Basic Math Review for Relay Techs

Prepared for Hands On Relay School

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Basic Math Review for Relay Techs

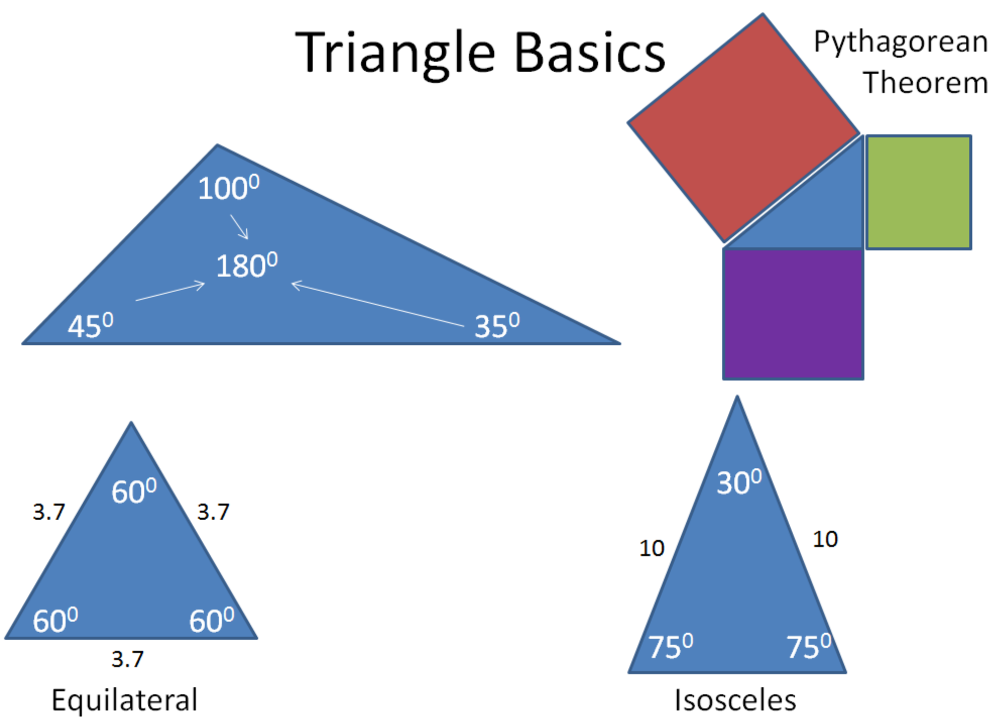
There are lots of resources for specific math concepts and functions used in relaying, but few general overviews for the technician. It is hoped that this document helps fill that void.

Geometry and Trigonometry

Many of the relationships in electrical quantities can be shown visually through geometry. Having a basic understanding of a few geometric concepts can help the technician understand electrical phenomena and provide tools for troubleshooting.

Basic triangle characteristics

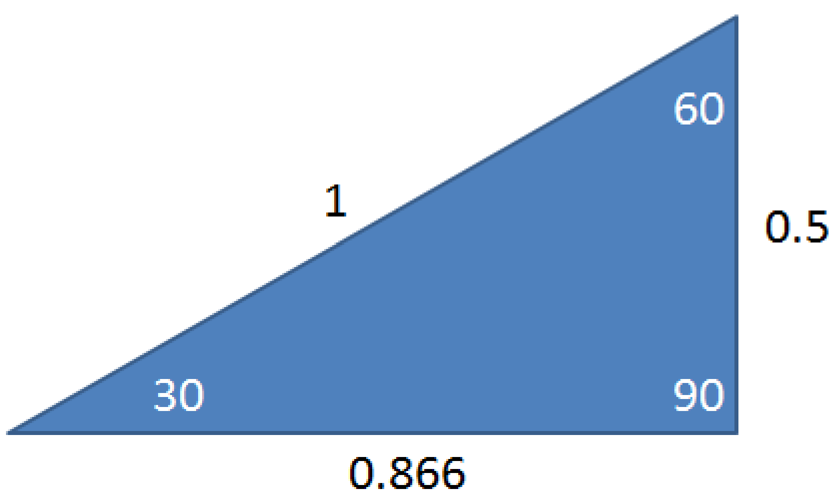
- Angles add up to 180^0
- **Equilateral** – all sides equal and angles 60^0
- **Isosceles** – two sides and angles equal
- **Pythagorean Theorem** – Square of hypotenuse is equal to the sum of the squares of the sides in a right triangle



While geometry can help us visualize the relationships, trigonometry helps us determine the values of these quantities. It's helpful to have a working familiarity with the characteristics of triangles with angles of 30^0, 60^0, 90^0, and 120^0. These angles appear often in electrical theory – 3 phases, wye-delta shifts, phase-to-phase to phase-to-neutral angle, and more.

Remember: **Sine** = Opposite over Hypotenuse SOH
Cosine = Adjacent over Hypotenuse CAH
Tangent = Opposite over Adjacent TOA

Sin30 = 0.500	Cos30 = 0.866	Tan30 = 0.577
Sin60 = 0.866	Cos60 = 0.500	Tan60 = 1.732
Sin90 = 1.000	Cos90 = 0.000	Tan90 = NA (can't divide by 0)
Sin120 = 0.866	Cos120 = -0.500	Tan120 = -1.732

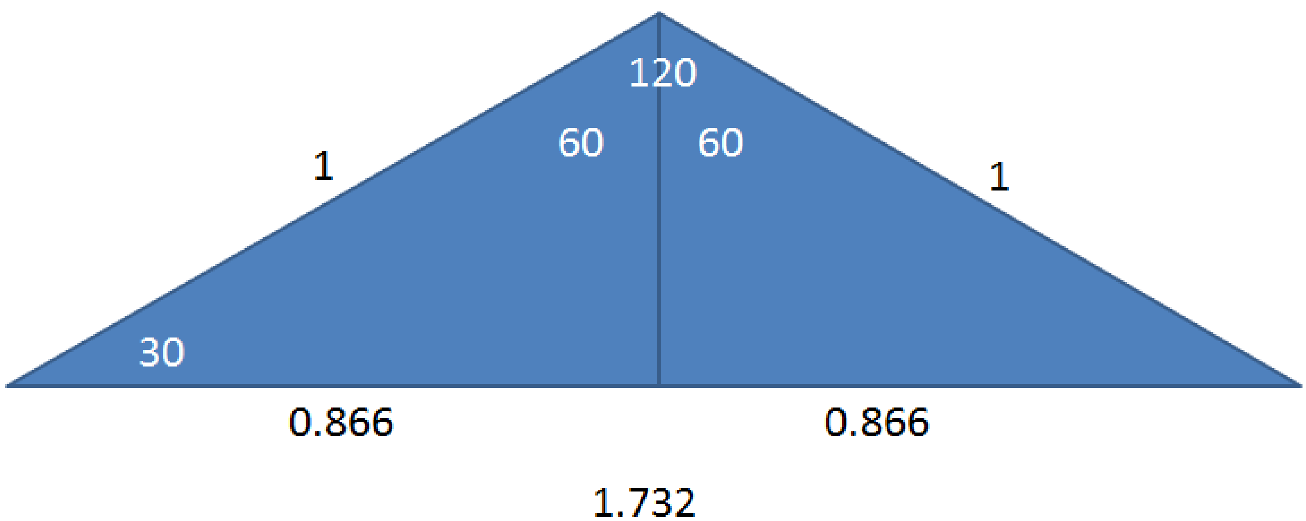


Notice the relationship between 30^0 and 60^0. In a 30-60-90 right triangle the sine of 30 equals the cosine of 60 – they are the same relative side of the triangle. These angles are "complementary," they add up to 90^0. Another helpful concept is supplementary angles, which add up to 180^0. Also notice that 120^0 is like 60^0 except the cosine and tangent are negative.

With this information, we can show why phase-to-phase voltage is $\sqrt{3}$ times phase-to-neutral voltage:

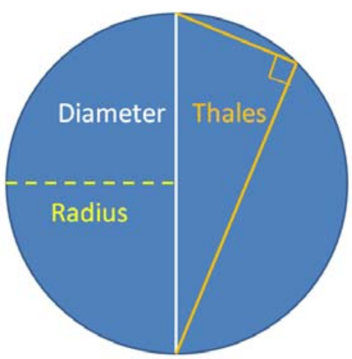
An isosceles triangle has two equal sides, and if we bisect the angle between them, it bisects the opposite side at 90^0 (revisit your high school geometry book for proofs).

Bisect the 120^0 angle between two wye phasors with magnitude 1 to get two 30-60-90 triangles whose hypotenuse is 1. The side adjacent the 30^0 angle is 0.866 (cosine function). Twice 0.866 is 1.732. So the phase-to-phase quantity is 1.732 times the phase-to-neutral quantity.



Circles

Knowledge of circles is helpful when dealing with impedance elements which often have circular characteristics.
Some things to know:
The circumference is the distance or length around the edge of the circle.
The radius is the length from the center to the circumference.
A chord connects 2 points on the circumference.
The diameter is the longest chord and passes through the center.
Thales Theorem states that the angle formed by two chords originating from the end points of a diameter inside a semi-circle is a right angle.
The circumference is equal to the diameter times **π. C = πd = 2πr; π = 3.1416**
We usually think of angles or arcs of a circle in **degrees** – 360^0 in a circle. Another way to describe angles and arcs is in **radians**, which use the **πr** relationship:
if **r** = 1, the full circumference = **2π**, because **C = 2πr**. Some formulas, like those for torque and frequency, use radians.



90^0 = 1/2π radians

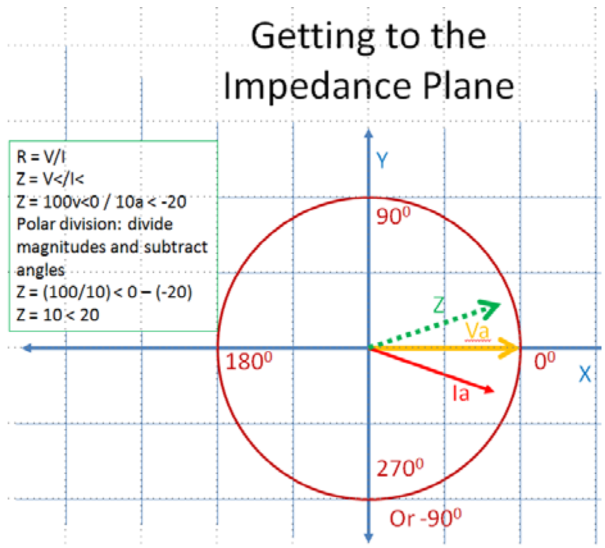
180^0 = π radians

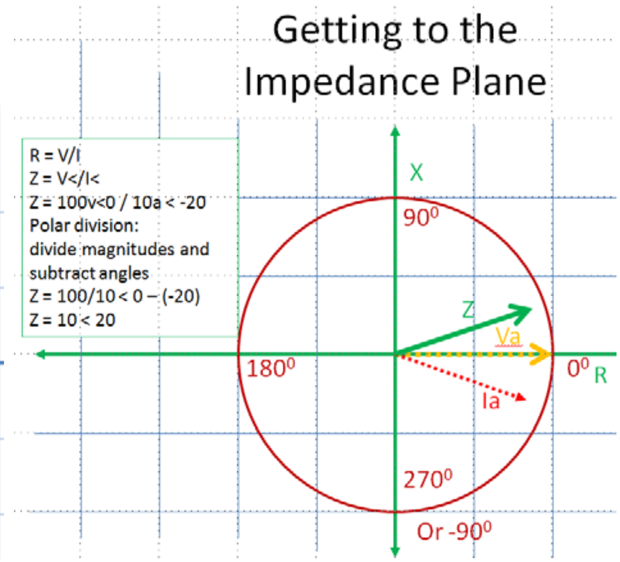
360^0 = 2π radians

Cartesian Planes

We use several Cartesian planes to visualize the concepts used in protective relaying. A Cartesian plane is a grid of squares with a horizontal axis that locates a zero point with positive to the right and negative to the left, generically labeled the X axis, and a vertical axis through the zero point with positive values up and negative values down, labeled Y. We plot vectors and phasors on this grid.
A **vector** is a visual representation of a magnitude with direction – 120 volts at 10^0, for example. It's a line segment with an arrowhead on one end.
Phasors are vectors with a rotational component and we use them to represent voltage and current. In a balanced three phase electrical system, the phases are 120^0 apart.
Phasors and symmetrical components won't be covered here.

The most basic plane is an X/Y plane on which we plot voltage and current phasors. Sometimes a circle is plotted around the origin to help visualize the rotational nature of phasors. We can also add numbers to show a reference and various angles in relation to it. Most common in relaying is 360 degrees marked out counterclockwise from 3 o'clock. In metering, the angles are clockwise, so 90^0 is at 6 o'clock. Sometimes 0 is at 12 o'clock. Know the reference and angular direction whenever you're working with phasor quantities.
Another helpful plane is the **power plane**, or P – Q plane. This assists in visualizing and analyzing power quantities – watts in and out, vars in and out, with a phasor to indicate the total VA and power factor. It works much like the power triangle. The axes represent functions – no longer directly measured single quantities, but watts (V x I x cosθ) and vars (V x I x sinθ). (Be aware that generation engineers show inductive vars as positive, in quadrant I.)
A third plane, and one that people involved in protection see often, is the **impedance plane**, the R – X plane, showing resistance and reactance. This is the plane used for mho circles for distance and loss of field protection.





Z is a function of V and I. When you go to the impedance plane, the axes are now resistance and reactance.

Another plane is for differential relaying where restraint and operate are the axes. These are functions of the two currents. Each manufacturer chooses the functions – restraint can be the largest current, the average of the two currents, etc.

There are also some proprietary planes, like Schweitzer's "Alpha Plane" that creates a visual representation of their 87L protection algorhythm. They have defined the functions that determine that plane.

It is important to keep straight what the axes represent – is it a directly measured unit like amps or volts, or a function of them.

Ratios

Ratios are used to compare two quantities. They can be expressed as comparisons: 2:3 or as fractions: 2/3. We see them in transformers – 115kV/12.47kV. The full description of transformer parameters is:

a (turns ration) = $N_1 / N_2 = V_1 / V_2 = I_2 / I_1$.

Like fractions, ratios can be simplified by dividing 115/12.47 = 9.2/1. This can simplify calculations. Ratios are used to define CTs (2000/5 = 400/1), and PTs (115,000/115 = 1000/1).

Per Unit Values and Percent

Creating a base of 1 for calculations can make life easier for engineers, rather than using primary values. Using ratios, an electrical system can be converted to a base of 1. A simple example is in the basic electromechanical overcurrent relay, the IAC or CO. The relay has several "taps" that determine the operating current of the relay. The time curve is a function of this tap. We test at 2 times tap, 3 times tap, etc. We could also say we test at 2 per unit, 3 per unit, etc. For injections, we then have to convert back to primary values: if tap = 2.0, then 3 x tap = 6amps.

Generally for larger projects and studies, engineers use 100 MVA as the "base." The "actual," 100MVA is converted to 1MVA for the "base." There are base values for voltage, current, impedance and power.

If you have a 40 MVA transformer with high side voltage of 115kV and rated current of 200.8 amps, these can become your base values. If the measured current is 100.4 amps, it can be described as 0.5 per unit current (half of 200.8 amps).

In one type of differential relay, there are settings for MVA, voltage of winding 1 and 2, and CT ratios. These values are used to derive the "tap," the rated secondary current. In the 40 MVA example above:

40,000,000 VA $\div$ 115,000 V $\div \sqrt{3}$ = 200.8 amps $\div$ 40 (CTR) = 5.02,

which is our "tap" for winding 1.

Suppose that the minimum operating current is 0.5 per unit. We multiply 5.02 by 0.5 (0.5 times tap) and get 2.51 amps, our minimum pickup for winding 1.

Another way of thinking about per unit is to call it percent: 50% of 5.02 is 2.51. In the overcurrent example, we test at 200%, 300%, etc. Unrestrained differential pick up is often in per unit as well. Like our overcurrent above, it is multiples of tap, often as high as 10 or 12 per unit. A setting of "10.0" for unrestrained in the example above would require injection of 50.2 amps.

Another place it's used (but not called "per unit") is harmonics. We have a fundamental frequency, called the 1st harmonic. Twice as fast is the 2nd harmonic, three times as fast is the 3rd harmonic, and so on. This makes it easy to think about without getting caught up in the specifics of what the fundamental actually is – the waveform relationship is the same regardless of the fundamental frequency. But when you go to inject you need to know that the 2nd harmonic is 2 per unit of the fundamental, or in our electrical system, 2 x 60 = 120 Hz.

In some generator relays, volts per hertz are set in percentages, a variation on per unit. The pickup might be "110", meaning that the relay should pick up at 110% of base voltage. If base (secondary) is 66 v, the volts per hertz element should pick up at 72.6 volts when you inject at 60Hz.

Another generator element, directional power, can be set in per unit values. If the pickup setting is "-.020", we need to know what the base current and voltage are to correctly inject for pickup. If base or nominal current is 3.14, then the element should pick up at 0.0628 at 180 degrees *at Vnom* (3.14 x -0.020 = 0.0628; the negative sign indicates reverse power flow).

Decimals

Keep track of your prefixes as you calculate:

1M = 1,000K = 1,000,000

1µ = 0.001m = 0.000,001

Significant Digits

A calculator shows that $\sqrt{3}$ = 1.732050808

It calculates that 120 v / $\sqrt{3}$ = 69.2820323

Most test sets will differentiate at millivolt or -amp level but not microvolts or– amps.

Think about how many decimal places you realistically need – usually 69.3 or 69.28 volts is close enough

Algebra

- Basic identities: If A = B and B = C, then A = C
- Properties:
  - Distributive: a(b+c) = ab+ac
  - Associative: a+(b+c) = (a+b)+c and a(bc) = (ab)c
  - Commutative: a+b = b+a and ab = ba

Factoring

- 18 = 1x18 = 2x9 = 3x6
- 1,2,3,6,9, and 18 are the factors of 18
- You can use them in equations and expressions to simplify
- a/18 = b/3; since 3 is a factor of 18 you can multiply both sides by 3 to simplify
- a/6 = b/1 or a/6 = b
- We use multiples of 10 as factors a *lot*

Equations

- Solving equations: do the same thing to both sides
- a+2 = 5
  - subtract 2 from both: a+2-2=5-2, a = 3
- 3a = 6
  - divide both sides by 3, 3a/3 = 6/3, a = 2
- The same applies to addition and multiplication

Examples

- V=IR Ohm's Law
  - I=V/R
  - R = V/I
- $A^2 + B^2 = C^2$ Pythagorean Theorem
  - $A^2 = C^2 – B^2$
  - $B^2 = C^2 – A^2$
- Cosine = Adjacent / Hypotenuse Trig
  - Adjacent = Cosine x Hypotenuse
  - Hypotenuse = Adjacent / Cosine

Order of Operations

Please **E**xcuse **M**y **D**ear **A**unt **S**ally

- Parentheses – complete operations within any parentheses
- Exponents – resolve any squares or square roots, etc.
- Multiplication – complete any multiplication
- Division – complete any division
- Addition – combine anything
- Subtraction – subtract anything

Examples

- $\mathbf{A^2 + B^2 = C^2}$, where A = 3 and B = 4
- No parentheses, so exponents: $A^2 = 9$, $B^2 = 16$
- No multiplication or division, so addition: $9 + 16 = 25 = C^2$
- No subtraction, so find the square root of 25 to get C = 5
- $\mathbf{a((5 - 3)^2) + a(5 + 3) = 24}$
- Parentheses: 5-3=2; 5+3 = 8 so we have: $a(2^2) + 8a = 24$
- Exponents: $2^2 = 4$ so we have: 4a + 8a = 24
- Addition: 12a = 24
- a = 2

Resources

The Relay Testing Handbook	Chris Werstiuk
Protective Relaying Quick Reference	Power Engineers

Several pocket references for power calculations, etc.

Decimal Prefixes

Pico (p) 10^{-12} = 0.000000000001
Nano (n) 10^{-9} = 0.000000001
Micro (μ) 10^{-6} = 0.000001
Milli (m) 10^{-3} = 0.001
Kilo (k) 10^{3} = 1,000
Mega (M) 10^{6} = 1,000,000
Giga (G) 10^{9} = 1,000,000,000

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