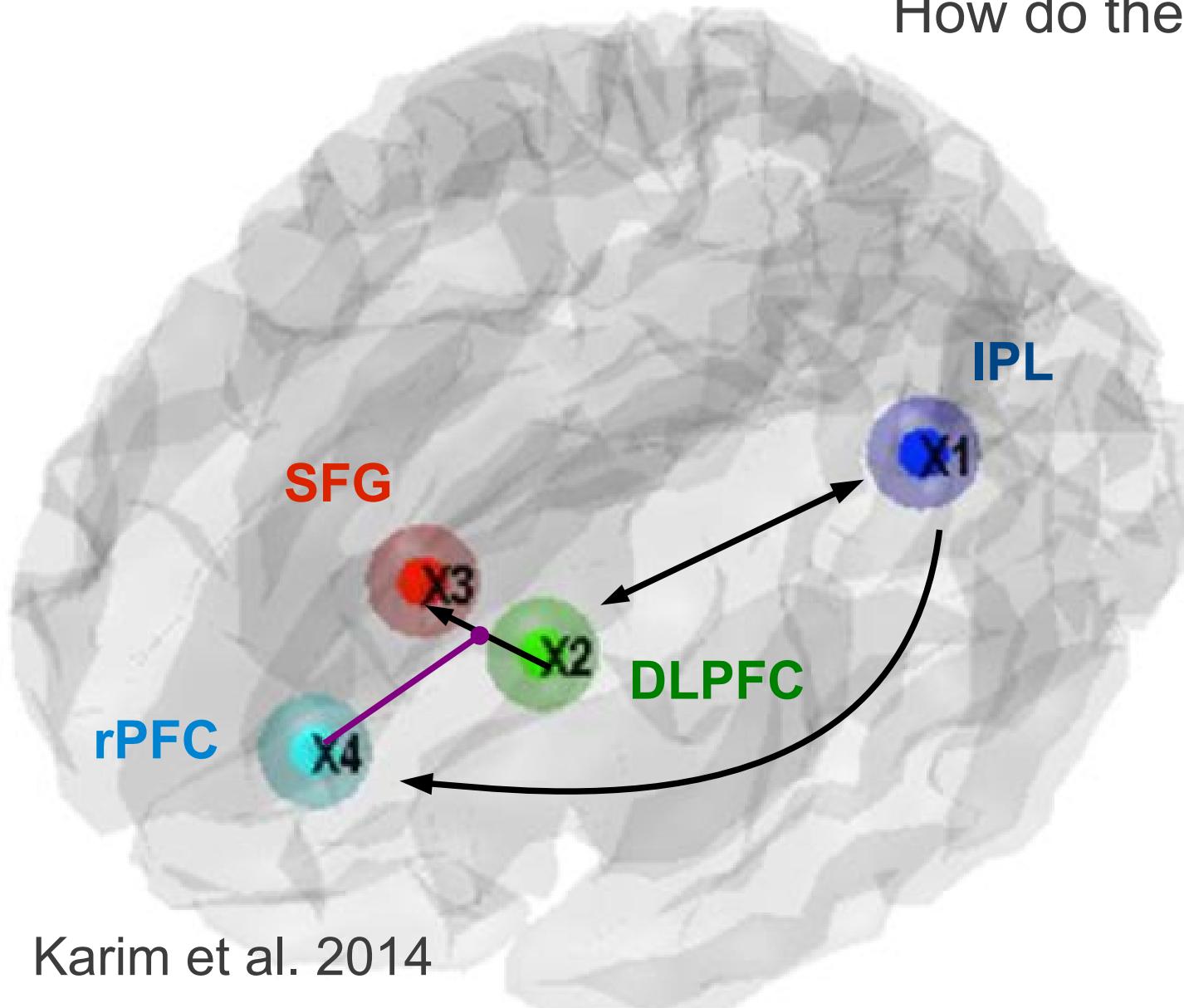


# The nonlinear hierarchical model which best describes the data



# The nonlinear hierarchical model which best describes the data

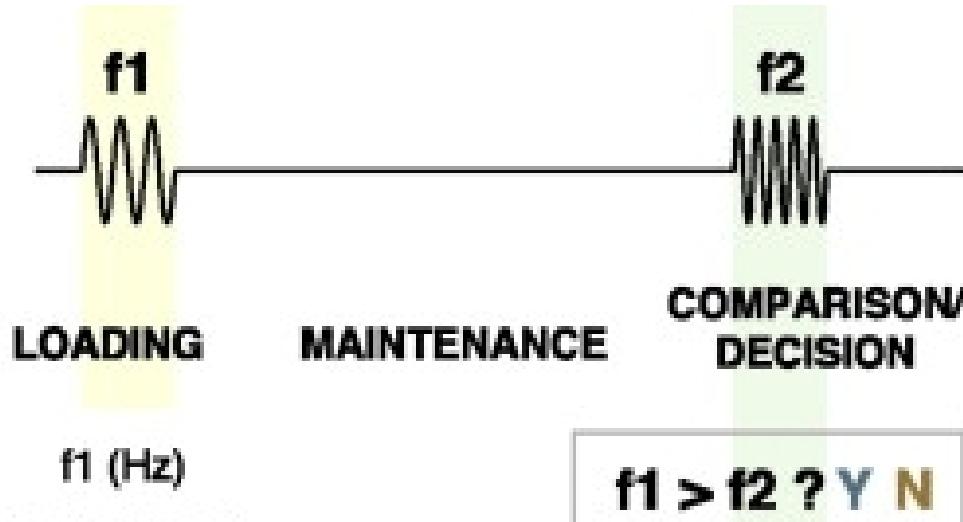
How do these regions interact?



# outline

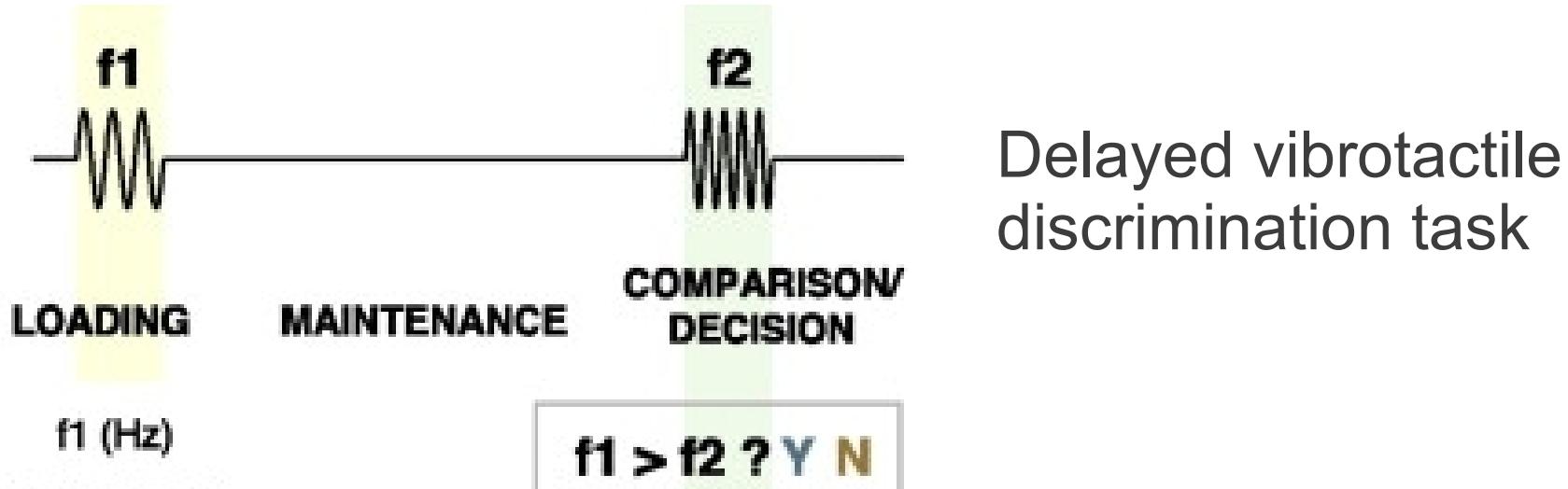
- Same or Different? Effective connectivity  
Behavioral, fMRI, DCM
- $f_2 > f_1$ ? Time-order effect  
Behavioral, Bayesian approach

# Modeling time-order effect



Delayed vibrotactile  
discrimination task

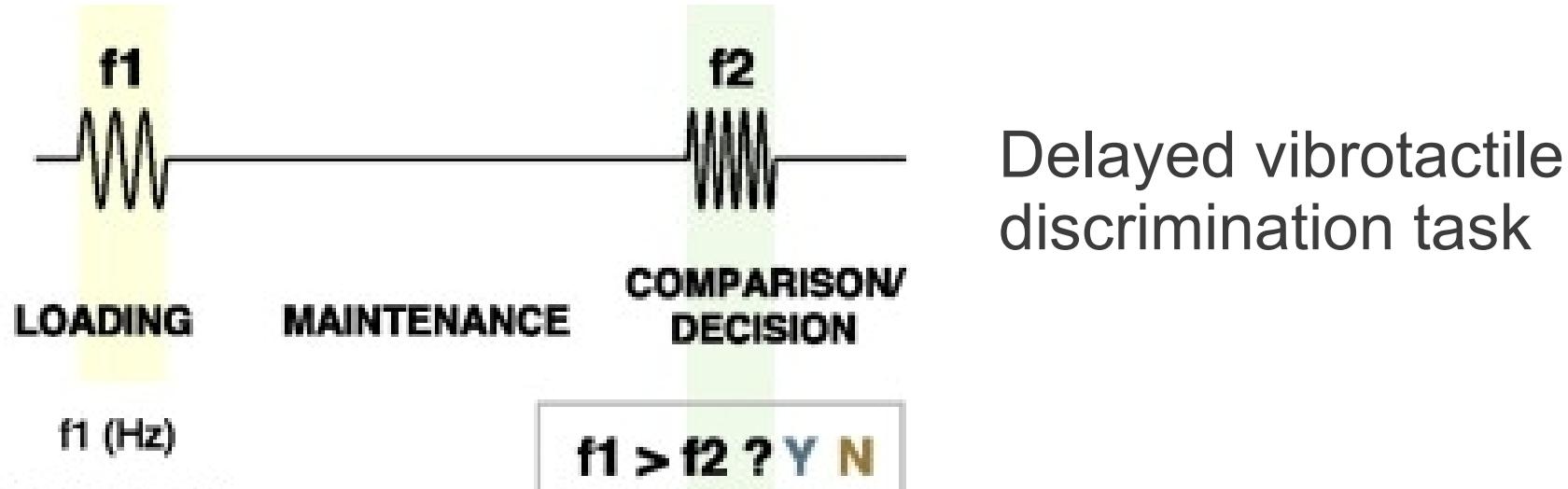
# Modeling time-order effect



$f_1=A$ ;  $f_2=B$   $\longrightarrow$  Accuracy AB

$f_1=B$ ;  $f_2=A$   $\longrightarrow$  Accuracy BA  $\neq$  Accuracy AB

# Modeling time-order effect

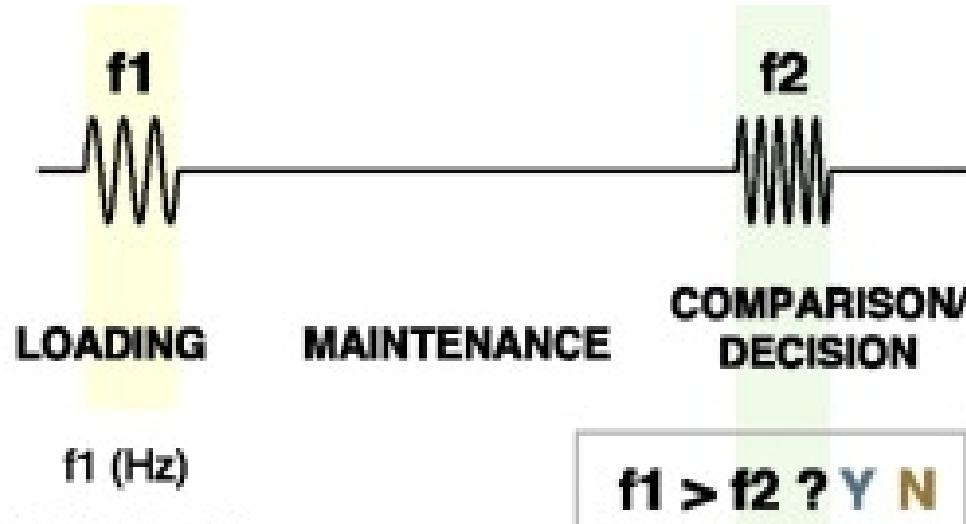


$f_1=A; f_2=B \longrightarrow$  Accuracy AB

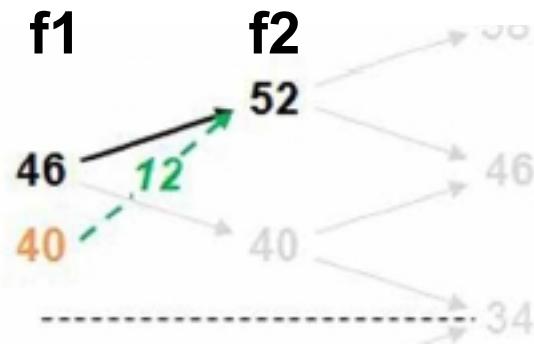
$f_1=B; f_2=A \longrightarrow$  Accuracy BA  $\neq$  Accuracy AB

Time-order effect

# Modeling time-order effect



Delayed vibrotactile discrimination task

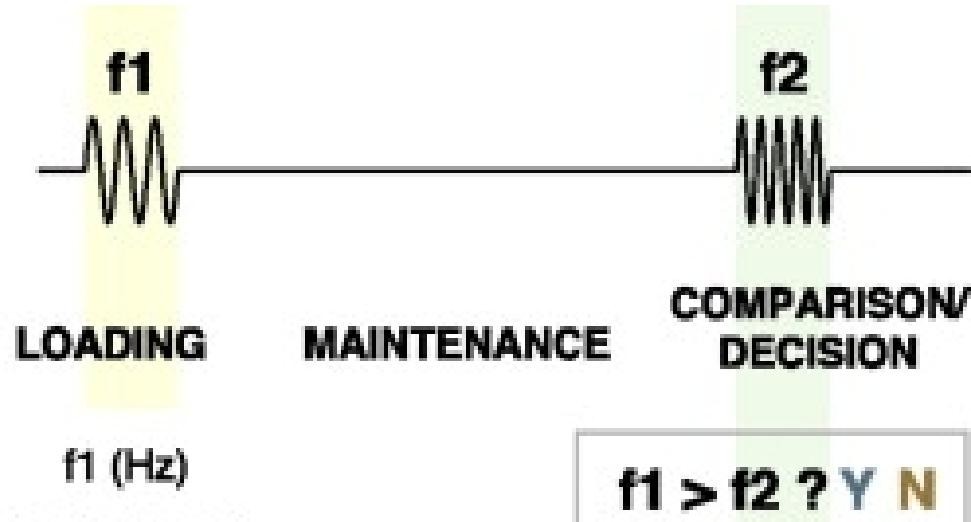


Global mean = 34 Hz

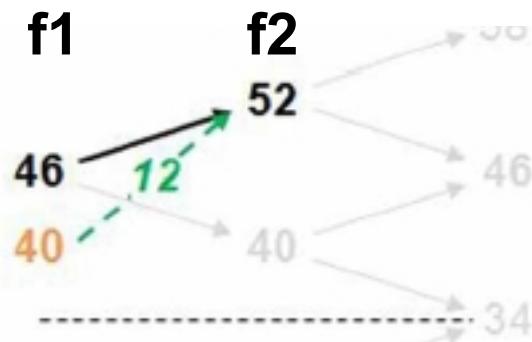
Preferred order  
improves accuracy

Karim et al. 2013

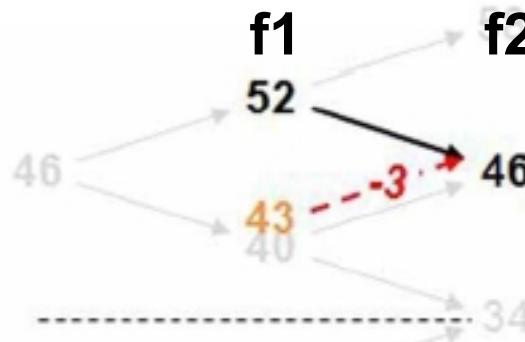
# Modeling time-order effect



Delayed vibrotactile discrimination task



Preferred order improves accuracy  
↑



Nonpreferred order reduces accuracy  
↓

Global mean = 34 Hz

Karim et al. 2013

# Bayesian approach

$$p(x|s) = p(x) p(s|x) / p(s)$$

# Bayesian approach

Gaussian assumption:  $p(x|s) = p(x) p(s|x) / p(s)$

Prior belief:

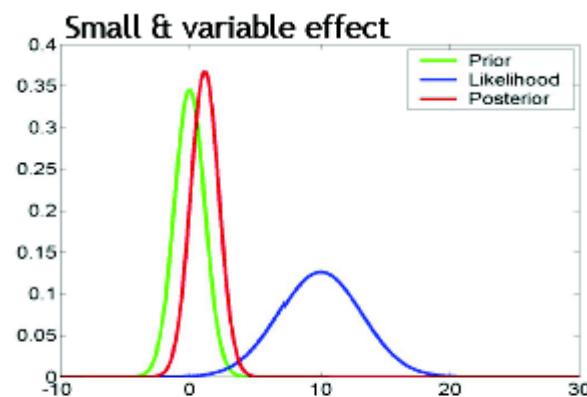
$$p(x) = \frac{1}{\sqrt{2\pi}\sigma_0} \exp\left(-\frac{(x-x_0)^2}{2\sigma_0^2}\right)$$

Likelihood:

$$p(s|x) = \frac{1}{\sqrt{2\pi}\sigma_s} \exp\left(-\frac{(x-x_s)^2}{2\sigma_s^2}\right)$$

Posterior belief:

$$p(x|s) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\hat{x})^2}{2\sigma^2}\right)$$



# Bayesian approach

Gaussian assumption:  $p(x|s) = p(x) p(s|x) / p(s)$

Prior belief:

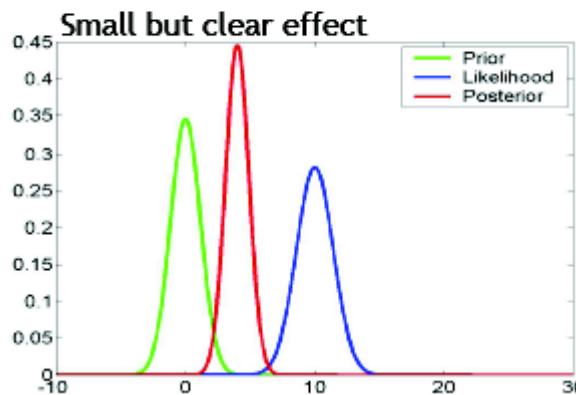
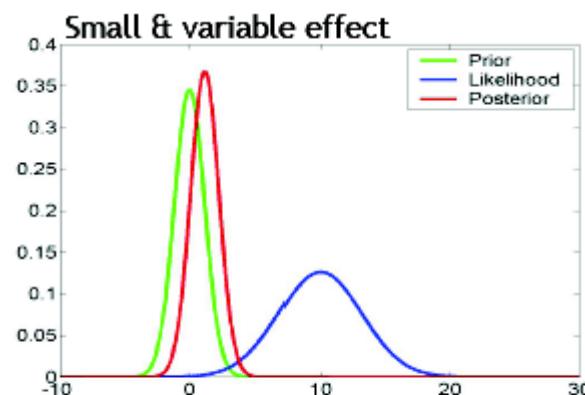
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Likelihood:

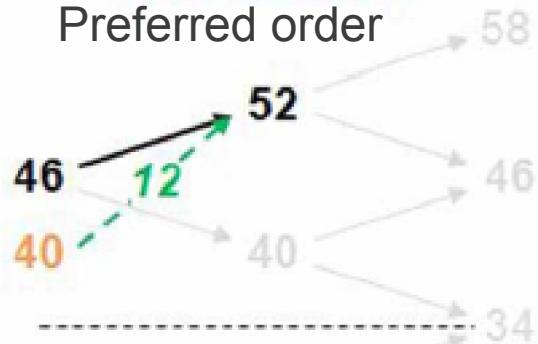
$$p(s|x) = \frac{1}{\sqrt{2\pi}\sigma_s} \exp\left(-\frac{(x-x_s)^2}{2\sigma_s^2}\right)$$

Posterior belief:

$$p(x|s) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\hat{x})^2}{2\sigma^2}\right)$$

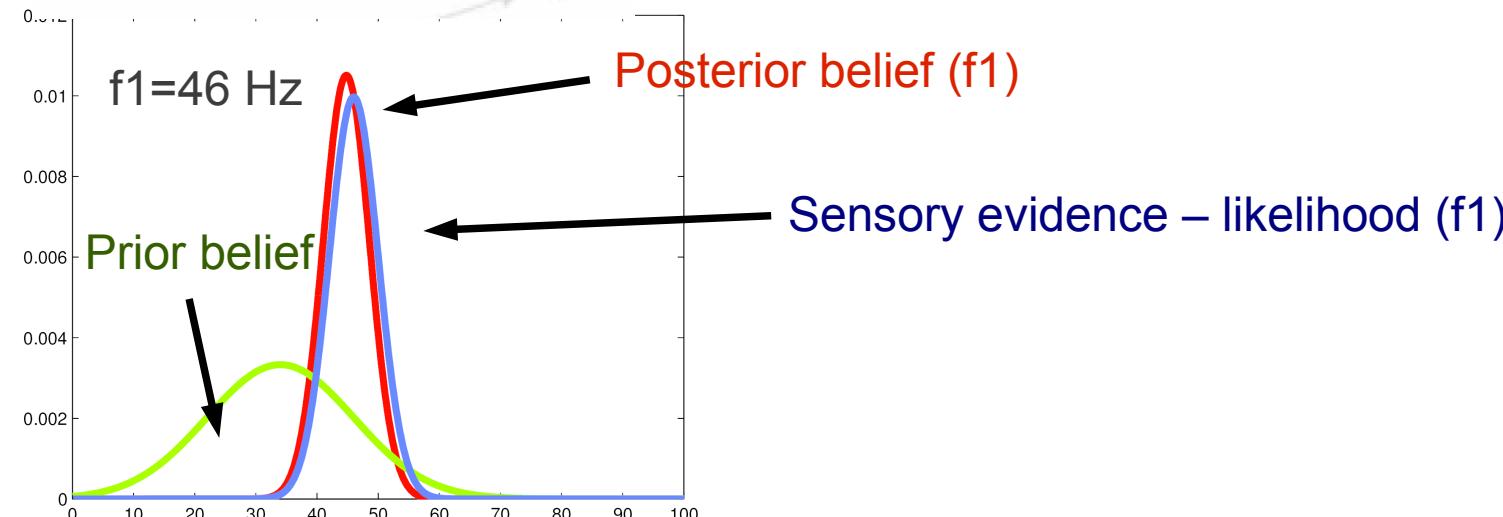


Preferred order



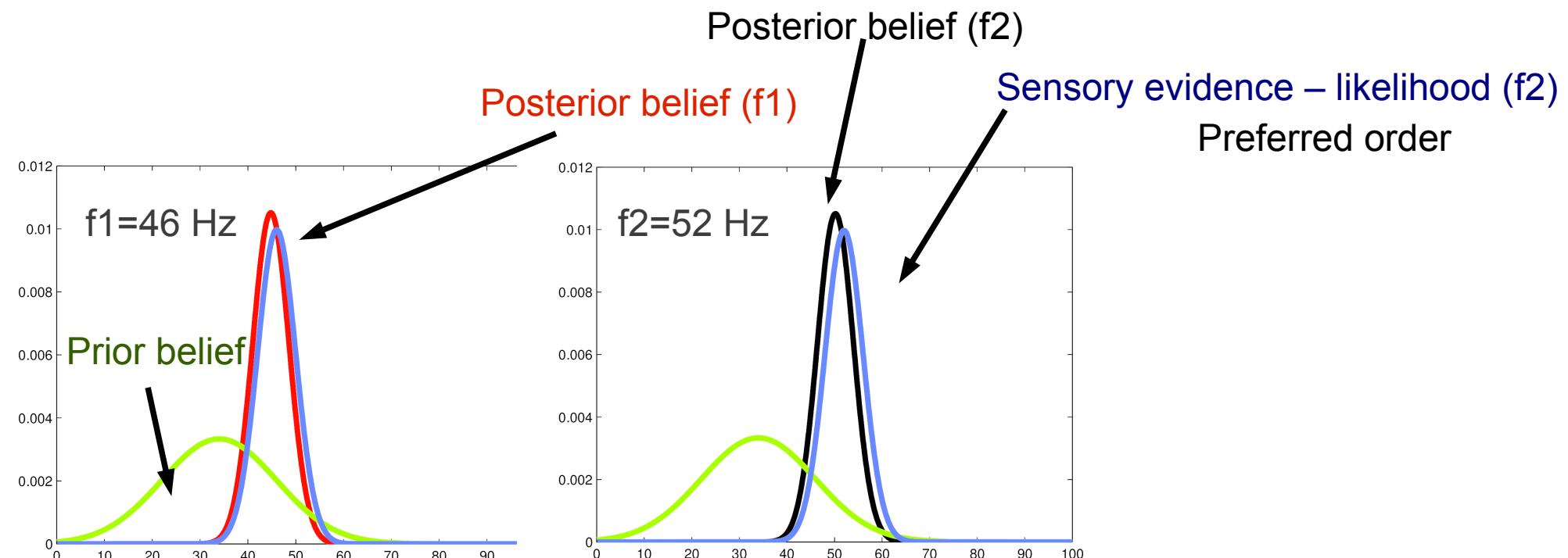
Without precision fade

$$p(x|s) = p(x) p(s|x) / p(s)$$



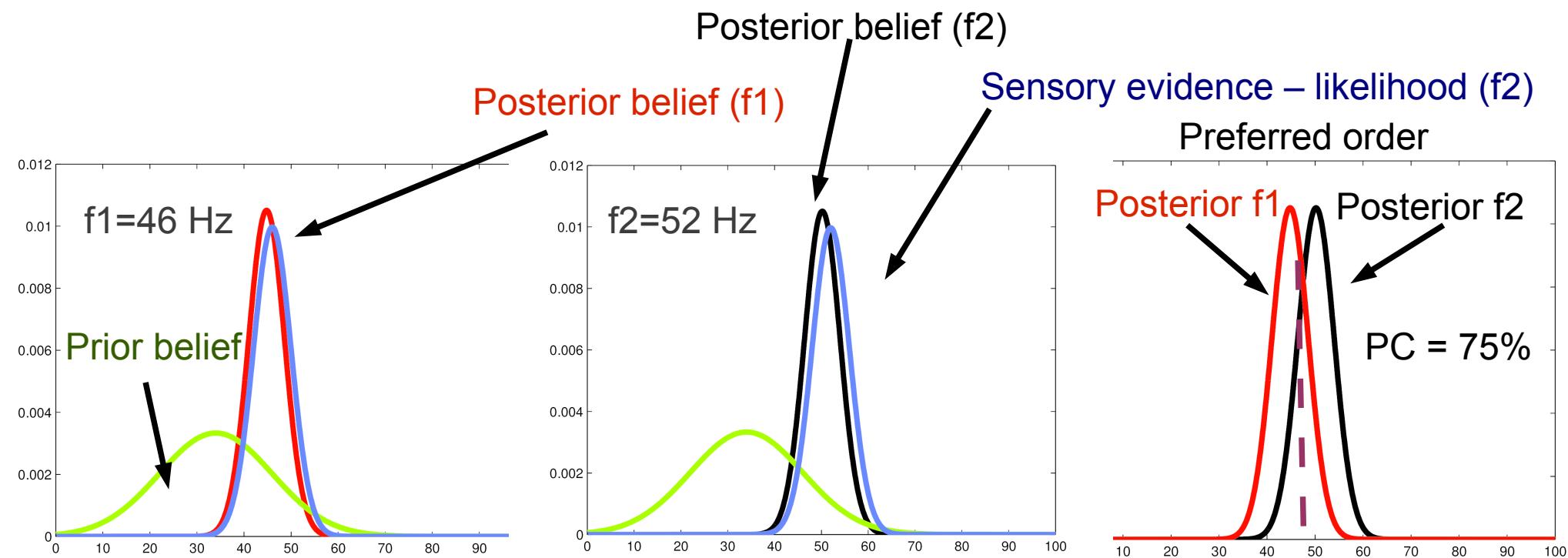
# Without precision fade

$$p(x|s) = p(x) p(s|x) / p(s)$$



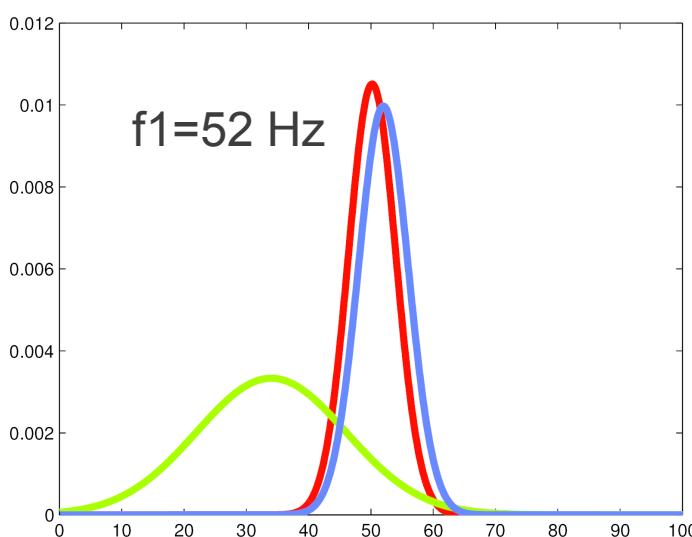
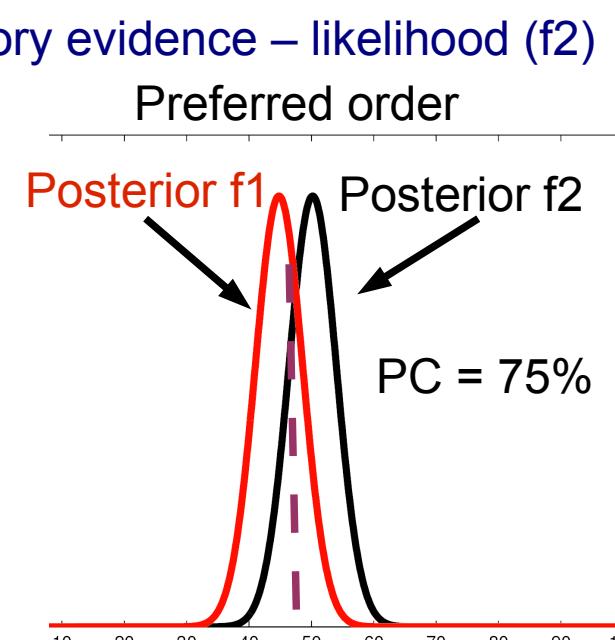
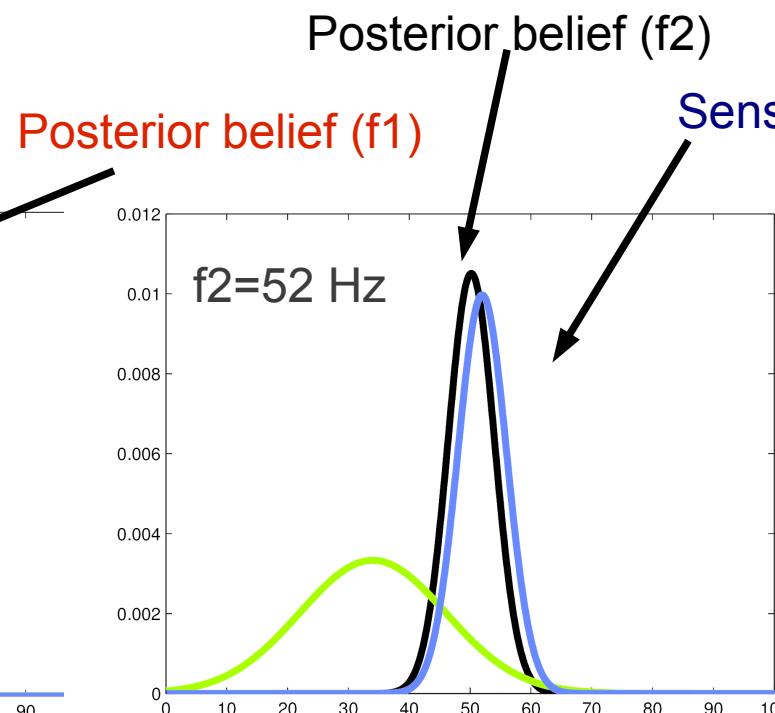
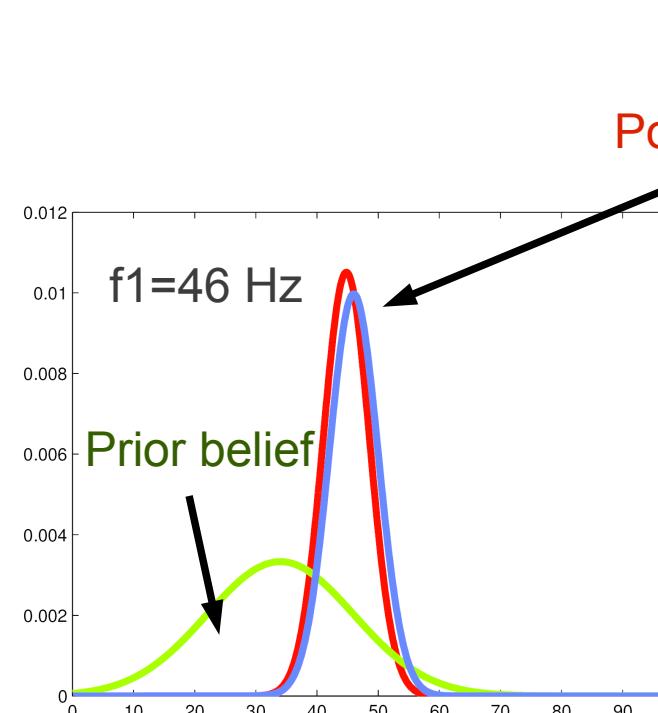
# Without precision fade

$$p(x|s) = p(x) \cdot p(s|x) / p(s)$$



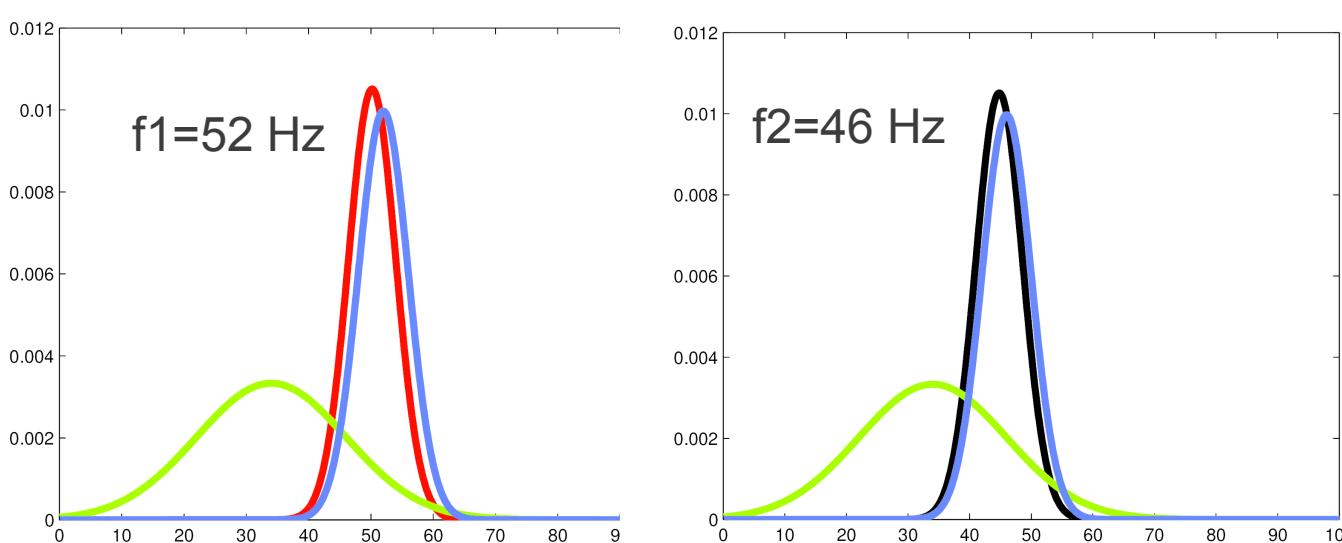
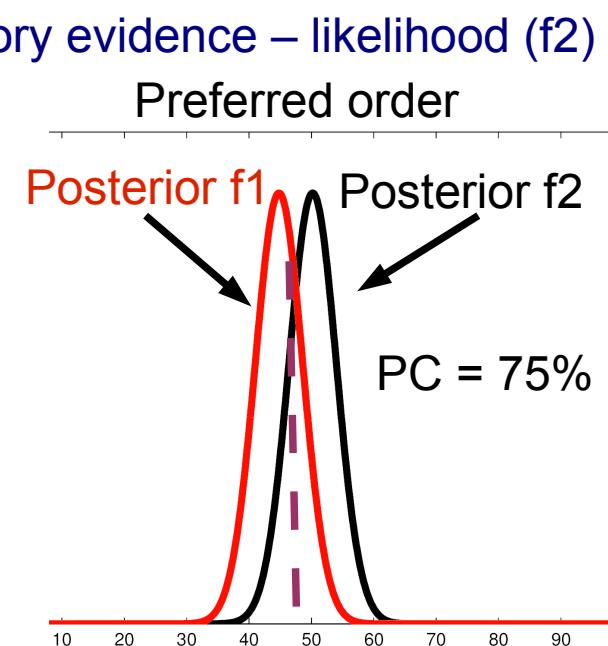
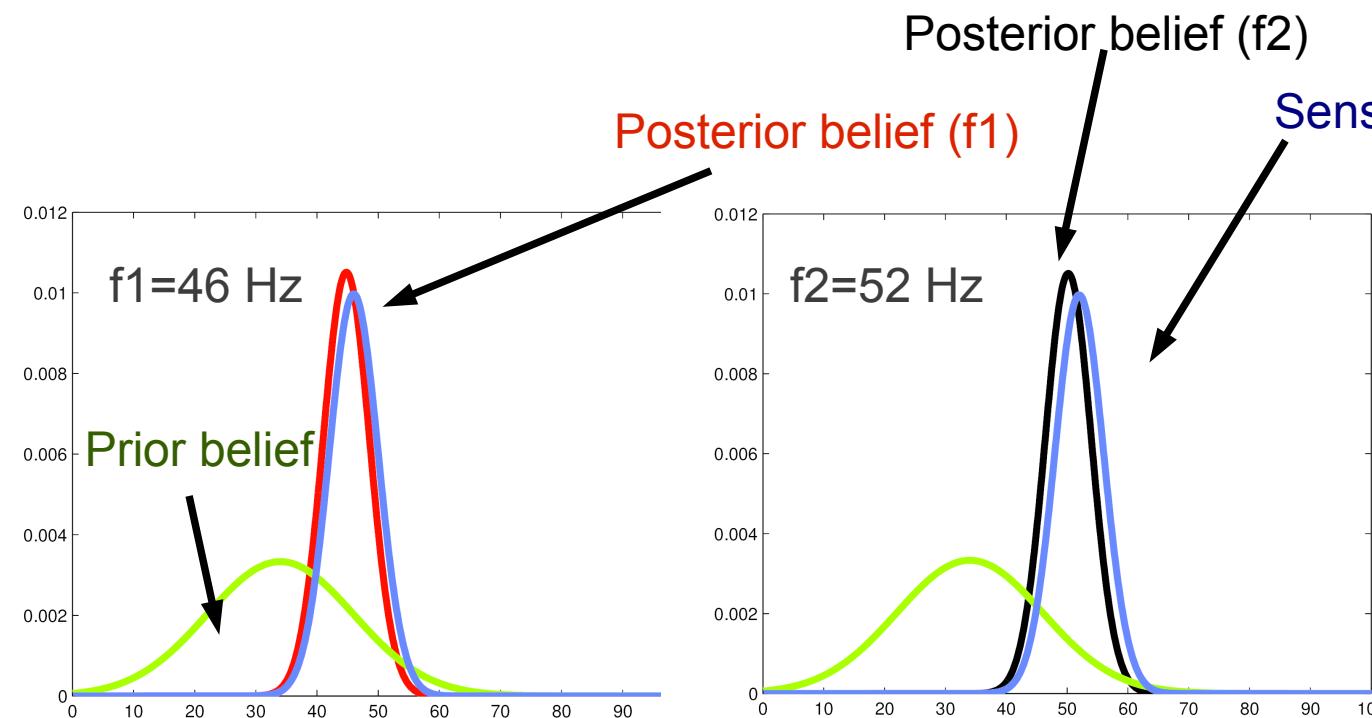
# Without precision fade

$$p(x|s) = p(x) p(s|x) / p(s)$$



# Without precision fade

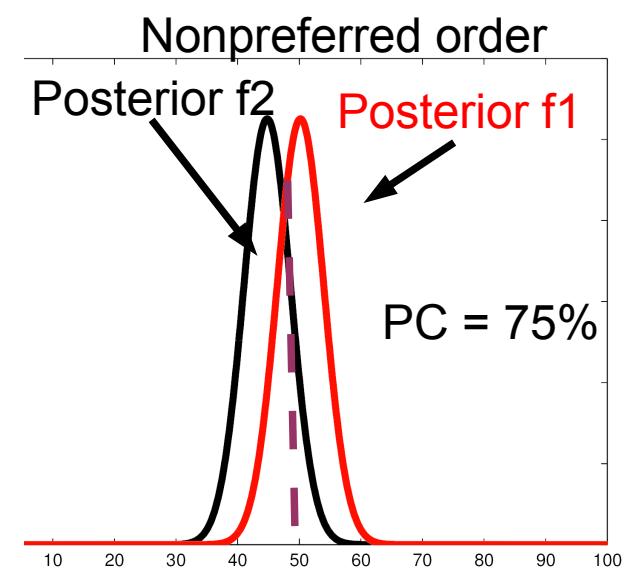
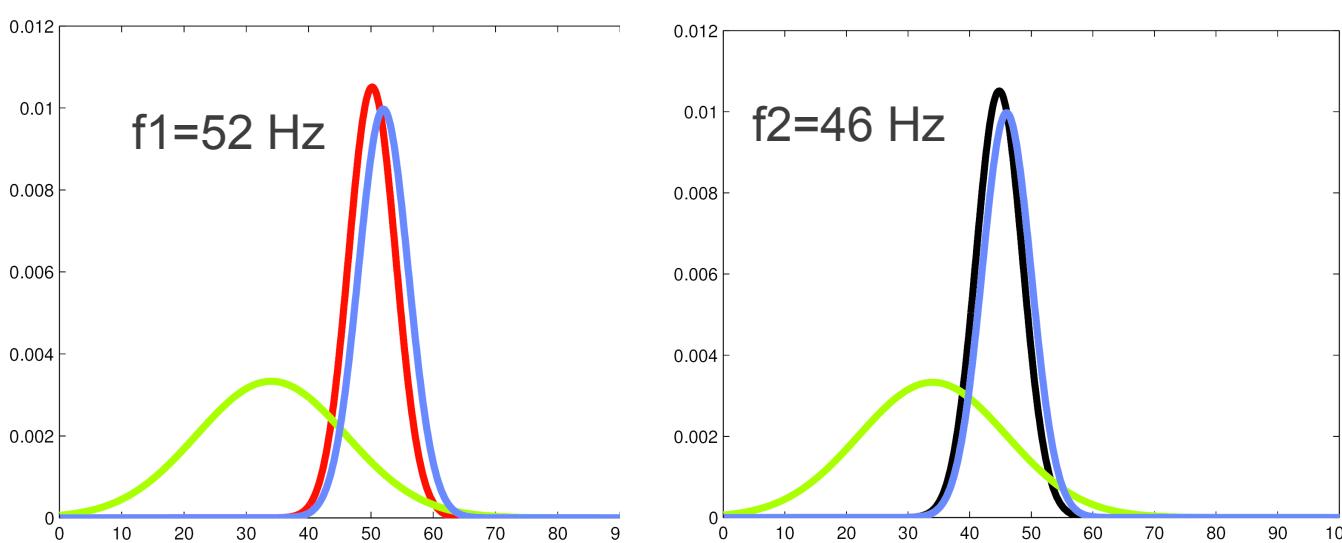
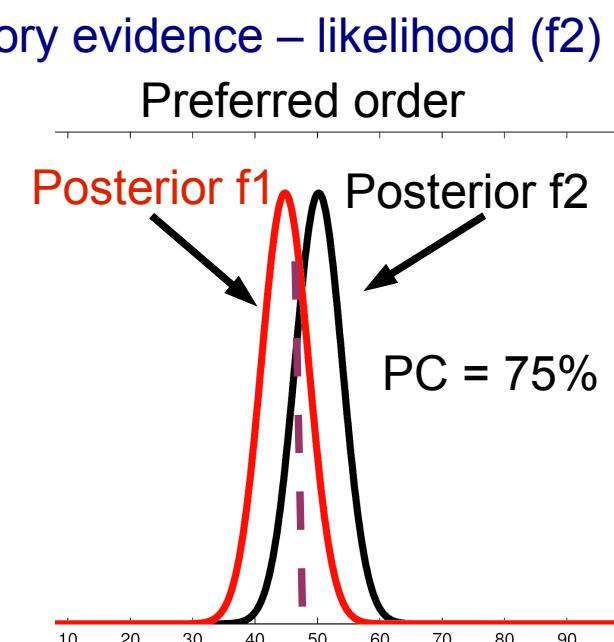
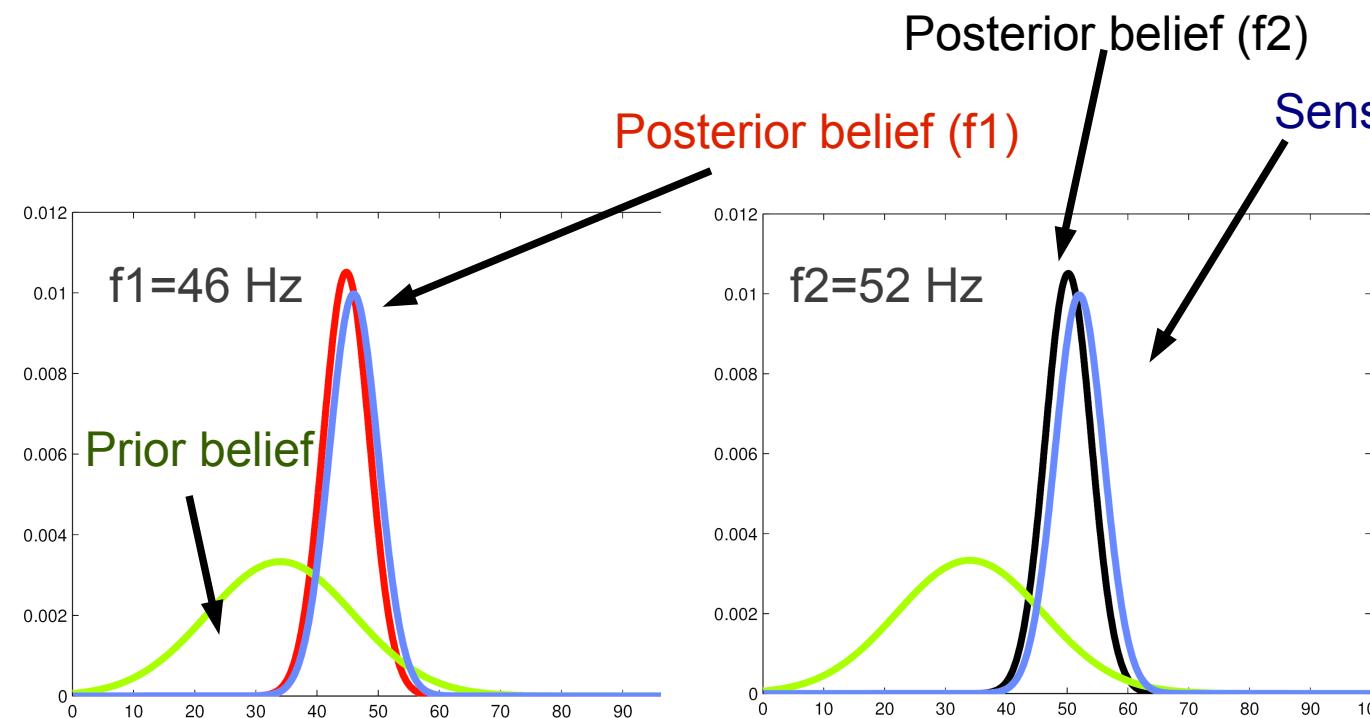
$$p(x|s) = p(x) p(s|x) / p(s)$$



Nonpreferred order

# Without precision fade

$$p(x|s) = p(x) p(s|x) / p(s)$$

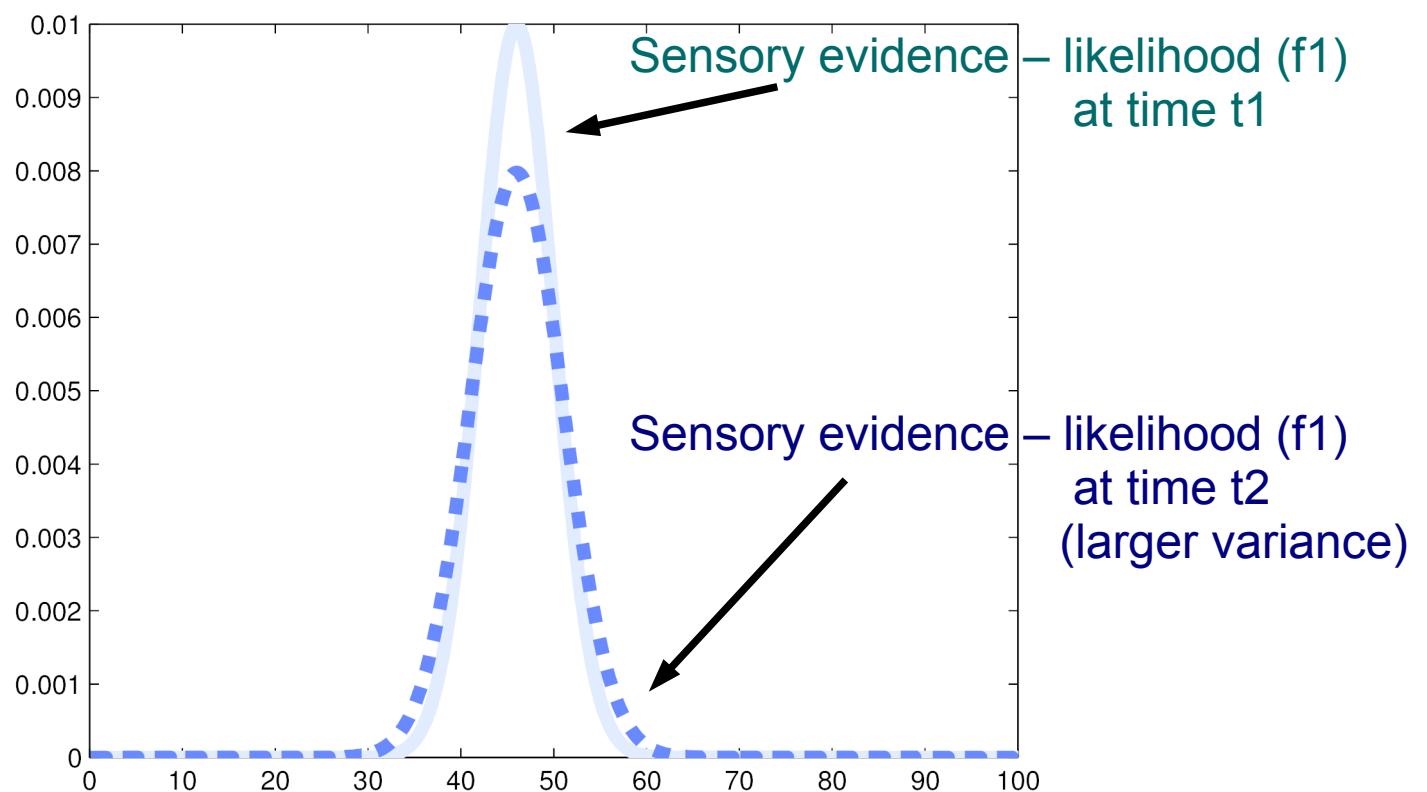
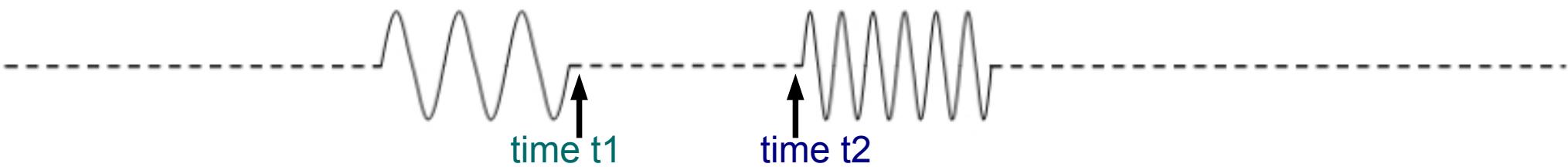


## Precision fade

The variance of the sensory evidence of stimulus  $f_1$  grows in time

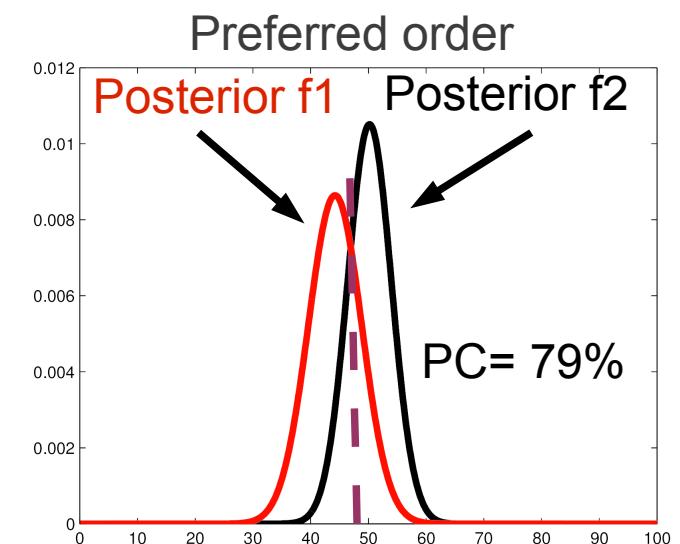
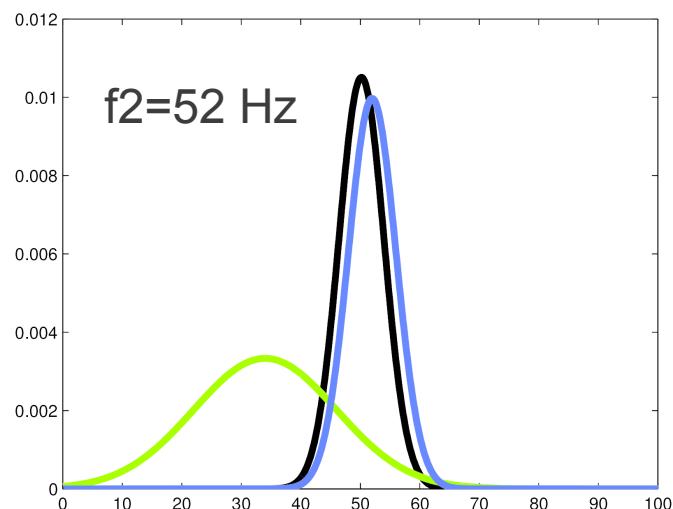
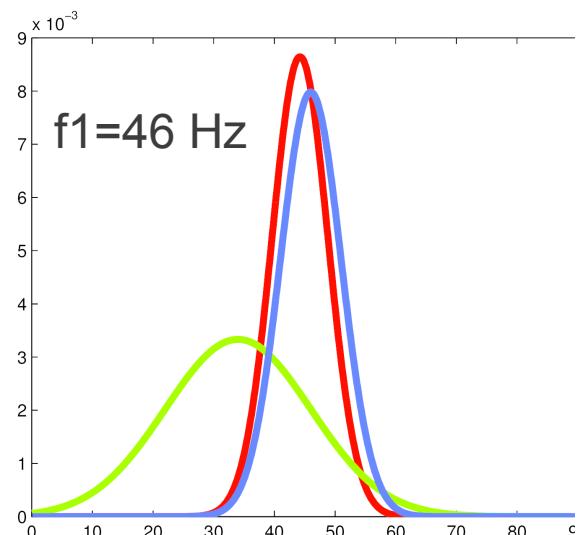
## Precision fade

The variance of the sensory evidence of stimulus  $f_1$  grows in time



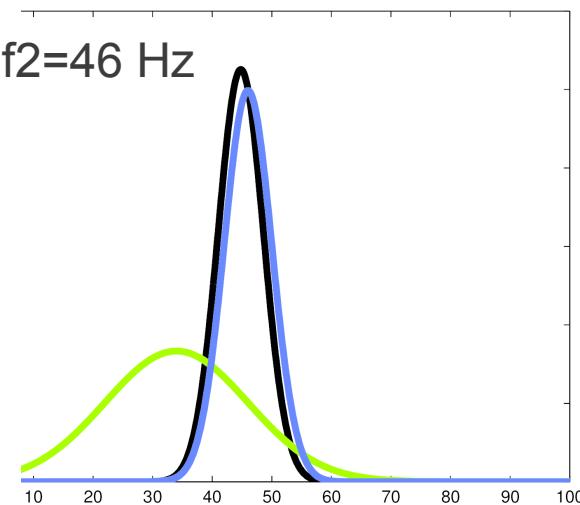
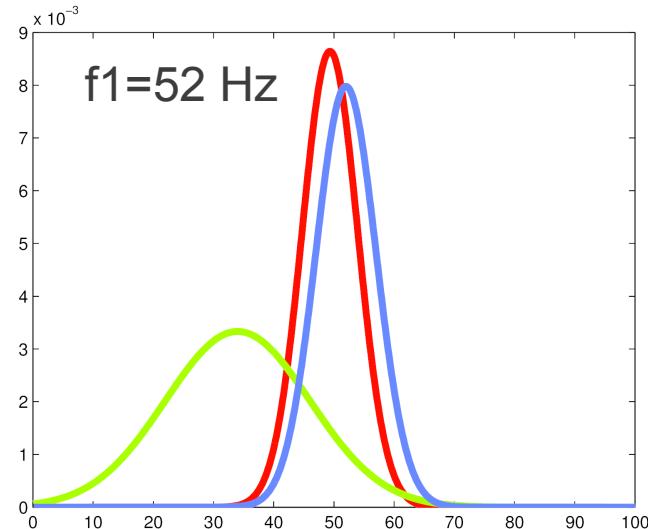
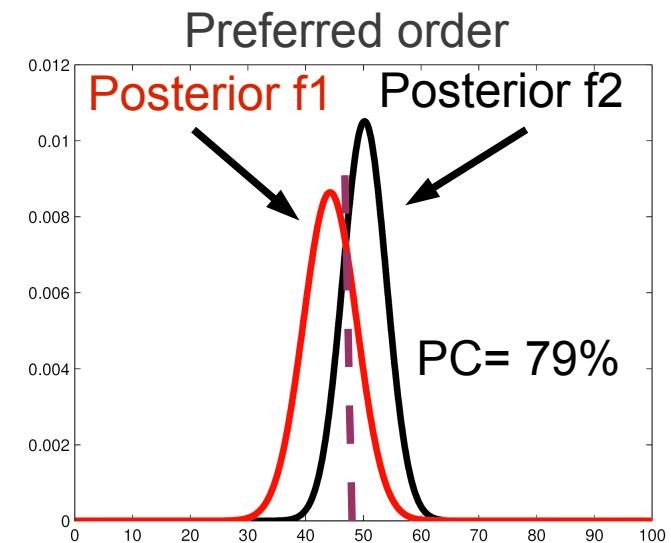
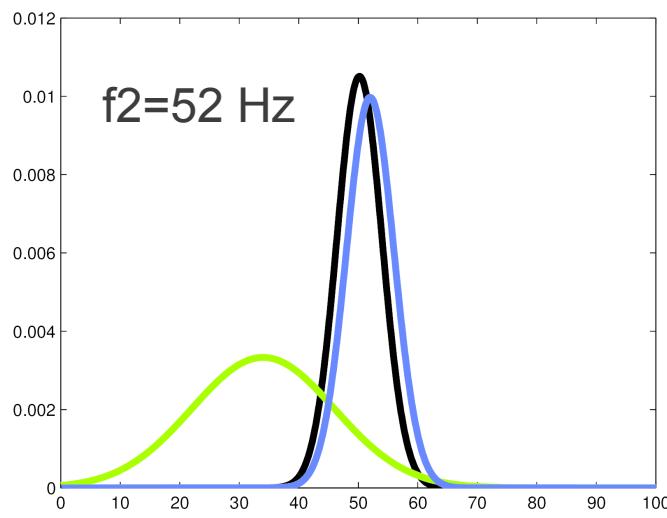
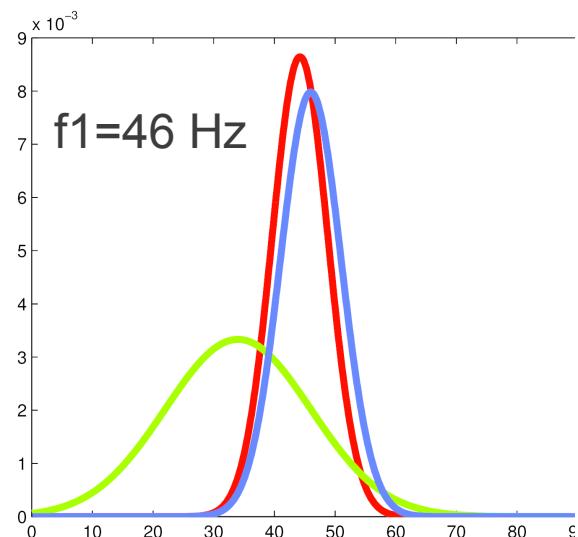
# Precision fade

$$p(x|s) = p(x) \cdot p(s|x) / p(s)$$



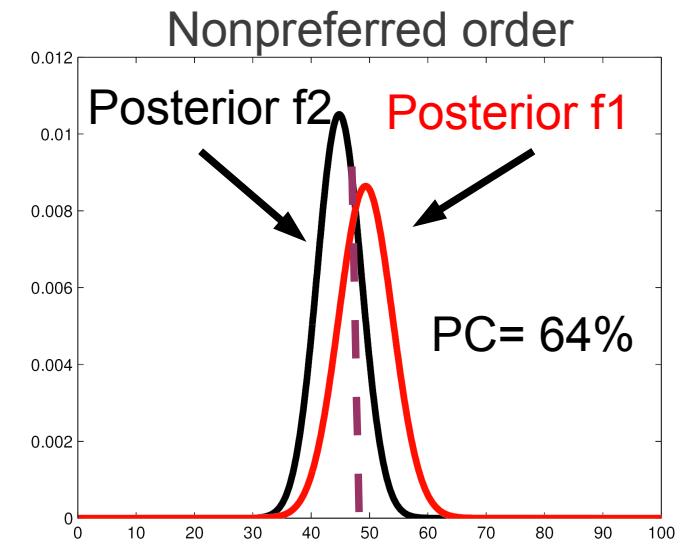
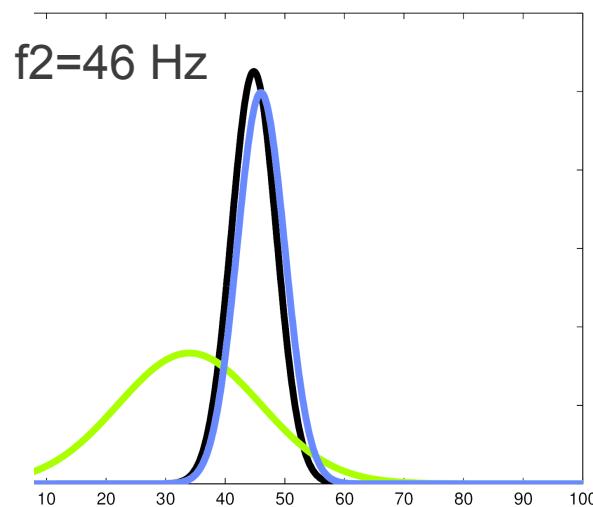
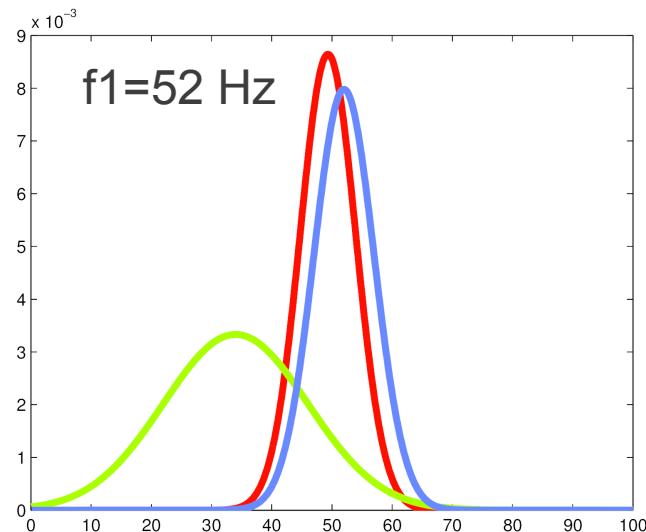
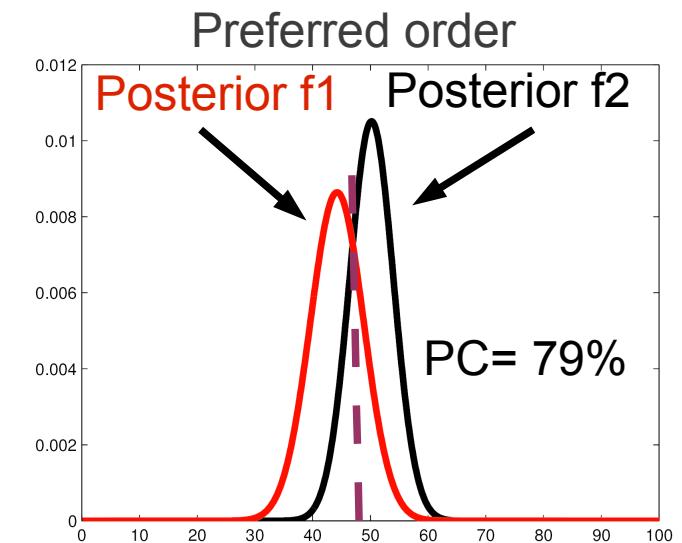
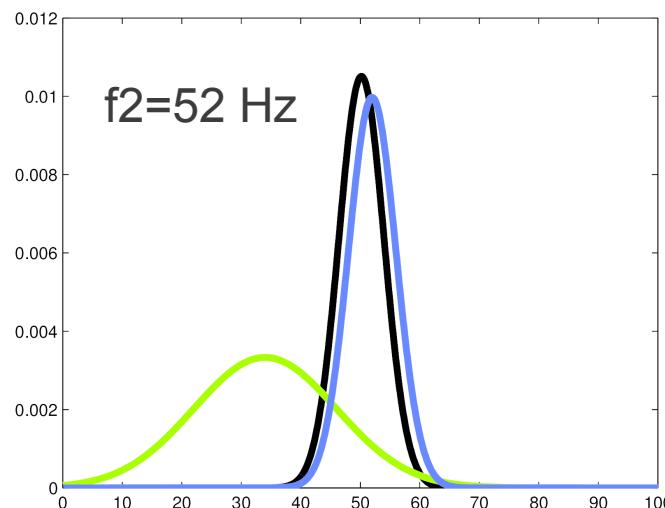
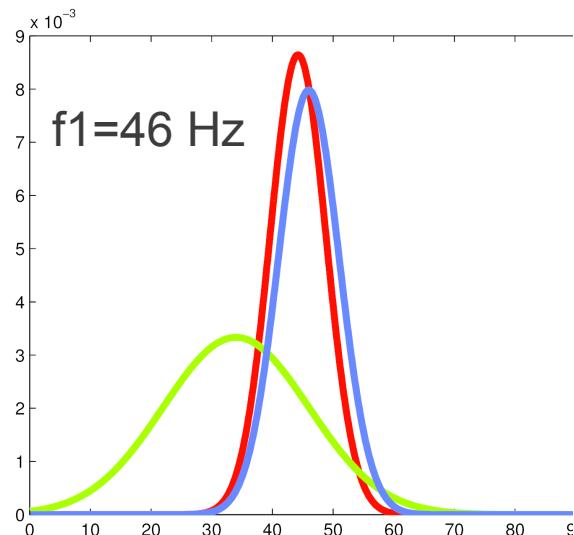
# Precision fade

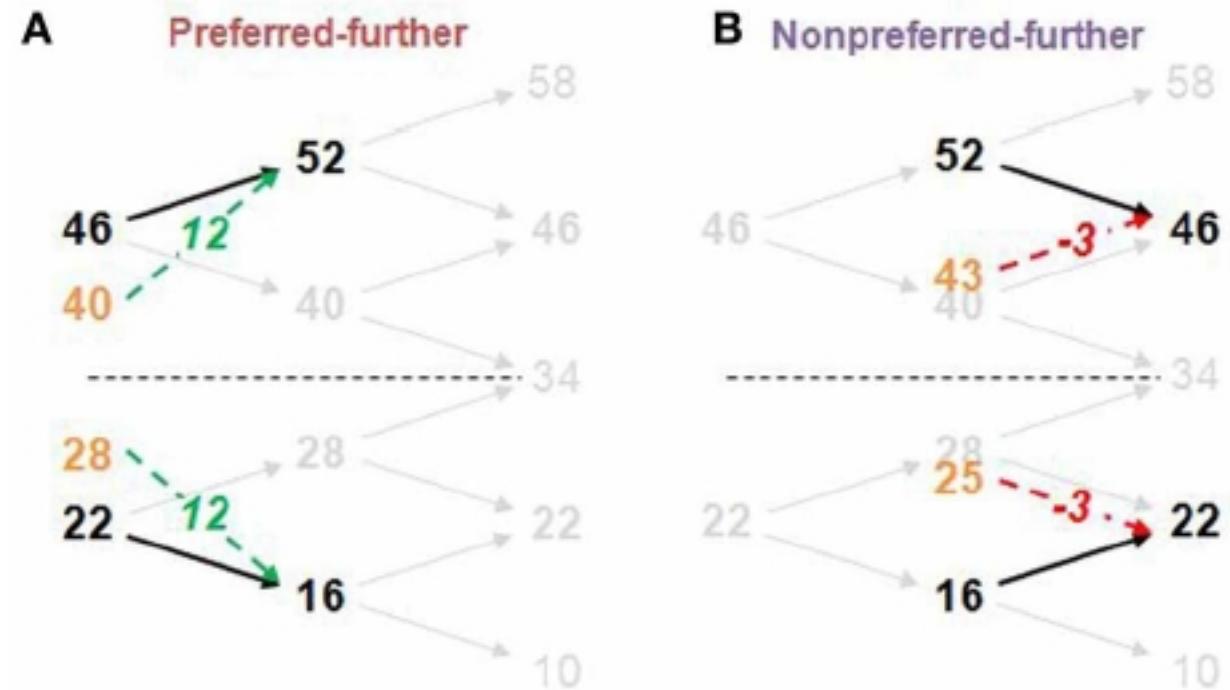
$$p(x|s) = p(x) \cdot p(s|x) / p(s)$$



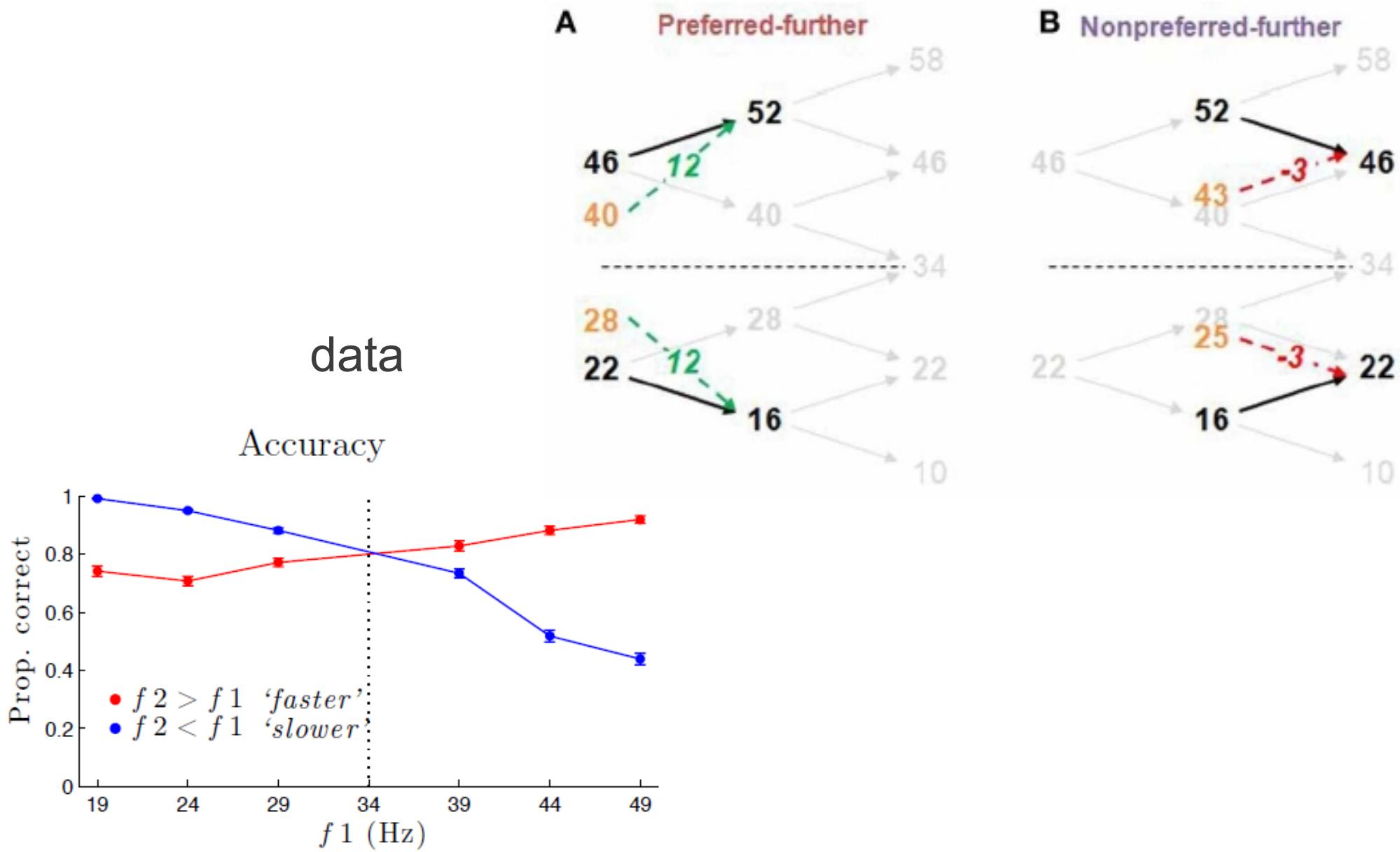
# Precision fade

$$p(x|s) = p(x) p(s|x) / p(s)$$



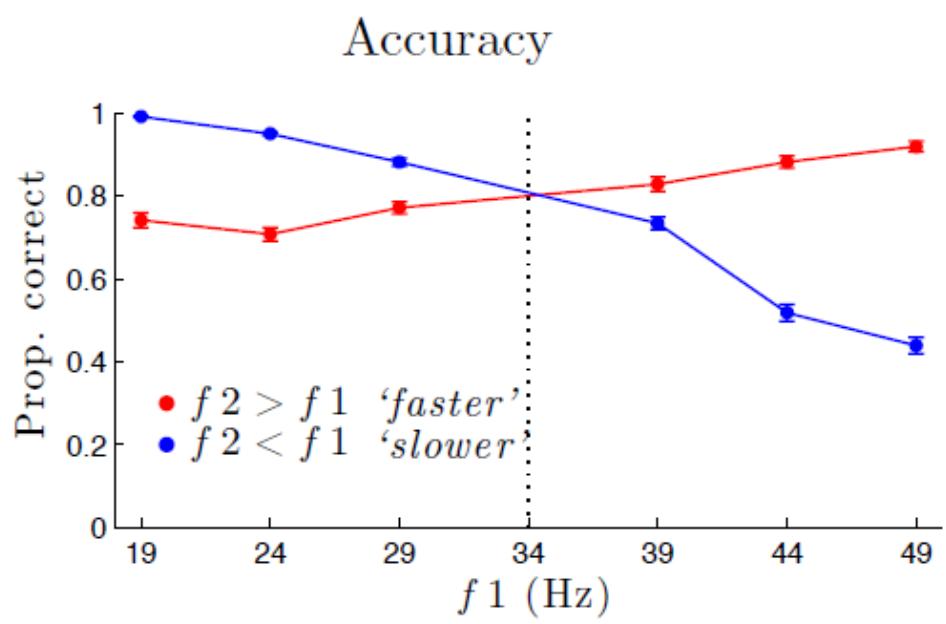


Karim et al. 2013

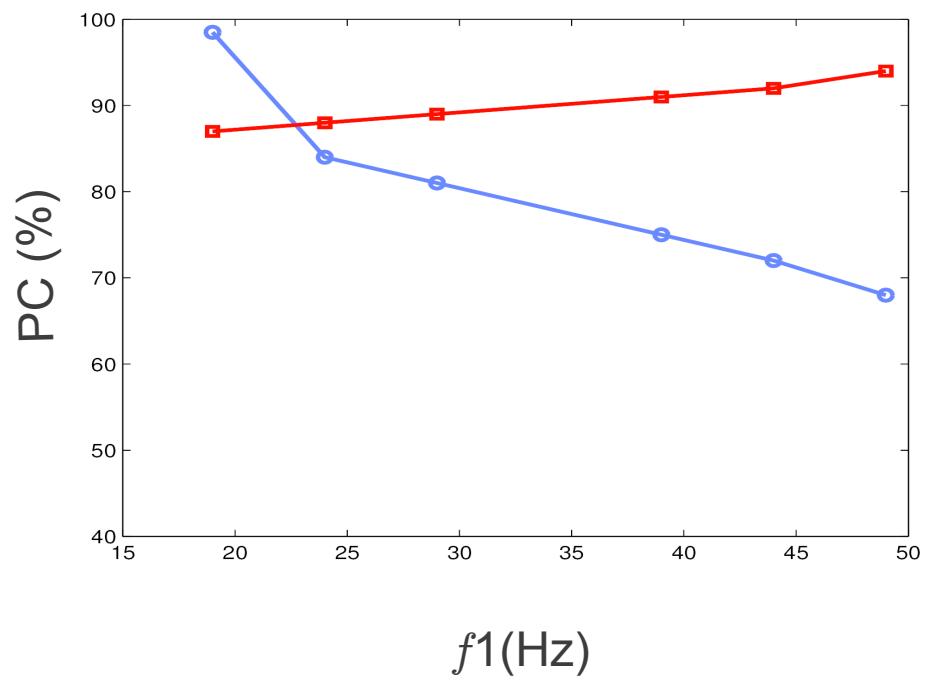


Langdon et al. 2012

data



model



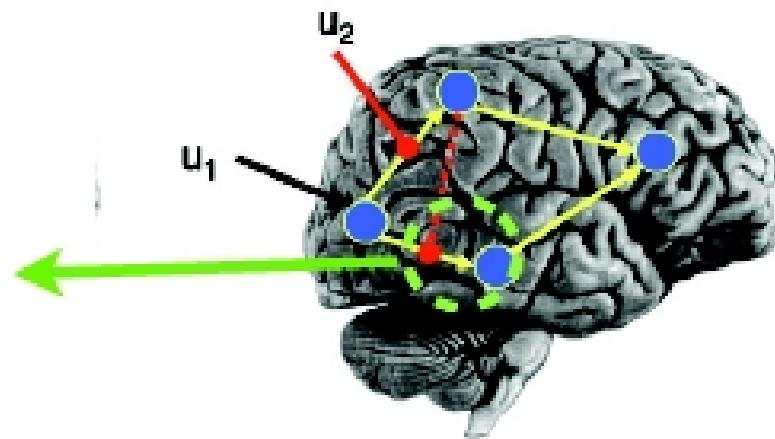
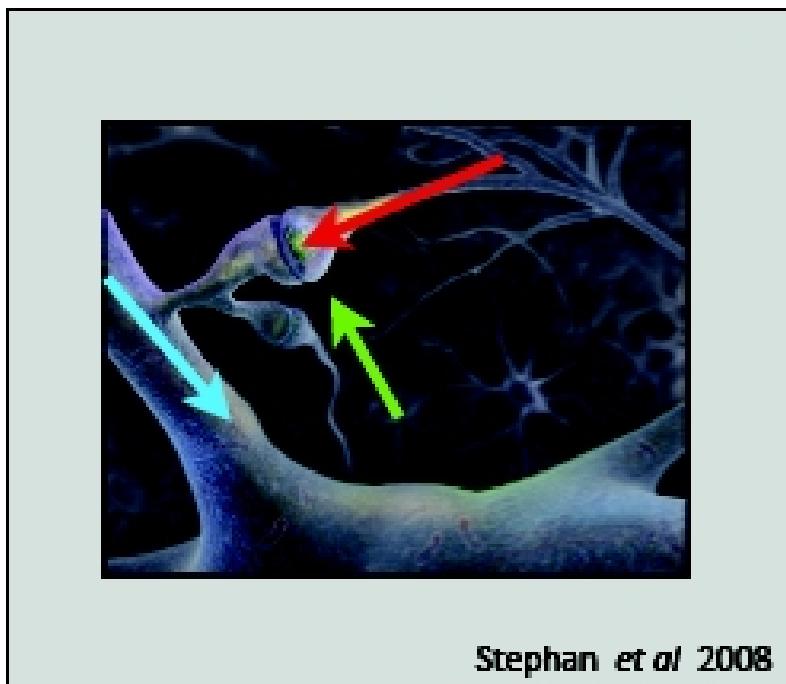
# Conclusions

- Vibrotactile decision-making task depends on a constellation of prefrontal cortical areas
- The data is best described by nonlinear hierarchical models
- Time-order effect = estimation + precision fade

Thank you

Thank you

## nonlinear DCM



Nonlinear state equation

$$\frac{dx}{dt} = \left( A + \sum_{i=1}^m u_i B^{(i)} + \sum_{j=1}^n x_j D^{(j)} \right) x + Cu$$

The nonlinear term in the D matrix embodies a very basic form of neurophysiological gating, namely voltage dependent gating via NMDA receptors