PHASOR FUNDAMENTALS

HANDS-ON RELAY SCHOOL 2019

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This presentation is a revision to *Phasor Diagrams*, presented at the Hands-On Relay School for many years by Ron Alexander.

Much of the content in this presentation is based on *J. Lewis Blackburn’s* fantastic reference: *Protective Relaying Principles and Applications* (multiple editions).

Our Primary Objective for this presentation is to enhance attendee knowledge by:
- Reviewing Phasor fundamentals
- Examining In-Service/Load checks

Our Secondary Objective (time permitting) will add:
- Reviewing Fault Phasors
- Using phasors to determine phase shift across three-phase transformer banks
Phasors – Why?

- Tool for understanding the power system during load and fault conditions.
- Assists a person in understanding principles of relay operation for testing and analysis of relay operations.
- Allows technicians to simulate faults that can be used to test relays.
- Common language of power protection engineers and technicians.
- Provides both mathematical and graphical view of System conditions.
Phasor Definitions

- A line used to represent a complex electrical quantity as a vector. (Google)

- A rotating vector representing a quantity, such as an alternating current or voltage, that varies sinusoidally. (Collins Dictionary)

- A vector that represents a sinusoidally varying quantity, as a current or voltage, by means of a line rotating about a point in a plane, the magnitude of the quantity being proportional to the length of the line and the phase of the quantity being equal to the angle between the line and a reference line. (Dictionary.com)
Phasor Representation

Common pictorial form for representing electrical and magnetic phasor quantities uses the Cartesian coordinates with x (the abscissa) as the axis of the real quantities and y (the ordinate) as the axis of imaginary quantities. (see figures below)
Here we can see a plot of an electrical quantity and its phasor representation.

Note the phasor has a constant (usually RMS) magnitude that rotates while the actual electrical quantity varies sinusoidally over time.
Multiple Phasors

- Here we can see two phasors of the same frequency rotating.
- Note that their phase relationship to each other is constant – in this case, the blue phasor leads the red phasor by some constant angle that does not change.
PLOTTING PHASORS

CARTESIAN COORDINATE SYSTEM
Phasor Representation

- Consider an example phasor ‘c’ having a magnitude of $120V_{\text{RMS}}$ and a phase angle of 30 degrees:
  - Rectangular Form: $c = x + jy$
    - $c = 104 + j60 \text{ V}$
  - Polar Form: $c = |c| \angle \theta$
    - $c = 120 \angle 30^\circ \text{ V}$
  - Complex Form: $c = |c| (\cos \theta + j \sin \theta)$
    - $c = 120(\cos(30) + j \sin(30)) \text{ V}$
  - Exponential Form: $c = |c| e^{j\theta}$
    - $c = 120e^{j30} \text{ V}$
Phasor Conversion

• Rectangular <> Polar Conversion:
  • Rectangular Form: \( c = x + jy \)
    - \( c = 104 + j60 \)
  • Polar Form: \( c = c \angle \theta \)
    - \( c = 120 \angle 30^\circ \)

• Trig Functions (right triangles only):
  • \( \sin(\theta) = \text{opposite} / \text{hypotenuse} \)
  • \( \cos(\theta) = \text{adjacent} / \text{hypotenuse} \)
  • \( \tan(\theta) = \text{opposite} / \text{adjacent} \)

• Pythagorean Theorem
  - \( c^2 = a^2 + b^2 \)
**Phasor Conversion**

- **Rectangular to Polar Conversion:**
  - **Rectangular Form:** $c = x + jy$
  - $c = 104 + j60$
  - $c^2 = a^2 + b^2$
  - $c^2 = 104^2 + 60^2 \Rightarrow c = 120$
  - $\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$
    - $\tan(\theta) = \frac{60}{104} \Rightarrow \theta = 30^\circ$
  - **Converted:** $c = 120 \angle 30^\circ$

![Diagram showing phasor conversion](image)
Phasor Conversion

- Polar to Rectangular Conversion:
  - Polar Form: \( c = |c| \angle \theta \)
  - \( c = 120 \angle 30^\circ \)
  - \( \sin(\theta) = \frac{o}{h} \)
    - \( o = h \times \sin(30) = 60 \)
  - \( \cos(\theta) = \frac{a}{h} \)
    - \( a = h \times \cos(30) = 104 \)
  - Converted: \( c = 104 + j60 \)
Two ‘Operators’ related to phasors are commonly used in the Power world: ‘\( j \)’ and ‘\( a \)’

- Mathematically ‘\( j \)’ is an imaginary number representing the imaginary (reactive) portion of a phasor: \( j = \sqrt{-1} \)
- Graphically, it is a ‘rotator’ constant with an angle of 90°
- It can also be viewed as a ‘unit phasor’ always having a value of \( 1 \angle 90° \)

The ‘\( a \)’ Operator, commonly used when working with Symmetrical Components.

- Graphically it is a ‘rotator’ constant with an angle of 120°
- It is also a ‘unit phasor’ with a value of \( 1 \angle 120° \)
Combining Phasors

- Two common operations are performed with phasors:
  - Adding / Subtracting
  - Multiplying / Dividing

- It’s generally easier to add/subtract in rectangular form and easier to multiply/divide in polar form.

- When adding/subtracting in rectangular form, add/subtract the real and reactive components (respectively):
  - Example: \((2+j3) + (3+j4) = (5+j7)\)

- When multiplying/dividing in polar form, multiply/divide the magnitude, and add/subtract the angle:
  - Example: \((10 \angle 30^\circ) * (5 \angle 45^\circ) = 50 \angle 75^\circ\)
Adding Phasors Graphically

- You can also add vectors graphically by connecting them head to tail.
- The resultant is the phasor originating at the origin of the first arrow and ending at the head of the last arrow.
- Here, if we have the following:
  - $V_a = 120 \angle 0^\circ V$
  - $V_b = 120 \angle -120^\circ V$
  - $V_c = 120 \angle 120^\circ V$
  - $V_a + V_b + V_c = 0$
Adding Phasors Graphically

- In this example, the $V_{AB}$ voltage is $V_A - V_B$. We subtract $V_B$ by reversing the $V_B$ phasor and adding it to $V_A$.
- $V_{AB} = V_A - V_B = V_A + (-V_B)$
Sinusoidal Waveforms

- Note that adding or subtracting sinusoidal waveforms simply produces a resultant sinusoidal waveform of the same frequency.
Which One is Easier?
Phasors – Conjugation

- Most commonly used in power calculation: $P = EI$ where the conjugate of $I$ is sometimes used and shown as: $P = EI^*$

- Conjugates:
  - $c = x - jy$
  - $c = c \angle -\Phi$
  - $c = |c|(\cos\Phi - j\sin\Phi)$
  - $c = |c|e^{-j\Phi}$
Phasors – Conjugation

Calculating: \( P = E \times I \) we get:
- \( P = 120 \angle 0^\circ \text{V} \times 0.42 \angle -45^\circ \text{A} = 50.9 \angle -45^\circ \text{VA} \)
- Placing this phasor on one of our previous Power Flow diagrams gives →→→

But Blackburn and many others use +Q in the 1st Quadrant.

If we calculate P using the conjugate of the current: \( P = E \times I^* \)
- \( P = 120 \angle 0^\circ \text{V} \times 0.42 \angle +45^\circ \text{A} = 50.9 \angle +45^\circ \text{VA} \)
- This places P in the 1st Quadrant.
Phasors – Conjugation

It’s extremely important to remember that phasor diagrams are descriptors of circuit information. How they are placed on a diagram does not change the electrical qualities of a circuit.

[Diagram of a circuit with phasors labeled +Q, -Q, +P, and -P.]
Phase Sequence

- Phase Sequence is the order in which phasors pass a reference point.
- In Symmetrical Components, you will hear the terms: positive sequence, negative sequence, and zero sequence.
  - Positive Sequence = ABC
  - Negative Sequence = ACB.
  - Zero sequence is all 3 phases rotating together at the same angle.
Resistor AC Response

\[ I = \frac{V}{R} \]
\[ Z = R \]
\[ i_m \sin \omega t \]
\[ v_m \sin \omega t \]

\[ I = \frac{I_m}{\sqrt{2}}, \quad V = \frac{V_m}{\sqrt{2}} \]

Unity Power Factor Waveform

Angle in Degrees

Magnitude

Phasor Diagram
### Inductor AC Response

\[
I = \frac{V}{X_L}
\]

\[
X_L = \omega L
\]

\[
i_m \sin(\omega t - 90°)
\]

\[
I = \frac{I_m}{\sqrt{2}}, \quad V = \frac{V_m}{\sqrt{2}}
\]

**Phasor Diagram:**

- **Diagram:**
  - Voltage
  - Current
  - Power

**90 Degree Lagging Waveform:**

- **Graph:**
  - Magnitude
  - Angle in Degrees

**Equations:**

- \( I = \frac{V}{X_L} \)
- \( X_L = \omega L \)
- \( i_m \sin(\omega t - 90°) \)
- \( I = \frac{I_m}{\sqrt{2}}, \quad V = \frac{V_m}{\sqrt{2}} \)
Capacitor AC Response

\[ I = \frac{V}{X_C} \]
\[ X_C = \frac{1}{\omega C} \]

\[ i_m \sin(\omega t + 90^\circ) \]

\[ v_m \sin\omega t \]

\[ I = \frac{I_m}{\sqrt{2}}, \quad V = \frac{V_m}{\sqrt{2}} \]
Phasor Nomenclature

Circuit Diagram

Open Phasor Diagram

\[ V_{bc} \]

\[ V_{ab} \]

\[ V_{cd} \]

\[ V_{ad} \]

\[ I_T = I_{ab} = I_{bc} = I_{cd} \]
Phasor Nomenclature

Circuit Diagram

Open Phasor Diagram

$V_{ab}$ $V_{bc}$ $V_{cd}$

$V_{R}$ $V_{L}$ $V_{C}$

$I_{ab}$ $I_{bc}$ $I_{cd}$

$I_T$

$V_{ad}$ $V_{bc}$ $V_{cd}$

$I_T = I_{ab} = I_{bc} = I_{cd}$
Phasor Nomenclature

- Phasor diagrams can be drawn ‘open’ or ‘closed’.

![Circuit Diagram](image)

- Open Phasor Diagram
  - \( V_{bc} \)
  - \( V_{ad} \)
  - \( V_{ab} \)
  - \( I_T = I_{ab} = I_{bc} = I_{cd} \)

- Closed Phasor Diagram
  - \( V_{bc} \)
  - \( V_{ab} \)
  - \( V_{cd} \)
  - \( I_T = I_{ab} = I_{bc} = I_{cd} \)
Polarity

- Current into the primary polarity and out of the secondary polarity are (essentially) in-phase.
- Voltage drop from polarity to non-polarity on both windings are (essentially) in-phase.
Polarity and Phasors

- When dealing with Phasors, we make assumptions about current and voltage polarity on our circuit diagram so polarity is also extremely important.
- Consider the following circuit: we’ll do ‘in-service’ checks of the relays in the bottom current circuit.
Phasors in Practice

- Here we’ve zoomed in on the current circuit we are interested in:
- We’ll assume for convenience that the normal load flow for this circuit is from the Main Bus out into the line – the ‘up’ direction on this print.
Phasors in Practice

Now we’ll plan on where we will take secondary current readings. The test switches are made for this so we’ll use terminals 11/12, 15/16, and 19/20 on test switch 3L.
Phasors in Practice

- We’ll also plan on checking the voltages to the relays for proper magnitude and polarity.
- We’ll use the same 3L test switch, terminals 1-8.
Phasors in Practice

- Which of the following phasor diagrams is correct for the circuit we are doing in-service on?
  - It is indeterminate / uncertain; both could be correct, or wrong...
- Why?
  - We haven’t completed our circuit diagram (which we know should accompany our phasor diagram).
Phasors in Practice

- Since we already assumed a load flow direction, now we just need to show on our circuit diagram our assumed polarity at our measurement point.
- This determines how we ‘jack in’ to the current circuit using a phase-angle meter. In this diagram we will insert test equipment with assumed current ‘in’ on terminals 11, 15, and 19.
Phasors in Practice

- Let’s show/document our measurement points for the voltages too.
- $V_{AN} = (3L)\ 1-7, \ V_{BN} = (3L)\ 3-7, \ V_{CN} = (3L)\ 5-7$
In-Service Sheet

- Here is an in-service sheet based on our work and our measurements based on our circuit diagram.
- What does it tell us?
- It tells us our ‘assumed’ current direction was correct, and load is flowing from Chemawa to Santiam. It also tells us the load at Santiam is moderately Inductive.
Phasors in Practice

- Revisiting this slide, it appears Phasor Diagram #1 is correct – at least when it is paired with the circuit diagrams we’ve previously looked at.
- But is Phasor Diagram #2 right or wrong?
- Let’s consider that a 2\textsuperscript{nd} Relay Technician took the phasor readings for diagram #2 and let’s consider that they used the following circuit diagrams.

![Phasor Diagram #1](image1)

![Phasor Diagram #2](image2)
Phasors in Practice

- For Relay Technician #2 they assumed the direction of load flow was from Santiam to Chemawa – reverse to Technician #1.
- Based on assumed current direction, Tech #2 ‘jacked in’ to the current circuit using a phase-angle meter with assumed current ‘in’ on terminals 12, 16, and 20.
Phasors in Practice

- Tech #2 used the same ‘across’ process for the voltages.
- $V_{AN} = (3L)\ 1-7, \ V_{BN} = (3L)\ 3-7, \ V_{CN} = (3L)\ 5-7$
In-Service Sheet

- Here is the in-service sheet for Tech #2.
- What does it tell us?
- It tells us our ‘assumed’ current direction was incorrect, and load is actually flowing from Chemawa to Santiam. It also tells us the load at Santiam is moderately Inductive.
Now we can see why the circuit diagram is vital in making a determination of whether phasors are correct... or not...

- In this case, interpretation of the phasors proved both to be correct, when each is properly tied to its circuit diagram.

- Phasors without an associated circuit diagram are essentially unprovable and of questionable value...
Phasor Fundamentals Review

- Important Phasor fundamentals:
  - Phasor Representation
  - Phasor Nomenclature
  - Combining Phasors
  - Circuit Diagrams
  - Phase Rotation and Phase Sequence
  - Polarity

- As Blackburn notes in his book:
  - Phasor fundamentals are essential aids in selection, connection, operation, performance, and testing of protection for power systems.
Fault Phasors

Faults are unavoidable in the operation of a power system. Faults are caused by:

- Lightning
- Insulator failure
- Equipment failure
- Trees
- Accidents
- Vandalism such as gunshots
- Fires
- Foreign material

Faults are essentially short circuits on the power system and can occur between phases and ground in virtually any combination:

- One phase to ground
- Two phases to ground
- Three phase to ground
- Phase to phase

These fault types are (relatively) easy to understand; only one requires some consideration (Phase-to-Phase Fault) but even it is not hard once you consider some fault Fundamentals.
Transmission line faults are almost always inductive; let’s examine why:

- Transmission line impedance is based on the length of line, the type of conductor, and the type and spacing of the structures the line is installed on.
- When the line is loaded with normal shunt loads, the transmission line impedance is (for the most part) negligible and the loading and phase angle is determined by the load impedance; thus the load angles can be inductive/resistive/capacitive.
- When a line faults, the load impedance is bypassed (shunted) and the fault current is limited by the source voltage and the impedance of the transmission line itself.
Fault Phasors

It’s important to note that given a uniform transmission line the fault angle will not vary for a fault anywhere on the transmission line.
Fault Phasors

- Let’s look at some standard fault phasors.

- To begin with, we’ll examine an ideal unfaulted system:
  - We’ll use $V_{AN}$ as our reference.

![Diagram of an unfaulted system with phasors $V_{CN}$, $I_C$, $V_{BN}$, $I_B$, and $I_A$.](image)
1LG Fault – AG

- Assume a fault angle of 70°
- Assume 50% voltage collapse

Key points:
- $V_{AN}$ is reduced by 50%
- $I_A$ increases and lags the Fault Voltage by the line angle, 70°
2LG Fault - BCG

- Assume a fault angle of 70°
- Assume 50% voltage collapse

Key points:
- $V_{BN}$ and $V_{CN}$ are reduced by 50%
- $I_B$ and $I_C$ increase and lags their Fault Voltage by the line angle, 70°
3LG Fault - ABCG

- Assume a fault angle of 70°
- Assume 50% voltage collapse

Key points:
- All voltages are reduced by 50%
- All currents increase and lag their Fault Voltage by the line angle, 70°
2L Fault – BC

Assume a fault angle of 70°
Assume 50% voltage collapse

Key points:
- The fault voltage is reduced by 50%
- Fault current increases and lags the Fault Voltage by the line angle, 70°
- It should be recognized that the fault voltage and fault current are not line-to-ground quantities, but phase-to-phase.
2L Fault – BC

- Assume a fault angle of 70°
- Assume 50% voltage collapse

Key points:
- The fault voltage is reduced by 50%
- Fault current increases and lags the Fault Voltage by the line angle, 70°
2L Fault – BC

- Assume a fault angle of 70°
- Assume 50% voltage collapse

Key points:
- The fault voltage is reduced by 50%
- Fault current increases and lags the Fault Voltage by the line angle, 70°
- It must be recognized that the fault voltage and fault current are not line-to-ground quantities, but phase-to-phase.
2L Fault – BC

- Sum of Voltage Phasors must equal zero when ground is not involved.
- This can only happen when $V_{BN}$ and $V_{CN}$ rotate.
2L Fault – CB

- Assume a fault angle of 70°
- Assume 50% voltage collapse

Key points:
- The fault voltage is reduced by 50%
- Fault current increases and lags the Fault Voltage by the line angle, 70°
Phasors and Transformers
## Phasors and Transformers

### Low Voltage Connections

<table>
<thead>
<tr>
<th>L.V. Lines ON 5, 8, 11</th>
<th>Neut. ON 14</th>
<th>Vector Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Volts</strong></td>
<td><strong>Amp.</strong></td>
<td><strong>Connect</strong></td>
</tr>
<tr>
<td>7970</td>
<td>218</td>
<td>15 to 16, 18 to 19, 21 to 22</td>
</tr>
<tr>
<td>7200</td>
<td>241</td>
<td>16 to 17, 19 to 20, 22 to 23</td>
</tr>
<tr>
<td>13800</td>
<td>125</td>
<td>15 to 16, 18 to 19, 21 to 22</td>
</tr>
<tr>
<td>12470</td>
<td>139</td>
<td>16 to 17, 19 to 20, 22 to 23</td>
</tr>
</tbody>
</table>

![Diagrams](image1.png)
Phasors and Transformers

Note use of upper case for high voltage winding and lower case for lower voltage winding.

‘Clock number’ is a multiple of 30° in the lagging direction.

- 1 = 30 degree lag
- 11 = 330 degree lag (30 degree lead)
Let’s try to plot some phase shifts through a sample transformer connection.
Closing our Phasor Diagram

- Is this correct?
  - No. There is no VAN / X1-X0 to use for our comparison to the primary to determine phase shift.
  - You can sometimes ‘close’ a phasor diagram this way but you have to know when it is ok.
Closing our Phasor Diagram

- How to close a Delta Connection:
- Now we can see that the secondary reference voltage lags the primary voltage by 30°
- What is our connection type then?
  - Yd1

[Diagram of phasor connections showing secondary voltages and a closed diagram]
Practice Connections

- Here are some practice connections for you to try on your own.

- Phase Shift answers are listed in the notes.
Phasors: Faults and Transformers

- Our Secondary Objective:
  - Reviewing Fault Phasors
  - Using phasors to determine phase shift across three-phase transformer banks

- As Blackburn notes in his book:
  - Phasor fundamentals are essential aids in selection, connection, operation, performance, and testing of protection for power systems.

- Thanks for your time and participation!