# Fundamentals of Short-Circuit Protection for Transformers

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Abstract—This paper reviews principles of protection against internal short circuits in transformers of various constructions. Transformer fundamentals are reviewed as pertaining to protection. In particular, the electromagnetic circuit of a transformer is reviewed that links the terminal currents, winding currents, fluxes, and ampere-turns (ATs) in a set of balance equations for a given transformer. These balance equations are used to explain the sensitivity of protection to various types of transformer faults. The paper shows that the classical transformer differential compensation rules have roots in the first principles-they reflect the AT balance of the protected transformer. The rule of building transformer differential protection equations following the AT balance is used in this paper to derive differential equations for autotransformers; power zig-zag, Scott-T, and Le-Blanc transformers; and phase shifters. The restricted earth fault (REF) and negative-sequence transformer differential (87TQ) functions are explained as a means to detect ground faults near the neutral and turn-to-turn faults, respectively.

### I. INTRODUCTION

The differential protection principle applied to power transformers requires accounting for transformer winding connections. With reference to transformers of standard winding connections, these protection rules are commonly referred to as ratio matching, vector group compensation, and zero-sequence removal.

Many protection engineers face difficulties applying these principles to transformers of nonstandard or unusual connections (e.g., phase shifters, Scott-T transformers, power zig-zag transformers) under unusual locations or connections of current transformers (CTs) or under special configurations, such as a tertiary delta winding not accounted for in the differential scheme. When offered a choice of CT locations in a nonstandard transformer, an engineer may lack the expertise to select an optimum location, given the targeted protection sensitivity and the intended protection method.

In addition, it is typically challenging for some practitioners to analyze the coverage and sensitivity of a given short-circuit protection scheme. For example: Is the buried tertiary covered by differential protection? Is the restricted earth fault (REF) sensitive to interwinding faults? Do the sensitivity and coverage of a given differential scheme depend on the type of transformer core (three leg versus five leg) or the grounding method for wye-connected windings?

This paper explains principles of short-circuit protection for transformers and autotransformers by deriving proper balance equations for differential protection from the ampereturn (AT) equations of a healthy transformer. A number of standard and nonstandard winding configurations are considered, assuming different core types, as well as locations and connections of CTs. This enables an in-depth understanding of the known current differential compensation rules for standard transformers and teaches how to develop proper protection equations for any arbitrary transformer.

### **II. TRANSFORMER FUNDAMENTALS**

It is good to go back to basic principles to refresh ourselves on how a transformer works before discussing protection concepts. To maintain a high level of focus, we will limit the discussion to sinusoidal alternating current.

### A. Refresher on Transformer Theory

In its most basic form, a transformer is two or more coils of conductor in close proximity to each other, such that the magnetic fields generated by the current in each coil are linked. Almost universally, a magnetic core is included to maximize the coupling of the fields between the coils.

But let us step back one more level. Faraday's law tells us that a voltage (electromotive force) is created when either a conductor is moved through a magnetic field or a magnetic field is varied through the conductor. Equation (1) gives the special case of sinusoidal current:

$$\mathbf{v}(\mathbf{t}) = -\mathbf{n} \boldsymbol{\cdot} \boldsymbol{\omega} \boldsymbol{\cdot} \boldsymbol{\Phi} \boldsymbol{\cdot} \cos(\boldsymbol{\omega} \boldsymbol{\cdot} \mathbf{t}) \tag{1}$$

where:

n = number of turns in the coil.

 $\omega$  = angular velocity of the sinusoidal function.

 $\Phi$  = magnetic flux.

The amount of magnetic flux,  $\Phi$ , in (1) is given by (2):

$$\Phi = \frac{\mathbf{n} \cdot \mathbf{i}}{\Re} \tag{2}$$

where:

i = current.

 $\Re$  = reluctance in the magnetic path.

In (2),  $n \cdot i$  is the magnetomotive force (mmf) that drives the flux and is measured in ATs. Equation (2) is the magnetic circuit corollary to Ohm's law for electric circuits.

An inductor is a single conductor arranged in a coil, such that the magnetic field generated by current in each turn results in magnetic flux cutting the other turns in the coil. In an air core inductor, the reluctance is relatively high, so the magnetic flux is relatively low for a given current level. The result is a relatively low induced voltage in the coil that opposes the current flow [note the minus sign in (1)]. Because this opposing voltage is proportional to the current flowing in the coil, we characterize this as impedance. Now, if we introduce a magnetic core for the coil that creates a low-reluctance path for the magnetic flux, the amount of flux per ampere is high, and therefore the voltage opposing the current flow per ampere is high. The coil now appears as an almost open circuit. The current that flows in this case is the so-called magnetizing current that we often neglect in our calculations.

Next, let us introduce a second coil on the magnetic core. This creates our basic two-winding transformer. Because the two coils are on the same core, the flux cutting them is nearly the same. Also, because of the concentrating effect of the magnetic core, the volts per turn in both coils are nearly the same. We use this characteristic to create the familiar transformer, which provides different voltage levels at the coil terminals by the ratio of the number of turns in each coil.

This is a good time to introduce the concepts of mutual flux  $(\Phi_M)$  and leakage flux  $(\Phi_L)$ . The coupling between the coils is never perfect. In Fig. 1, the flux shown as  $\Phi_{PL}$ , generated by  $n_P \cdot i_P$ , does not couple to the secondary coil. Similarly, the flux shown as  $\Phi_{SL}$ , generated by  $n_S \cdot i_S$ , does not couple with the primary coil. This flux is called leakage flux, and it results in a voltage being induced in each coil that opposes the flow of current, per (1). This is represented as the leakage impedance in the equivalent circuit of the transformer.



Fig. 1. Fluxes in a transformer.

### B. Magnetic Circuit Parameters

It is also useful to review magnetic circuits before going on. One way to help electrical engineers understand the magnetic circuit is to equate its basic parameters to its equivalent electrical circuit parameters. Referring to (2):

- The flux, Φ, in a magnetic circuit is equivalent to the current in an electric circuit.
- The AT quantity, n i, in a magnetic circuit is equivalent to the voltage in an electric circuit.
- An unsaturated magnetic core has low reluctance,  $\Re$ , so it is like a conductor (reluctance is the magnetic equivalent of resistance in an electric circuit).
- Air has high reluctance,  $\Re$ , so it is like an open circuit.

Fig. 2 is an electrical analogy to the magnetic circuit shown in Fig. 1.  $n_p \cdot i_p = AT_p$  is analogous to a voltage source in Fig. 2. Similarly,  $n_s \cdot i_s = AT_s$  is also analogous to a voltage source in Fig. 2. The right core leg in Fig. 1 is a lowreluctance path that shorts the top and bottom of the magnetic circuit together. According to Kirchhoff's voltage law, the analogy in Fig. 2 is that the two voltage sources must sum to zero around the voltage loop. Similarly,  $AT_P$  and  $AT_S$  must sum to zero in the magnetic circuit along the closed loop core path.



Fig. 2. Electrical analogy to Fig. 1.

Now, let us close the electrical circuit on the secondary terminals in Fig. 1 through a load impedance. Because there is a voltage across the secondary terminals, current  $i_S$  flows through the load impedance, and  $i_P$  flows in the primary coil, such that the ATs sum to zero in the core leg (neglecting the magnetizing current, which sets up the mutual flux). This allows a power transfer between the windings through the magnetic core.

# C. Three-Phase Systems

Most of the systems that we deal with are three-phase systems. Similar to electrical systems that can be built as three-wire delta or four-wire wye (star) systems, the magnetic circuit of a three-phase transformer can be built as a threelegged core or with a low-reluctance return path. Examples of this type of core include four-legged, five-legged, shell type, and three single-phase cores. To help maintain the familiar concepts of three-wire versus four-wire systems, we will refer to all core construction that provides a low-reluctance return path as a four-legged core. As discussed in Section II, Subsection B, a return (fourth) core leg acts as a magnetic short circuit that forces the AT to sum to zero on each core leg, neglecting the leakage flux:

$$AT_1 = AT_2 = AT_3 = 0$$
 (3)

where:

AT = the sum of ampere-turns in all coils on the same core leg.

1, 2, 3 = the three core legs with coils.

A three-legged core requires further analysis. We use symmetrical component theory to analyze unbalanced threephase electrical systems. This technique is useful in understanding the differences between a three-legged and four-legged core. Symmetrical components let us break down a set of three-phase currents into the following components:

- Positive sequence
- Negative sequence
- Zero sequence

The positive- and negative-sequence components of the current are balanced and sum to zero. The AT and resulting flux from these components of the current flowing in the coils of the transformer also sum to zero, so there is no need for a return path for flux. Thus, there is no difference between a three-legged and four-legged core for the positive- and negative-sequence components of the phase currents.

On the other hand, zero-sequence currents are in phase with each other and equal in magnitude. Recall that 1x10 flows in each phase per symmetrical component theory. Because the zero-sequence currents do not sum to zero, they require a return path to flow. This zero-sequence return path must exist for the currents in the electrical circuit and for the fluxes in the magnetic circuit, or the zero-sequence current cannot flow. As we might expect, the difference between a three-legged and four-legged core lies in what happens during zero-sequence unbalances on the power system.

If the zero-sequence current in all windings provides balancing ATs (i.e., if there is equal 1x10 AT in each of the three primary coils versus each of the three secondary coils), the fluxes sum to zero on each core leg, and there is no need for the fourth core leg. For example, in many cases, a delta winding is included. A delta winding appears as an electrical short circuit to zero-sequence voltage on the associated wyeconnected winding. Thus, for any zero-sequence current flowing in the wye-connected windings, there is an easy path for compensating ATs to flow in the delta winding. In other words, a circulating current in the delta winding provides the zero-sequence flux that balances the flux along each core leg to zero. In this case, the AT on each core leg cancels by nature, and there is little flux that needs a return path.

But, in some cases, there will be zero-sequence ATs developed in one three-phase set of coils that are not completely compensated by zero-sequence current in the other three-phase set of coils. The flux that is driven by these ATs, per (2), closes through the high-reluctance path outside the magnetic core (i.e., the insulating oil and transformer tank). Going back to (1), a coil with a high-reluctance path, such as air, results in relatively low levels of flux. Low levels of flux in the coil result in relatively low electrical impedance for that coil. This low electrical impedance is only associated with zero-sequence unbalance. The phenomenon of zero-sequence current flowing in wye-connected windings of a three-legged core transformer without a compensating delta winding is commonly referred to as a phantom delta, phantom tertiary, or tank delta.

To summarize, a three-legged core transformer with wye and delta windings is a low-impedance path for zero-sequence current because of the electrical short circuit of the delta winding. A three-legged core transformer without a delta winding becomes a low-impedance path for zero-sequence current flow because of the low magnetizing impedance of the oil/air path outside the transformer core.

This seems counterintuitive, because oil and air have low relative permeability. However, it is this low permeability of oil and air that results in a high-reluctance path. A highreluctance path yields a low flux value from the zero-sequence current flow. A low flux value means a low opposing voltage is induced and therefore a low-inductance value. From a zerosequence current point of view, the transformer windings have a low impedance.

# D. AT Equations for Three-Legged Core Transformers

It is desirable to be able to ignore the core construction when designing transformer protection. To do this, we can sum the ATs around the three magnetic circuit loops created by the three core legs. Two sets of equations are possible for the transformer shown in Fig. 3. Creating the loops around a pair of core legs in a clockwise direction yields:

$$(AT_{P1} + AT_{S1}) - (AT_{P2} + AT_{S2}) = 0 = AT_1 - AT_2 (AT_{P2} + AT_{S2}) - (AT_{P3} + AT_{S3}) = 0 = AT_2 - AT_3$$
(4)  
 (AT\_{P3} + AT\_{S3}) - (AT\_{P1} + AT\_{S1}) = 0 = AT\_3 - AT\_1

Creating the loops around a pair of core legs in a counterclockwise direction yields:

$$(AT_{P1} + AT_{S1}) - (AT_{P3} + AT_{S3}) = 0 = AT_1 - AT_3 (AT_{P2} + AT_{S2}) - (AT_{P1} + AT_{S1}) = 0 = AT_2 - AT_1$$
(5)  
 (AT\_{P3} + AT\_{S3}) - (AT\_{P2} + AT\_{S2}) = 0 = AT\_3 - AT\_2

In any case, ATs on any pair of core legs are equal at all times for three- and four-legged cores, as well as for transformers built from single-phase units.



Fig. 3. ATs on a three-legged core.

### **III. SHORT CIRCUITS IN TRANSFORMERS**

Now that we have refreshed our knowledge on transformer theory, let us consider various faults a transformer may experience. As we can see from Fig. 4, transformer faults can be divided into three basic categories—winding-to-ground faults (1, 4, 8), winding-to-winding faults (2, 5), and turn-to-turn faults on the same winding (3, 6, 7). Examining each of these categories separately, we will discuss the magnitude of the fault current, the method for detecting the fault, and the sensitivity of each method.



Fig. 4. Internal transformer faults.

# A. Characteristics of Partial Winding Faults

With reference to Fig. 5, consider a turn-to-turn fault (S1 closed, S2 opened) or a winding-to-ground fault close to the grounded neutral (S1 opened, S2 closed). The H winding has a short circuit on the leg labeled 1; the X winding represents the healthy winding(s), k is the fractional number of shorted turns (few percent), and  $i_F$  is the fault current. No assumption is made regarding the winding connection or core type.



Fig. 5. A general model of a partial winding fault.

The difference in ATs on the affected core leg before and during the fault amounts to the ATs produced by the fault current  $(i_F)$  flowing through the number of shortened turns,  $(k \cdot n_H)$ . As explained in Section II, the ATs are equal between all three legs, meaning the fault current produces equal ATs across the other two legs of the core.

As explained later in Section IV, transformer differential protection monitors ATs of an unfaulted transformer. Therefore, the differential current reflects the unmonitored extra ATs produced by a partial winding fault, which is:

$$\mathbf{i}_{\text{DFF}} \sim \mathbf{k} \cdot \mathbf{n}_{\text{H}} \cdot \mathbf{i}_{\text{F}} \tag{6}$$

However, the exact level of the fault current (and thus the level of the differential current) is a complex function depending on the resistance and inductance of the shorted portion of the winding, distribution of fluxes due to the winding location with respect to other windings on the magnetic core, and possible local saturation of the core. These factors are beyond the scope of this paper.

### B. Winding-to-Ground Faults

Winding-to-ground faults can be subdivided into two categories—namely faults farther away from the neutral point (faults that have a greater driving voltage) and faults close to the neutral point.

Winding-to-ground faults farther from the neutral point have lower fault current in the faulted loop but result in greater change in phase currents on the windings that are on the same core as the faulted winding. The large change in phase current is due to the increase in the voltage driving the fault current. The end result is that protection elements using phase currents as operating quantities, such as phase differential, can readily detect this type of fault and speedily isolate the transformer, thereby limiting damage.

Winding-to-ground faults close to the transformer neutral (see Section III, Subsection A) are associated with a low driving voltage. The result of this is that the increase in differential current is small, as per (6), but the current in the faulted loop is large. If not isolated in a timely manner, this current will create considerable damage.

As seen in Fig. 5, the current in the faulted loop circulates (returns) via the transformer neutral, and if we measure this loop current, we could reliably detect this type of fault. One method uses the neutral current in conjunction with zero-sequence current calculated from phase currents to create a zero-sequence differential element. This is known as the REF method and is explained in Section VII.

# C. Turn-to-Turn Faults

Turn-to-turn faults also produce a relatively small change in differential current, while fault current in the loop is relatively large and can cause significant damage if not detected and isolated rapidly (see Section III, Subsection A).

These faults are detectable via a negative-sequence differential function, as described in Section VI.

# D. Winding-to-Winding Faults

Winding-to-winding faults produce a large change in the phase currents of windings that are involved in the fault. This means protection elements using phase currents as operating quantities can readily detect these faults. Typically, phase differential elements are used to detect and isolate these types of faults.

### IV. TRANSFORMER DIFFERENTIAL PROTECTION

This section makes a connection between the well-known current compensation rules for transformer differential protection and AT balance equations of the protected transformer. It also explains why a differential function that follows the practical current compensation rules ensures detection of internal transformer short circuits.

Going back to first principles and balancing the ATs allow for the development of current compensation rules for nonstandard transformer connections, as illustrated in Section V.

Consider the wye-delta power transformer in Fig. 6.



Fig. 6. Sample wye-delta transformer.

Neglecting the magnetizing branch of the core, we can write the following AT equations for the three legs of the core:

$$AT_{1} = n_{H} \bullet i_{HA} + n_{X} \bullet i_{X1}$$

$$AT_{2} = n_{H} \bullet i_{HB} + n_{X} \bullet i_{X2}$$

$$AT_{3} = n_{H} \bullet i_{HC} + n_{X} \bullet i_{X3}$$
(7)

Kirchhoff's current law allows writing the current balance equations for the nodes of the delta winding:

$$\begin{split} i_{XA} & -i_{X1} + i_{X3} = 0 \\ i_{XB} & -i_{X2} + i_{X1} = 0 \\ i_{XC} & -i_{X3} + i_{X2} = 0 \end{split} \tag{8}$$

Note that we cannot solve for the delta-winding currents, because any common component in these currents cancels and cannot be measured at the transformer bushings. Instead, we eliminate the delta-winding currents in the equations from (8) using the AT equations in (7). This yields the following:

$$n_{X} \bullet i_{XA} + n_{H} \bullet (i_{HA} - i_{HC}) - (AT_{1} - AT_{3}) = 0$$
  

$$n_{X} \bullet i_{XB} + n_{H} \bullet (i_{HB} - i_{HA}) - (AT_{2} - AT_{1}) = 0$$
  

$$n_{X} \bullet i_{XC} + n_{H} \bullet (i_{HC} - i_{HB}) - (AT_{3} - AT_{2}) = 0$$
(9)

As explained in Section II, ATs on any leg pair are equal regardless of the core type. By pairing core legs, we can write AT balance equations around the resulting magnetic circuit loops, similar to the way we would around an electrical circuit loop using Kirchhoff's voltage law.

Given the equations from (5), the balance equations in (9) for the transformer in Fig. 6 become:

$$n_{X} \bullet i_{XA} + n_{H} \bullet (i_{HA} - i_{HC}) = 0$$
  

$$n_{X} \bullet i_{XB} + n_{H} \bullet (i_{HB} - i_{HA}) = 0$$
  

$$n_{X} \bullet i_{XC} + n_{H} \bullet (i_{HC} - i_{HB}) = 0$$
(10)

Note that  $\frac{n_{\rm H}}{n_{\rm X}} = \frac{V_{\rm H}}{\sqrt{3} \cdot V_{\rm X}}$  and therefore:

$$\begin{split} & i_{XA} + \frac{V_{H}}{V_{X}} \bullet \frac{1}{\sqrt{3}} \left( i_{HA} - i_{HC} \right) = 0 \\ & i_{XB} + \frac{V_{H}}{V_{X}} \bullet \frac{1}{\sqrt{3}} \left( i_{HB} - i_{HA} \right) = 0 \\ & i_{XC} + \frac{V_{H}}{V_{X}} \bullet \frac{1}{\sqrt{3}} \left( i_{HC} - i_{HB} \right) = 0 \end{split}$$
(11)

Equations in (11) hold true as long as the transformer conforms to the assumed model in Fig. 6. Departures from this model, including turn-to-turn faults, interwinding faults, ground faults, or tap changer operation altering the turn ratio, will cause an unbalance. Therefore, expressions in (11) are used as operating signals of a transformer differential protection element:

$$i_{DIF(1)} = \left| i_{XA} + \frac{V_{H}}{V_{X}} \cdot \frac{1}{\sqrt{3}} (i_{HA} - i_{HC}) \right|$$

$$i_{DIF(2)} = \left| i_{XB} + \frac{V_{H}}{V_{X}} \cdot \frac{1}{\sqrt{3}} (i_{HB} - i_{HA}) \right|$$

$$i_{DIF(3)} = \left| i_{XC} + \frac{V_{H}}{V_{X}} \cdot \frac{1}{\sqrt{3}} (i_{HC} - i_{HB}) \right|$$
(12)

where: || denotes magnitude estimation (filtering).

The restraining signal matches the differential signal but responds to the sum of magnitudes rather than the sum of vectors:

$$\begin{split} \dot{i}_{RST(1)} &= \left| \dot{i}_{XA} \right| + \frac{V_{H}}{V_{X}} \cdot \frac{1}{\sqrt{3}} \left| \dot{i}_{HA} - \dot{i}_{HC} \right| \\ \dot{i}_{RST(2)} &= \left| \dot{i}_{XB} \right| + \frac{V_{H}}{V_{X}} \cdot \frac{1}{\sqrt{3}} \left| \dot{i}_{HB} - \dot{i}_{HA} \right| \\ \dot{i}_{RST(3)} &= \left| \dot{i}_{XC} \right| + \frac{V_{H}}{V_{X}} \cdot \frac{1}{\sqrt{3}} \left| \dot{i}_{HC} - \dot{i}_{HB} \right| \end{split}$$
(13)

The equations in (12) and (13) have been used for decades for the protection of wye-delta transformers. Protective relays are connected to CTs, which can be connected either in wye or delta. Using phase-to-phase currents (delta-connected CTs) in the wye winding and phase currents (wye-connected CTs) in the delta winding is commonly referred to as vector compensation, a phase shift, or delta compensation. Furthermore, using the phase-to-phase current (deltaconnected CTs) in the wye winding also removes the zerosequence current component and is therefore commonly referred to as zero-sequence removal. The application of the voltage ratio is referred to as ratio matching or tap compensation.

Considering the vector diagram in Fig. 6, these rules of transformer current compensation yield the diagram in Fig. 7 for applications with electromechanical relays. In applications with microprocessor-based relays, these rules lead to selection of the following compensation matrices:

$$\Gamma_{\rm H} = \frac{1}{\sqrt{3}} \bullet \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}, \ T_{\rm X} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
(14)



Fig. 7. Application with external CT compensation.

Regardless of the relay technology (electromechanical, static, microprocessor-based), the rules of phase shifting, zerosequence removal, and ratio matching have roots in matching ATs between pairs of the magnetic core legs. As such, the resulting differential currents signify the physical balance in the protected transformer, and when out of balance, they point to an internal short circuit. This creates a solid foundation of dependability of transformer differential protection beyond the security-oriented view of balancing the transformer currents for different phase shifts and ratios.

Development of the balance equations neglects the magnetizing current. The magnetizing current during inrush and overexcitation demonstrates itself as the differential signal and requires a separate treatment, typically in the form of harmonic blocking or restraining.

# V. TRANSFORMER DIFFERENTIAL PROTECTION BEYOND THE COMMON WYE OR DELTA CONNECTIONS

This section considers transformer connections beyond the standard wye or delta windings and progresses from autotransformers, through power zig-zag, Scott-T, and Le-Blanc transformers, to extended delta phase shifters in order to illustrate the derivation of proper transformer differential protection equations.

# A. Autotransformers

Consider the autotransformer in Fig. 8 with the series (S) and common (C) windings. The ATs on each of the core legs equal:

$$AT_{1} = n_{S} \bullet i_{HA} + n_{C} \bullet (i_{HA} + i_{XA})$$

$$AT_{2} = n_{S} \bullet i_{HB} + n_{C} \bullet (i_{HB} + i_{XB})$$

$$AT_{3} = n_{S} \bullet i_{HC} + n_{C} \bullet (i_{HC} + i_{YC})$$
(15)

$$AT_{1} = (n_{S} + n_{C}) \cdot i_{HA} + n_{C} \cdot i_{XA}$$

$$AT_{2} = (n_{S} + n_{C}) \cdot i_{HB} + n_{C} \cdot i_{XB}$$

$$AT_{3} = (n_{S} + n_{C}) \cdot i_{HC} + n_{C} \cdot i_{XC}$$
(16)



Fig. 8. An autotransformer connection.

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Equations from (16) explain the benefits of autotransformers compared with two-winding transformers the transformation ratio is increased by the common turns without adding extra turns in the series winding. This reduces both the copper and iron requirements and yields a more economical design with reduced losses, size, and weight.

Balancing the ATs per (5) leads to the following differential protection equations:

$$\begin{aligned} \mathbf{i}_{\mathrm{DIF}(1)} &= \left| \left( \mathbf{i}_{\mathrm{XA}} - \mathbf{i}_{\mathrm{XC}} \right) + \frac{\mathbf{V}_{\mathrm{H}}}{\mathbf{V}_{\mathrm{X}}} \bullet \left( \mathbf{i}_{\mathrm{HA}} - \mathbf{i}_{\mathrm{HC}} \right) \right| \\ \mathbf{i}_{\mathrm{DIF}(2)} &= \left| \left( \mathbf{i}_{\mathrm{XB}} - \mathbf{i}_{\mathrm{XA}} \right) + \frac{\mathbf{V}_{\mathrm{H}}}{\mathbf{V}_{\mathrm{X}}} \bullet \left( \mathbf{i}_{\mathrm{HB}} - \mathbf{i}_{\mathrm{HA}} \right) \right| \\ \mathbf{i}_{\mathrm{DIF}(3)} &= \left| \left( \mathbf{i}_{\mathrm{XC}} - \mathbf{i}_{\mathrm{XB}} \right) + \frac{\mathbf{V}_{\mathrm{H}}}{\mathbf{V}_{\mathrm{X}}} \bullet \left( \mathbf{i}_{\mathrm{HC}} - \mathbf{i}_{\mathrm{HB}} \right) \right| \end{aligned}$$
(17)  
here: 
$$\frac{\mathbf{V}_{\mathrm{H}}}{\mathbf{V}_{\mathrm{X}}} = \left( 1 + \frac{\mathbf{n}_{\mathrm{S}}}{\mathbf{n}_{\mathrm{C}}} \right).$$

Note that the differential protection equations for autotransformers follow the same current compensation rules as a wye-wye transformer. The appropriate compensation matrices in a microprocessor-based relay are:

$$T_{\rm H} = T_{\rm X} = \frac{1}{\sqrt{3}} \cdot \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$
(18)

Consider an autotransformer built with three single-phase banks or on a four-legged core. Such an autotransformer can be protected on a per-phase basis, taking advantage of the observation that ATs in each single-phase core equal zero, per (3):

$$i_{DIF(1)} = \begin{vmatrix} i_{XA} + \frac{V_{H}}{V_{X}} \bullet i_{HA} \end{vmatrix}$$

$$i_{DIF(2)} = \begin{vmatrix} i_{XB} + \frac{V_{H}}{V_{X}} \bullet i_{HB} \end{vmatrix}$$

$$i_{DIF(3)} = \begin{vmatrix} i_{XC} + \frac{V_{H}}{V_{X}} \bullet i_{HC} \end{vmatrix}$$
(19)

The appropriate compensation matrices in a microprocessor-based relay are:

$$T_{\rm H} = T_{\rm X} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
(20)

In some applications, CTs are available at the neutral side of the low-voltage winding. In such cases, we may apply a bus-like protection zone between the H, X, and neutral CTs because of the metallic connection between the windings. These zones monitor the electric circuit only, and therefore they are blind to turn-to-turn faults. In order to detect turn faults, the differential function must effectively monitor both the electric and magnetic circuits of the transformer.

Next, consider an autotransformer with a tertiary delta winding, as shown in Fig. 9.



Fig. 9. An autotransformer connection with a tertiary delta winding.

The electromagnetic balance for this transformer is:

$$\begin{aligned} AT_1 &= \left(n_S + n_C\right) \bullet i_{HA} + n_C \bullet i_{XA} + n_Y \bullet i_{Y1} \\ AT_2 &= \left(n_S + n_C\right) \bullet i_{HB} + n_C \bullet i_{XB} + n_Y \bullet i_{Y2} \\ AT_3 &= \left(n_S + n_C\right) \bullet i_{HC} + n_C \bullet i_{XC} + n_Y \bullet i_{Y3} \end{aligned}$$
(21)

Following the example of a wye-delta transformer in Section IV, we eliminate the winding currents and obtain the following equations for the differential function:

$$\begin{aligned} \mathbf{i}_{\text{DIF}(1)} &= \left| \left( \mathbf{i}_{\text{XA}} - \mathbf{i}_{\text{XC}} \right) + \frac{\mathbf{V}_{\text{H}}}{\mathbf{V}_{\text{X}}} \bullet \left( \mathbf{i}_{\text{HA}} - \mathbf{i}_{\text{HC}} \right) + \sqrt{3} \frac{\mathbf{V}_{\text{Y}}}{\mathbf{V}_{\text{X}}} \bullet \mathbf{i}_{\text{YA}} \right| \\ \mathbf{i}_{\text{DIF}(2)} &= \left| \left( \mathbf{i}_{\text{XB}} - \mathbf{i}_{\text{XA}} \right) + \frac{\mathbf{V}_{\text{H}}}{\mathbf{V}_{\text{X}}} \bullet \left( \mathbf{i}_{\text{HB}} - \mathbf{i}_{\text{HA}} \right) + \sqrt{3} \frac{\mathbf{V}_{\text{Y}}}{\mathbf{V}_{\text{X}}} \bullet \mathbf{i}_{\text{YB}} \right| \tag{22}$$
$$\mathbf{i}_{\text{DIF}(3)} &= \left| \left( \mathbf{i}_{\text{XC}} - \mathbf{i}_{\text{XB}} \right) + \frac{\mathbf{V}_{\text{H}}}{\mathbf{V}_{\text{X}}} \bullet \left( \mathbf{i}_{\text{HC}} - \mathbf{i}_{\text{HB}} \right) + \sqrt{3} \frac{\mathbf{V}_{\text{Y}}}{\mathbf{V}_{\text{X}}} \bullet \mathbf{i}_{\text{YC}} \right| \end{aligned}$$

The appropriate compensation matrices in a microprocessor-based relay are:

$$\Gamma_{\rm H} = \Gamma_{\rm X} = \frac{1}{\sqrt{3}} \cdot \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}, \ \Gamma_{\rm Y} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
(23)

If the tertiary delta winding is not brought out (buried), the equations in (22) simplify to (17) as if the transformer were a two-winding transformer. The buried delta winding is still protected to a degree, because it takes part in the magnetic circuit of the transformer. The case of a buried delta is equivalent to the case of a power delta winding with the breaker opened. However, the degree of protection is limited by the fact that the power rating of the tertiary delta in autotransformer. This results in a small fault current when reflected from the delta winding to the high- and low-voltage windings of the autotransformer.

Also, ground faults on an ungrounded, buried delta winding cannot be detected based on current signals.

Let us go back to a single-phase or a four-legged core design in the context of a buried delta winding. Quite often, the winding currents in the delta winding are measured to polarize directional line protective relays  $[i_{Y1}, i_{Y2}, and i_{Y3} in (21)]$ . With the inside delta currents available, the autotransformer can be protected as a three-winding transformer, balancing the phase currents per (21), (20), and (19):

$$\begin{aligned} \dot{\mathbf{i}}_{\mathrm{DIF}(1)} &= \left| \dot{\mathbf{i}}_{\mathrm{XA}} + \frac{\mathbf{V}_{\mathrm{H}}}{\mathbf{V}_{\mathrm{X}}} \cdot \dot{\mathbf{i}}_{\mathrm{HA}} + \sqrt{3} \cdot \frac{\mathbf{V}_{\mathrm{H}}}{\mathbf{V}_{\mathrm{Y}}} \cdot \dot{\mathbf{i}}_{\mathrm{Y1}} \right| \\ \dot{\mathbf{i}}_{\mathrm{DIF}(2)} &= \left| \dot{\mathbf{i}}_{\mathrm{XB}} + \frac{\mathbf{V}_{\mathrm{H}}}{\mathbf{V}_{\mathrm{X}}} \cdot \dot{\mathbf{i}}_{\mathrm{HB}} + \sqrt{3} \cdot \frac{\mathbf{V}_{\mathrm{H}}}{\mathbf{V}_{\mathrm{Y}}} \cdot \dot{\mathbf{i}}_{\mathrm{Y2}} \right| \\ \dot{\mathbf{i}}_{\mathrm{DIF}(3)} &= \left| \dot{\mathbf{i}}_{\mathrm{XC}} + \frac{\mathbf{V}_{\mathrm{H}}}{\mathbf{V}_{\mathrm{X}}} \cdot \dot{\mathbf{i}}_{\mathrm{HC}} + \sqrt{3} \cdot \frac{\mathbf{V}_{\mathrm{H}}}{\mathbf{V}_{\mathrm{Y}}} \cdot \dot{\mathbf{i}}_{\mathrm{Y3}} \right| \end{aligned}$$
(24)

B. Power Zig-Zag Transformers

Consider the wye/zig-zag connection in Fig. 10.



Fig. 10. Wye/zig-zag power transformer connection.

The ATs on each of the core legs equal:

$$AT_{1} = n_{H} \cdot i_{HA} + \frac{n_{X}}{2} \cdot (i_{XA} - i_{XB})$$

$$AT_{2} = n_{H} \cdot i_{HB} + \frac{n_{X}}{2} \cdot (i_{XB} - i_{XC})$$

$$AT_{3} = n_{H} \cdot i_{HC} + \frac{n_{X}}{2} \cdot (i_{XC} - i_{XA})$$
(25)

Comparing the ATs on core leg pair 1 and 3 per (5) yields:

$$AT_{1} - AT_{3} = n_{H} \cdot (i_{HA} - i_{HC}) + \frac{n_{X}}{2} \cdot (2 \cdot i_{XA} - i_{XB} - i_{XC})$$
(26)

Considering that  $n_{\rm H} \sim \frac{V_{\rm H}}{\sqrt{3}}$  and  $n_{\rm X} \sim 2 \cdot \frac{V_{\rm X}}{3}$ , we obtain:

$$AT_{1} - AT_{3} = \frac{1}{\sqrt{3}} (i_{HA} - i_{HC}) + \frac{V_{X}}{V_{H}} \cdot \frac{1}{3} \cdot (2 \cdot i_{XA} - i_{XB} - i_{XC})$$
(27)

which yields the following differential signal:

$$i_{DIF(1)} = \left| \frac{1}{\sqrt{3}} (i_{HA} - i_{HC}) + \frac{V_X}{V_H} \cdot \frac{1}{3} \cdot (2 \cdot i_{XA} - i_{XB} - i_{XC}) \right|$$
(28)

For the other two relay elements, we use (28) and rotate the subscript indices:

$$i_{DIF(2)} = \left| \frac{1}{\sqrt{3}} (i_{HB} - i_{HA}) + \frac{V_X}{V_H} \cdot \frac{1}{3} \cdot (2 \cdot i_{XB} - i_{XC} - i_{XA}) \right|$$
(29)

$$i_{DIF(3)} = \left| \frac{1}{\sqrt{3}} (i_{HC} - i_{HB}) + \frac{V_X}{V_H} \cdot \frac{1}{3} \cdot (2 \cdot i_{XC} - i_{XA} - i_{XB}) \right|$$
(30)

The appropriate compensation matrices in a microprocessor-based relay are:

$$T_{\rm H} = \frac{1}{\sqrt{3}} \cdot \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}, \quad T_{\rm X} = \frac{1}{3} \cdot \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \quad (31)$$

# *C.* Summary of Compensation Matrices for Wye, Delta, and Zig-Zag Windings

The  $T_X$  matrix in (31) is often referred to as a double-delta connection, because it is equivalent to connecting the main CTs in delta and their secondary circuits in delta again when applying external compensation for electromechanical relays. The  $T_H$  matrix in (31) is referred to as a single-delta connection. The  $T_Y$  matrix in (23) is referred to as a wye connection. In general, transformer delta windings require a wye compensation matrix, wye windings require a single-delta compensation matrix, and zig-zag windings require a double-delta compensation matrix.

# D. Scott-T Transformers

The Scott-T transformers shown in Fig. 11 historically have been used to connect two- and three-phase systems. They are still used in applications with two-phase motors, such as in mining or railway systems and, recently, in power electronics.



### Fig. 11. A Scott-T transformer connection.

The transformer is made of two independent cores. We write AT balance equations for each core separately:

$$\frac{n_{\rm H}}{2} (i_{\rm HA} - i_{\rm HC}) + n_{\rm X} \bullet i_{\rm XD} = 0$$
 (32)

$$\frac{\sqrt{3} \cdot \mathbf{n}_{\mathrm{H}}}{2} \cdot \dot{\mathbf{i}}_{\mathrm{HB}} + \mathbf{n}_{\mathrm{X}} \cdot \dot{\mathbf{i}}_{\mathrm{XQ}} = 0$$
(33)

By replacing the turn counts with nominal voltages and introducing the differential currents, we obtain:

$$\mathbf{i}_{\text{DIF}(1)} = \left| \frac{\mathbf{V}_{\text{H}}}{2 \cdot \mathbf{V}_{\text{X}}} \cdot \left( \mathbf{i}_{\text{HA}} - \mathbf{i}_{\text{HC}} \right) + \mathbf{i}_{\text{XD}} \right|$$
(34)

$$\mathbf{i}_{\text{DIF}(2)} = \left| \frac{\sqrt{3} \cdot \mathbf{V}_{\text{H}}}{2 \cdot \mathbf{V}_{\text{X}}} \cdot \mathbf{i}_{\text{HB}} + \mathbf{i}_{\text{XQ}} \right|$$
(35)

where:

 $V_{\rm H}$  = the nominal voltage in the three-phase system (phase to phase).

 $V_X$  = the nominal winding voltage in the two-phase system (phase to ground).

The two differential currents can be created in a standard four-winding transformer relay using wye compensation matrices applied to the following four currents (windings):

$$\begin{bmatrix} i_{D} \\ i_{Q} \\ 0 \end{bmatrix}, \begin{bmatrix} i_{A} \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -i_{C} \\ 0 \\ i_{B} \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
(36)

The above currents are created by appropriately wiring the five currents of the Scott-T transformer to four current terminals of the relay. Three tap compensation factors are required for the first (Tap 1), fourth (Tap 2), and second and third (Tap 3) windings in (36), as per the scaling factors in (34) and (35).

Alternatively, we can create the A-phase minus C-phase currents by connecting secondaries of the main CTs and using the following three windings in a standard transformer relay:

$$\begin{bmatrix} i_{D} \\ i_{Q} \\ 0 \end{bmatrix}, \begin{bmatrix} i_{A} - i_{C} \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ i_{B} \\ 0 \end{bmatrix}$$
(37)

### E. Le-Blanc Transformers

Le-Blanc transformers also serve the purpose of feeding two-phase loads from three-phase systems but are built using a three-legged core, as shown in Fig. 12.



Fig. 12. Le-Blanc transformer connection.

The AT balance equations are:

$$AT_{1} = n_{H} \cdot i_{H1} - (\sqrt{3} - 1) \cdot n_{X} \cdot i_{XQ} + n_{X} \cdot i_{XD}$$

$$AT_{2} = n_{H} \cdot i_{H2} + n_{X} \cdot i_{XQ}$$

$$AT_{3} = n_{H} \cdot i_{H3} - (\sqrt{3} - 1) \cdot n_{X} \cdot i_{XQ} - n_{X} \cdot i_{XD}$$
(38)

Comparing the ATs in pairs of core legs per (5) allows eliminating the winding currents in the delta winding based on (8) and writing the differential protection equations. We further use nominal voltages defined for the Scott-T transformer above to eliminate the turn counts:

$$\mathbf{i}_{\text{DIF}(1)} = \left| \mathbf{i}_{\text{HA}} + \frac{2 \cdot \mathbf{V}_{\text{X}}}{\sqrt{3} \cdot \mathbf{V}_{\text{H}}} \mathbf{i}_{\text{XD}} \right|$$
(39)

$$\mathbf{i}_{\text{DIF}(2)} = \left| \mathbf{i}_{\text{HB}} + \frac{\mathbf{V}_{\text{X}}}{\sqrt{3} \cdot \mathbf{V}_{\text{H}}} \left( \sqrt{3} \mathbf{i}_{\text{XQ}} - \mathbf{i}_{\text{XD}} \right) \right|$$
(40)

$$\mathbf{i}_{\text{DIF}(3)} = \left| \mathbf{i}_{\text{HC}} - \frac{\mathbf{V}_{\text{X}}}{\sqrt{3} \cdot \mathbf{V}_{\text{H}}} \left( \sqrt{3} \mathbf{i}_{\text{XQ}} + \mathbf{i}_{\text{XD}} \right) \right|$$
(41)

In order to simplify implementation in a standard transformer relay, we can consider combining the second and third differential terms by subtracting (40) and (41) as follows:

$$i_{DIF(23)} = \left| i_{HB} - i_{HC} + \frac{2 \cdot V_X}{V_H} i_{XQ} \right|$$
 (42)

A multiwinding standard transformer differential relay with wye compensation matrices can be used to implement (39) through (41) or (39) and (42), as similarly explained for the Scott-T transformer above.

# F. Extended Delta Phase-Shifting Transformers

Consider the extended delta phase-shifting transformer (PST) in Fig. 13 with the series (S) and exciting (E) windings. The AT equations are:

$$AT_{1} = \mathbf{D} \cdot \frac{\mathbf{n}_{S}}{2} \cdot \mathbf{i}_{SA} - \mathbf{D} \cdot \frac{\mathbf{n}_{S}}{2} \cdot \mathbf{i}_{LA} + \mathbf{n}_{E} \cdot \mathbf{i}_{E1}$$

$$AT_{2} = \mathbf{D} \cdot \frac{\mathbf{n}_{S}}{2} \cdot \mathbf{i}_{SB} - \mathbf{D} \cdot \frac{\mathbf{n}_{S}}{2} \cdot \mathbf{i}_{LB} + \mathbf{n}_{E} \cdot \mathbf{i}_{E2}$$

$$AT_{3} = \mathbf{D} \cdot \frac{\mathbf{n}_{S}}{2} \cdot \mathbf{i}_{SC} - \mathbf{D} \cdot \frac{\mathbf{n}_{S}}{2} \cdot \mathbf{i}_{LC} + \mathbf{n}_{E} \cdot \mathbf{i}_{E3}$$

$$(43)$$

In (43), the source current works with half of the series windings multiplied by the tap changer position; the load current works with half of the series windings multiplied by the tap position but in the opposite direction; and the excitation current works with the number of excitation winding turns. The source, load, and excitation currents are tied together with Kirchhoff's current equations as follows:

.

$$i_{SA} + i_{LA} = i_{E3} - i_{E2}$$
  
 $i_{SB} + i_{LB} = i_{E1} - i_{E3}$   
 $i_{SC} + i_{LC} = i_{E2} - i_{E1}$ 
(44)



Fig. 13. Extended delta phase-shifting transformer (where S is source side, L is load side, D = 0 is neutral control, D = +1 is full advance control, and D = -1 is full retard control).

Comparing the ATs on each pair of core legs in (5) and substituting the excitation currents with the source and load currents per (44) yields:

$$\begin{split} \dot{i}_{DIF(1)} &= \left| n_{S} \frac{D}{2} \left( \left( i_{SA} - i_{LA} \right) - \left( i_{SC} - i_{LC} \right) \right) + n_{E} \left( i_{SB} + i_{LB} \right) \right| \\ \dot{i}_{DIF(2)} &= \left| n_{S} \frac{D}{2} \left( \left( i_{SB} - i_{LB} \right) - \left( i_{SA} - i_{LA} \right) \right) + n_{E} \left( i_{SC} + i_{LC} \right) \right| \quad (45) \\ \dot{i}_{DIF(3)} &= \left| n_{S} \frac{D}{2} \left( \left( i_{SC} - i_{LC} \right) - \left( i_{SB} - i_{LB} \right) \right) + n_{E} \left( i_{SA} + i_{LA} \right) \right| \end{split}$$

As expected, the transformer differential protection equations are dependent on the position of the tap changer, D. Note that the protection method in (45) does not require measuring the excitation currents. A standard multiwinding transformer relay can be used to implement equations from (45). First, let us rewrite (45) to a more convenient form:

$$\begin{split} \dot{i}_{DIF(1)} &= \left| n_{S} \frac{D}{2} \left( \left( i_{SA} - i_{SC} \right) + \left( i_{LC} - i_{LA} \right) \right) + n_{E} \left( i_{SB} + i_{LB} \right) \right| \\ \dot{i}_{DIF(2)} &= \left| n_{S} \frac{D}{2} \left( \left( i_{SB} - i_{SA} \right) + \left( i_{LA} - i_{LB} \right) \right) + n_{E} \left( i_{SC} + i_{LC} \right) \right| \quad (46) \\ \dot{i}_{DIF(3)} &= \left| n_{S} \frac{D}{2} \left( \left( i_{SC} - i_{SB} \right) + \left( i_{LB} - i_{LC} \right) \right) + n_{E} \left( i_{SA} + i_{LA} \right) \right| \end{split}$$

The six currents can be wired to four relay current terminals (windings) as follows:

$$\begin{bmatrix} i_{SA} \\ i_{SB} \\ i_{SC} \end{bmatrix}, \begin{bmatrix} -i_{LA} \\ -i_{LB} \\ -i_{LC} \end{bmatrix}, \begin{bmatrix} i_{SB} \\ i_{SC} \\ i_{SA} \end{bmatrix}, \begin{bmatrix} i_{LB} \\ i_{LC} \\ i_{LA} \end{bmatrix}$$
(47)

The first two windings use the following compensation matrix:

$$\frac{1}{\sqrt{3}} \cdot \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$
(48)

The latter two windings use the following matrix:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
(49)

Alternatively, external CT summation can be arranged, and a two-winding transformer relay can be used with the following currents:

$$\begin{bmatrix} i_{SA} - i_{LA} \\ i_{SB} - i_{LB} \\ i_{SC} - i_{LC} \end{bmatrix}, \begin{bmatrix} i_{SB} + i_{LB} \\ i_{SC} + i_{LC} \\ i_{SA} + i_{LA} \end{bmatrix}$$
(50)

and matrices (48) and (49), respectively.

The challenge is to implement a variable tap coefficient dependent on D for the first two windings in (47) or the first winding in (50). This variability can be accommodated with a coarse granularity via multiple settings groups. We need to remember that D can be negative (retard versus advance control).

# VI. NEGATIVE-SEQUENCE DIFFERENTIAL

As described in Section III, when a turn-to-turn fault occurs, the magnitudes of the phase currents of the associated winding do not change significantly. However, as a result of the fault, the symmetry of the phase currents on both the primary and secondary sides of the transformer (or any other piece of equipment) is disturbed. The change in symmetry during this type of fault favors using negative-sequence currents to detect it.

But before we examine the negative-sequence differential element, let us understand how the symmetrical components in the terminal currents transform into the differential currents of the standard phase differential element, limiting its ability to detect these types of faults. As explained in Section IV and commonly used in microprocessor-based relays, the phase differential function applies matrix compensation to transformer terminal currents:

$$\begin{bmatrix} I_{A} \\ I_{B} \\ I_{C} \end{bmatrix}_{C} = T \cdot \begin{bmatrix} I_{A} \\ I_{B} \\ I_{C} \end{bmatrix}$$
(51)

We can use (51) to examine how the sequence components (zero = 0, positive = 1, negative = Q) in the terminal currents are converted into sequence components of the compensated currents.

$$\begin{bmatrix} I_0 \\ I_1 \\ I_Q \end{bmatrix}_C = \mathbf{A} \cdot \mathbf{T} \cdot \mathbf{A}^{-1} \cdot \begin{bmatrix} I_0 \\ I_1 \\ I_Q \end{bmatrix}, \ \mathbf{A} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$
(52)

For example, the transformation matrix  $T_H$  in (14), when expressed for symmetrical components, is:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & e^{-j30^{0}} & 0 \\ 0 & 0 & e^{j30^{0}} \end{bmatrix}$$
(53)

Generally, the compensation matrix for a symmetrical component of any three-phase transformer is:

$$\begin{bmatrix} 0/1 & 0 & 0 \\ 0 & e^{j\delta} & 0 \\ 0 & 0 & e^{-j\delta} \end{bmatrix}$$
(54)

and can be interpreted as follows:

- $\delta$  is the nameplate phase shift of the transformer.
- The positive sequence in the terminal current shifted by the vector group angle transforms into the positive sequence in the compensated current ( $e^{j\delta}$ ).
- The negative sequence in the terminal current shifted by the minus vector group angle transforms into the negative sequence in the compensated current (e<sup>-jδ</sup>).
- The zero sequence is normally removed (value 0), except when the unity transformation matrix (49) is used as applicable to delta windings (value 1).
- The three sequence components are decoupled [the off-diagonal elements in (54) are zeros].

The phase-restraining signal is a sum of the magnitudes of the compensated phase currents. Considering (54), this means that both the positive- and negative-sequence components in the terminal currents produce restraint for the phase differential function.

For a balanced system, the differential current is approximately zero, and the restraining signal is proportional to the positive-sequence current (load). This means that the sensitivity of the element is directly impacted by the load current. With this in mind, let us return to the turn-to-turn fault. We know that this type of fault creates a disturbance of the symmetry of both primary- and secondary-side phase currents, which manifests itself as a negative-sequence current. If the transformer is lightly loaded (i.e., the restraint current is small), then this element has good sensitivity and may detect turn-to-turn faults. However, as the load increases, the sensitivity of the phase differential element is reduced, and the ability to detect turn-to-turn faults is diminished.

If we use pure negative-sequence current to build a differential element, we have the advantage of making the element highly sensitive and relatively independent from load.

In this respect, note that the compensated currents are summed to form the phase differential current. Considering (54), this means that the sequence components in the differential currents are sums of appropriately shifted and ratio-matched sequence components in the terminal currents. For example, in the case of the wye-delta transformer in Section IV, the negative-sequence component in the differential current is:

$$\mathbf{I}_{\mathrm{DIF}(\mathbf{Q})} = \left| \mathbf{I}_{\mathbf{X}(\mathbf{Q})} + \frac{\mathbf{V}_{\mathrm{H}}}{\mathbf{V}_{\mathrm{X}}} \bullet \mathbf{e}^{j30^{0}} \bullet \mathbf{I}_{\mathrm{H}(\mathbf{Q})} \right|$$
(55)

Note that (55) is nothing more than a component of the phase differential signals in (12). As such, it does not by itself bring any significant gain in sensitivity.

Sensitivity is enhanced because we eliminate the impact of load current (positive sequence) and restrain (55) only with the negative-sequence current:

$$\mathbf{I}_{RST(Q)} = \left| \mathbf{I}_{X(Q)} \right| + \frac{\mathbf{V}_{H}}{\mathbf{V}_{X}} \bullet \left| \mathbf{I}_{H(Q)} \right|$$
(56)

A negative-sequence differential element defined by (55) and (56) is considerably more sensitive compared with the phase differential element, not because the operating signal (55) is higher but because the restraining signal (56) is load-independent and therefore considerably lower during turn-to-turn faults.

The wye-delta transformer example used above can be extended to any three-phase transformer, including autotransformers, multiwinding transformers, and phase shifters. The extension is possible because of the way the sequence components in the terminal currents blend into the sequence components in the phase differential currents from (54).

To demonstrate, consider the PST in Section V with the differential currents defined by (46). The negative-sequence differential current is:

$$I_{DIF(Q)} = \left| I_{S(Q)} + e^{j\Theta(D)} \bullet I_{L(Q)} \right|$$
(57)

where:

$$\Theta(D) = \text{angle}\left(\frac{j - \sqrt{3} \cdot \frac{D}{2} \cdot \frac{n_s}{n_E}}{j + \sqrt{3} \cdot \frac{D}{2} \cdot \frac{n_s}{n_E}}\right)$$
(58)

In summary, a negative-sequence differential element can be applied to any three-phase transformer as follows:

- The differential current is a vector sum of ratiomatched, negative-sequence terminal currents.
- Prior to adding to the negative-sequence differential current, the terminal negative-sequence currents are phase-shifted by the negative vector group angle.
- The restraining signal is a sum of the ratio-matched magnitudes of the terminal negative-sequence currents (other combinations are also possible, such as the "maximum of" approach).
- In the case of PSTs, angle shift is variable, but the operation of shifting the negative sequence is still a simple multiplication by a complex number (but variable depending on the tap changer position).

The above "recipe" for the transformer negative-sequence differential element may appear heuristic, but as demonstrated in this section, it is well-founded in the physics of a three-phase transformer. It is based on exactly the same principle of balancing ATs in the core of the transformer. The gain in sensitivity is possible because the matching through-fault negative-sequence current is used as a restraint.

Equation (55) can be implemented as a negative-sequence transformation of the phase differential currents, a sum of the negative-sequence components in the ratio-matched and compensated terminal currents [1], or a ratio-matched and phase-shifted sum of the negative-sequence components in the terminal currents [2]. Regardless of the method, the outcome is mathematically identical as long as the phasors are extracted using linear filters.

As is often the case, this advantage is also a weakness of the element in that false negative-sequence currents are also generated during through faults with CT saturation. However, external fault detection guards this sensitive protection function very well, allowing for secure, fast, and sensitive operation.

### VII. REF PROTECTION

### A. Common REF Schemes

REF protection is intended to detect partial winding faults to ground near the neutral terminal of a grounded wye or zigzag connected winding, or of an autotransformer. For these faults, the phase currents measured at the terminals of the transformer can be quite low, while the current in the shorted turns can be very high, quickly damaging the transformer (see Section III, Subsection A). REF schemes are also recommended for impedance-grounded systems where the ground fault current is limited to a much lower level than the phase fault currents. An REF scheme can provide much higher sensitivity than the phase differential elements to detect these low-level ground faults.

REF protection schemes take advantage of the fact that the current in the fault loop can be measured directly by the neutral terminal bushing CT. In a solidly grounded system, the ground current can be quite high, even when the phase currents are small.

Fig. 14 and Fig. 15 show the two main schemes used for REF protection, using the application example of an autotransformer [3][4]:

- Current-polarized directional ground element.
- High-impedance REF differential element.



Fig. 14. Current-polarized directional REF scheme.



Fig. 15. High-impedance REF scheme.

Both of these schemes work on the principle of Kirchhoff's current law, similar to a bus differential zone, and sum the zero-sequence currents in the transformer zone. Current must flow from a winding to ground in order for these schemes to detect the fault. They do not match ATs in the transformer cores and therefore cannot detect turn-to-turn faults. REF also cannot detect winding-to-winding (phase-to-phase) faults.

It is common to apply a similar current summation (buslike) differential zone with some uncommon transformers, such as PSTs, either in addition to or instead of transformer differential protection. For example, because of the difficulty in incorporating factor D from (46) in the transformer differential protection system, the PST in Fig. 13 may be protected by six current summation zones around each individual winding. To accomplish this, CTs would be required on each end of the delta excitation windings. Because this scheme does not monitor the magnetic circuit, it will not detect turn-to-turn faults. When using this differential protection approach, it is necessary to rely on sudden pressure relaying to detect turn-to-turn faults.

### B. Security and Dependability Issues With REF Schemes

The zero-sequence current at the terminals of the transformer is formed by the summation of three-phase currents and therefore can include false zero-sequence current when one of the phase CTs saturates. This is particularly relevant in dual-breaker transformer terminations when the maximum external fault current is not limited by transformer impedance and when the external fault current does not

include a genuine zero-sequence component (line-to-line and three-phase faults). Sensitive REF schemes must be designed to be secure for this condition. The high-impedance scheme in Fig. 15 is inherently immune to CT saturation by nature of its operation principle.

The current-polarized directional element in Fig. 14 requires additional logic for security. Security from false zero-sequence current is obtained by requiring that current be present above a threshold in the neutral of the transformer. Because zero-sequence current at the neutral of the transformer is measured by a single CT, it is not possible to have a false zero-sequence current in this signal.

The current-polarized directional scheme also requires additional logic for dependability. In a delta-wye transformer, the transformer can be energized from the delta winding with the wye-winding terminal open. In this case, no zero-sequence current can flow at the boundary of the zone, so a directional decision cannot be made. In this case, a bypass logic path allows tripping when there is zero-sequence current in the neutral and none present in the terminals of the transformer.

# VIII. DISCUSSION

Differential protection of transformers against internal short circuits is accomplished using differential current equations that effectively emulate the AT equations of the transformer and, by doing so, monitor both the electric and magnetic circuits of the transformers. Changes in the monitored electromagnetic circuits, such as turn-to-turn faults, ground faults, winding faults, and some open circuits, are therefore detected by the said protection equations within sensitivity limits of a given implementation.

For standard transformer connections, including wye, delta, and zig-zag windings, as well as autotransformers, the said transformer protection equations can be created using the known protection engineering principles of vector group compensation, ratio matching, and zero-sequence removal. Various heuristic rules are available to quickly select the proper compensation rules. When in doubt, the AT equations can be consulted to check or confirm the proper compensation rules.

For nonstandard transformer connections, such as PSTs, the AT balance allows developing proper transformer current differential equations. The resulting equations are always linear but different compared with what is required for the wye, delta, and zig-zag windings. Microprocessor-based relays can be used to implement these equations with some accuracy and sensitivity.

Traditional transformer differential protection faces sensitivity limits for turn-to-turn faults and ground faults near the neutral terminal in a grounded winding. Theoretically, these faults upset the AT balance, but because of the autotransformer effect, the differential current unbalance can be small if a small percentage of the coil is shorted. At the same time, a given relay needs to maintain security in light of possible CT errors or an on-load tap changer, typically by means of restraining, resulting in less sensitive protection. REF provides sensitive protection for ground faults near the neutral. This function monitors the electric circuit of the protected winding for currents leaking to ground, and therefore it responds to ground faults only.

Negative-sequence differential protection increases sensitivity to turn-to-turn faults. This method is based on the AT balance, but instead of monitoring three balance equations between the three-phase currents, it monitors a single composite current, known as the negative sequence. This is derived from the phase differential currents. Using the composite signal does not increase the operating signal, but it redefines the restraining signal, allowing for more sensitive operation. Similar to REF, this function is susceptible to CT errors and therefore benefits from external fault detector algorithms or similar security measures often made available in microprocessor-based relays.

Nonstandard transformers invite applications with bus-like differential zones. Like REF, these zones do not monitor the electromagnetic circuit of the transformer and therefore will not detect turn-to-turn faults and other faults that do not divert the current outside of the monitored bus-like zones.

Phase transformer differential protection supplemented with REF and negative-sequence functions provides good sensitivity to transformer faults based on current measurements. Sudden pressure protection complements the electrical protection when larger transformers are used or when the electrical functions cannot provide good sensitivity.

# IX. CONCLUSIONS

Differential protection of a transformer follows the AT balance of the unfaulted transformer. The AT balance brings the currents from the galvanically isolated winding networks to a common equivalent circuit, allowing application of Kirchhoff's current law to form a differential scheme.

The classical rules of current compensation for transformer differential protection (vector group compensation, ratio matching, zero-sequence removal) replicate the AT balance of the unfaulted transformer.

Following the rule of AT balance allows us to analyze sensitivity and limitations of a given protection method, as well as derive proper differential protection equations for nonstandard transformers (i.e., Scott-T, Le-Blanc, phase shifters).

REF protection complements the traditional phase differential scheme, providing good sensitivity to ground faults near the neutral point of grounded wye or zig-zag windings.

Similarly, the negative-sequence differential element supplements the phase differential by providing sensitive protection to turn-to-turn faults, particularly during heavy load conditions.

A combination of phase differential, REF, and negativesequence differential allows electrical (current-based) protection of transformers without sensitivity gaps as compared with sudden pressure relays.

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### XI. BIOGRAPHIES

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