PROBABILISTIC ANALYSIS ON DISCONTINUOUS SLOW CRACK GROWTH IN HIGH-DENSITY POLYETHYENE BY USING THE CRACK LAYER THEORY

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Abstract

It has been demonstrated that the crack layer (CL) model can reproduce the unique pattern of slow crack growth (SCG) particularly observed in the HDPE, the discontinuous SCG. The reliability assessment of the high-density polyethylene (HDPE) pipes requires the probabilistic analysis on their lifetime distribution. In this study, the existing deterministic CL model was used for the randomly-generated initial crack sizes for different locations, i.e., surface or internal crack. The corresponding lifetime distributions were closely analyzed and fitted by two kinds of probabilistic functions. The master curves for the scale and shape parameters regarding the applied stress and crack size distributions were also constructed.

Introduction

With consistent increasing of the use of thermoplastics in the infrastructures such as pipelines, the assessment of their lifetimes under the service conditions becomes an important issue. The lifetime expectancy of the high-density polyethylene (HDPE) pipes for transporting the natural gas and tap water is about 50 years, thus the accelerated failure testing methodology is required. Figure 1 illustrates the schematics of the variations of failure modes with regard to the applied hoop stress [1]. At the high hoop stress levels, the thermoplastic pipes fail in the ductile manner with the localized buckling. In this stage, the lifetime is highly dependent on the applied stress levels. At the intermediate stress range, the pipe undergoes the brittle fracture, associated with the crack initiation (CI), slow crack growth (SCG), and the dynamic fracture with instability [2]. Because this stress range normally includes their operating conditions, the field failures of the thermoplastic pipes are generally related to this failure mode. If the stress range is applied in much lower range, the chemical degradation due to the oxidative deteriorate or the diffused fluids itself becomes a dominating factor, and this type of failure is called environmental fracture. The failure in this stage occurs by the stress corrosion cracking (SCC), or the environmental stress cracking (ESC) mechanism [3, 4]. The extrapolating from the accelerated failure times to other conditions should be performed within the same failure mode.

The brittle fracture of thermoplastic pipes initiates from the pre-existing defects normally generated in the manufacturing process. The defects include the voids, residues of metal catalysts, dusts, and carbon blacks, and so on. The damage accumulation in the vicinity of these defects exceed a certain of damage levels, after so-called CI stage, the crack starts to grow in the quasi-equilibrium manner (SCG). In case of the HDPE, it has been widely observed that the SCG proceeds discontinuously in both creep and fatigue loading conditions. The unique behavior of the HDPE is closely related with the physical interaction between the crack tip and surrounding process

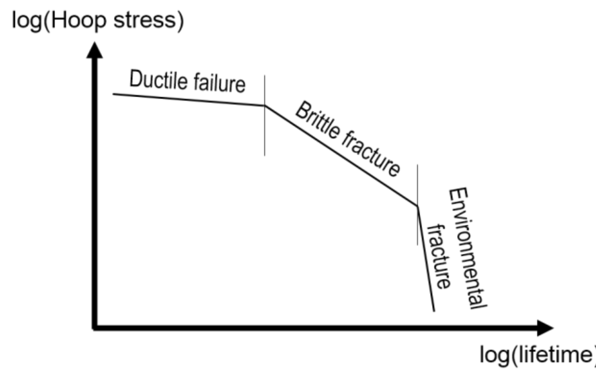

Figure 1. Hoop stress dependency of the failure modes in thermoplastic pipes.

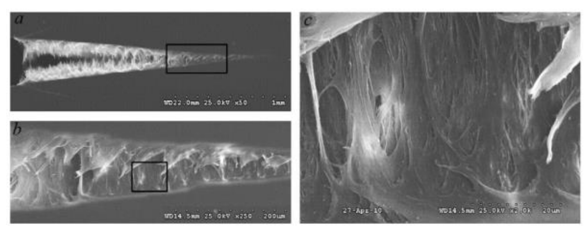

Figure 2. Typical crack layer system and process zone (PZ) observed in the HDPE [2].

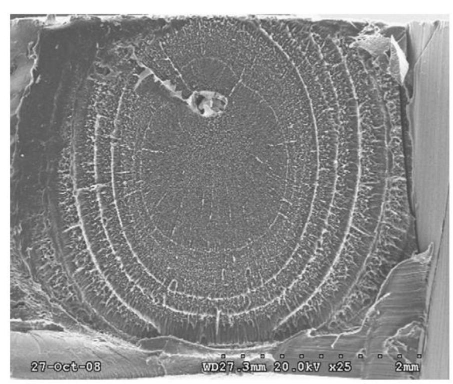

Figure 3. The fractured surface of HDPE pressurized pipe [2].

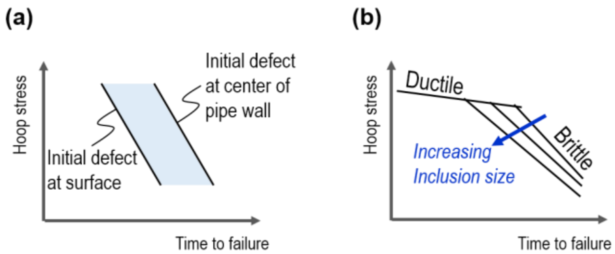

Figure 4. (a) lifetime range under the same initial crack size with different locations, (b) lifetime for the brittle fracture with increasing initial defect size.

zone (PZ) (Figure 2). In the crack layer (CL) theory, the variation of the potential energy regarding the crack and PZ growth is analyzed as follows,

$$-\frac{\partial G}{\partial l_{CR}}\bigg|_{l_{PZ}} = X^{CR} \text{ and } -\frac{\partial G}{\partial l_{PZ}}\bigg|_{l_{CR}} = X^{PZ}, \quad (1.1)$$

where the G denotes the Gibbs potential energy, l_{CR} and l_{PZ} are the crack and PZ lengths, respectively. Thus X^{CR} and X^{PZ} become the thermodynamic force for crack and PZ growth, respectively. The growth rate of the crack and PZ can be formulated by

$$\dot{l}_{CR} = k_{CR} X^{CR} \text{ and } \dot{l}_{PZ} = k_{PZ} X^{PZ} \quad (1.2)$$

where the k_{CR} and k_{PZ} stand for the kinetic coefficients for crack and PZ growth, respectively. Then, applying the time-increasing iteration gives the crack and PZ growth. It has been demonstrated that the CL model can accurately simulate the discontinuous SCG for the various specimen configurations.

Generally the initial crack inside the pipe wall shows much larger lifetime than that at the pipe surface, under the same initial crack size and applied hoop stress. Because the locations and sizes of the initial defect is apparently random (Figure 3), the pipe failed followed by the brittle fracture reveals the wide range of the lifetime distributions (Figure 4a). Also, the lifetime in the brittle fracture stage is apparently affected by the initial defect sizes (Figure 4b). Therefore, the investigation on the effect of distributions of initial defect sizes and locations on the lifetime distribution is essential for the reliability assessment in the HDPE pipes. In this study, the CL simulations, for the initial defect at center of pipe wall and at the surface, were conducted. The size of initial defect was randomly generated, and the consequent lifetime distributions were also characterized.

Results and discussions

Let us consider the HDPE pipe under the constant internal pressure. The initial defect with the length of a is located at the inner surface (case1) and the center of the pipe wall (case2) (Figure 5). The thickness of pipe wall is denoted by W. The case1 can be considered as a single edge notched tension (SENT) specimen under the constant remote stress (σ_∞), while the case2 becomes the center cracked tension (CCT) specimen under the σ_∞.

The crack layer (CL) growth simulations for the SENT (case1) and CCT (case2) are provided in Figure 6(a) and (b), respectively. The W and a were 10 and 2 mm, in both cases. The black solid curves and dotted red curves represent the crack and CL length (crack + PZ length), respectively. The discontinuous SCG simulations were successfully conducted. Under the remote stress of 6 MPa at the room temperature, the SENT specimen reveals the lifetime about 36.8 h, while the CCT shows 324.2 h. The much longer lifetime in case of the inner initial crack has been widely accepted results.

To randomize the initial defect size, the 500 random number following the normal distribution was generated. The mean values, standard deviation, and coefficient of variation (CV) of the initial defect size are denoted by $\bar{a}$, S_a and CV_a, respectively. Then, the independent 500 simulations for each initial crack size was conducted, and the resulting lifetime distributions can be obtained for case1 and case2. Figure 7a shows the probabilistic CL simulations for $\bar{a}$=2 mm and CV_a of 0.05. The resulting

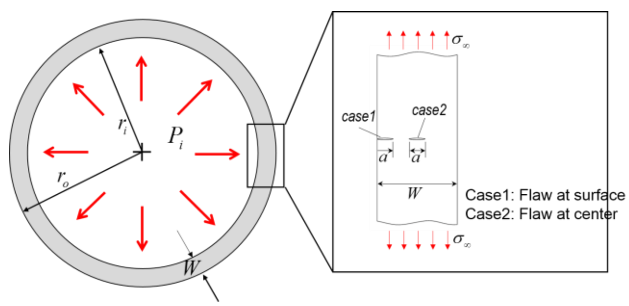

Figure 5. The pressurized HDPE pipe including the initial defect at the surface (case1) and center of the pipe wall (case2).

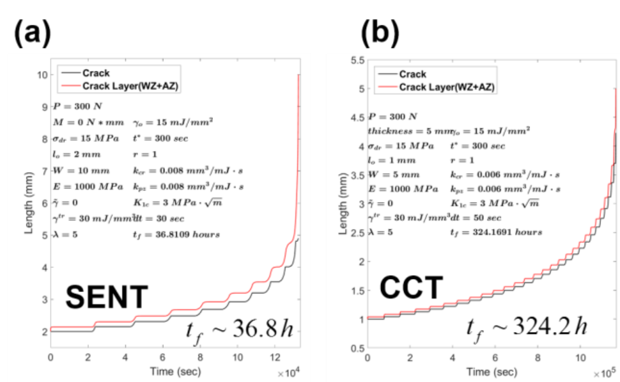

Figure 6. (a) the deterministic crack layer simulation for edge crack, and (b) for center crack.

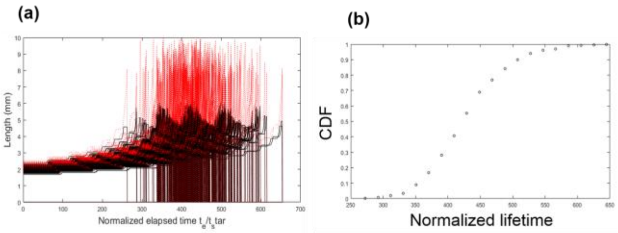

Figure 7. (a) probabilistic crack layer simulations from the normal distribution of initial crack size (b) corresponding cumulative distribution function (CDF) for the lifetime.

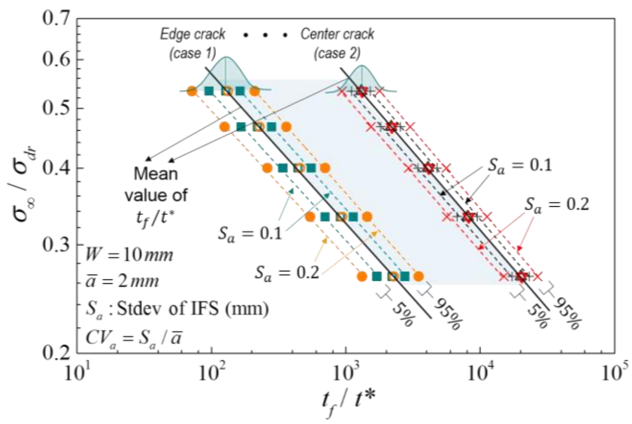

Figure 8. The applied stress – lifetime diagram for edge crack and center crack cases. The mean value of initial crack size is 2 mm. The 5% and 95% lifetime quantiles are also depicted.

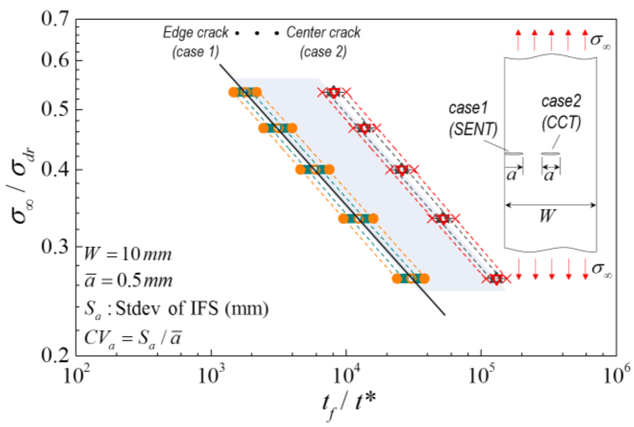

Figure 9. The applied stress – lifetime diagram for edge crack and center crack cases. The mean value of initial crack size is 0.5 mm. The 5% and 95% lifetime quantiles are also depicted.

lifetime distribution (cumulative distribution function, CDF) is also shown in Figure 7b. The stress-lifetime diagrams can be also obtained by simulating under the various stress levels. The linear relationship between the stress and lifetime in the log-log scale is in accordance with previous experimental results.

The center crack (case2) shows clearly much longer lifetime than the edge crack case (case1), and it can be thought that the actual lifetime would be inside between the two extreme cases. In case of the smaller initial defect size, 0.5 mm, the overall lifetime shifts to the right, and also the gap between the case1 and case2 become larger.

The lifetime distribution followed by the discontinuous SCG resembles the striations under the fatigue loading. To characterize the probabilistic lifetime distribution through the fatigue loading, the Birnbaum-Saunders (B-S) distribution function has been widely used. The B-S function is derived from that each

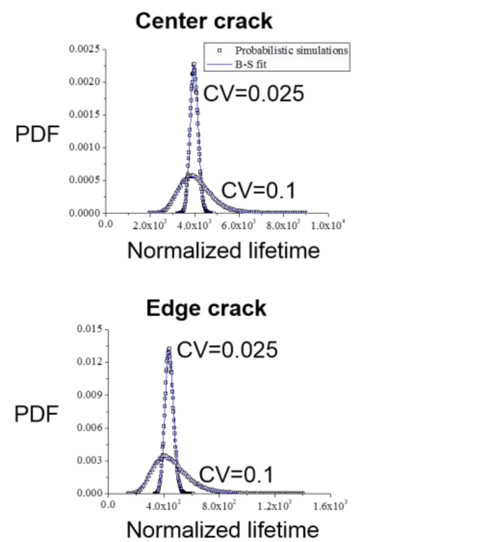

Figure 10. PDF fit by using the B-S function, in the edge crack (case1) and center crack (case2).

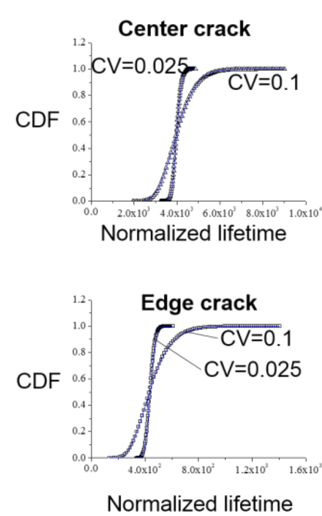

Figure 11. CDF fit by using the B-S function, in the edge crack (case1) and center crack (case2).

randomized striation lengths are accumulated to the critical level, i.e., to the fracture toughness. The cumulative distribution function (CDF) of the B-S function for the variable *x* is given by

$$CDF\left(x;\beta,\gamma\right)=\Phi\left\{\frac{1}{\gamma}\left(\sqrt{\frac{x}{\beta}}-\sqrt{\frac{\beta}{x}}\right)\right\}, \quad (1.3)$$

where the Φ is the CDF of standard normal distribution. The γ and β is shape and scale parameter of B-S function. Figure 10 and 11 reveal the probability density function (PDF) and the CDF. The symbols stand for the raw data from the 500 independent simulations, and blue continuous curves are the regressed B-S function. Even the B-S function is the two-parameter probabilistic function, the PDF and CDF can be accurately fitted for all cases. Thus, it can be demonstrated that the lifetime distributions followed by the discontinuous SCG behavior, can be accurately fitted by the B-S function.

Conclusions

It is well-known that the crack layer model can accurately simulate the discontinuous SCG behavior, which is normally observed for the HDPE materials. The existing crack layer model was applied to perform the reliability analysis regarding the initial defect location and sizes. The lifetime distributions with the two extreme crack location cases, i.e., surface crack and center initial crack, were obtained by the probabilistic crack layer simulations. It was demonstrated that the lifetime distribution followed by the discontinuous SCG behavior can be accurately fitted by the Birnbaum-Saunders distribution.

References

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