INTERNATIONAL POLYOLEFINS CONFERENCE
Houston, Texas • February 24-27, 2019
Presented by SPE South Texas Section, SPE Polymer Modifiers and Additives Division, and SPE Thermoplastic Materials and Foams Division

Probabilistic analysis on the discontinuous slow crack growth in high-density polyethylene by using crack layer theory

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2019. 2. 25

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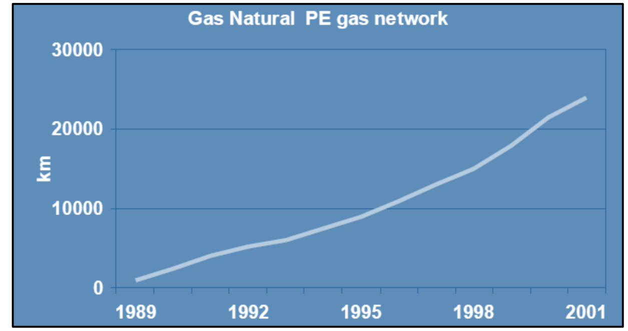
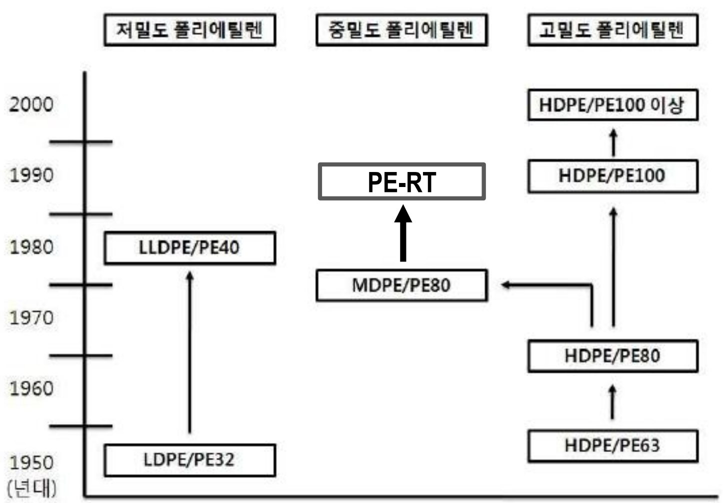
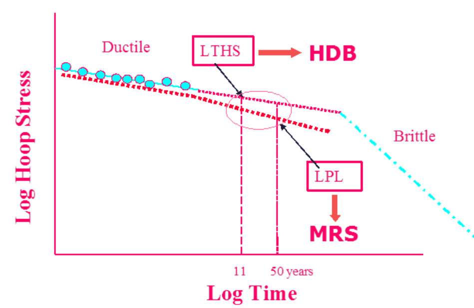
Introduction

Pipe-grade HDPE



- Light weight, flexibility, ease to install.
- Required lifetime : 50 years → Accelerated testing.
- Accurate lifetime estimation methodology is required to increase working pressure

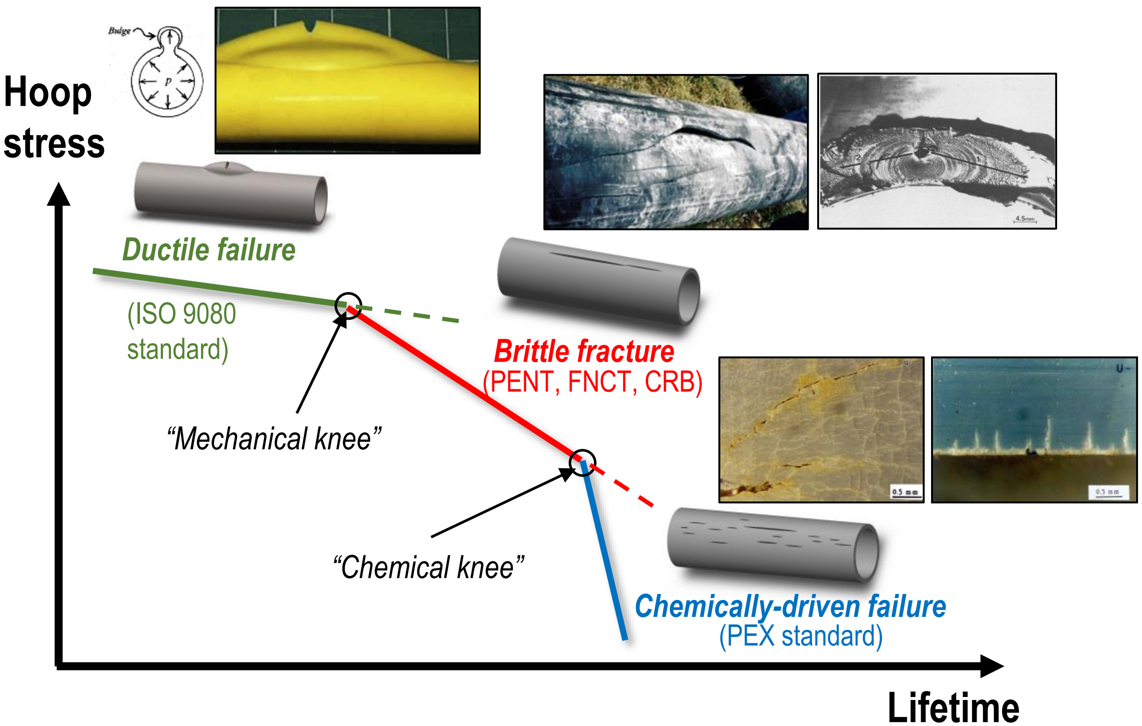




RANGE OF CALCULATED LPL VALUES, MPa	MINIMUM REQUIRED STRENGTH, MPa
12.5 – 15.9	12.5
10.0 – 12.4	10.0
8.0 – 9.9	8.0
6.3 – 7.9	6.3

ex: PE100 = MRS 100 bar (10 MPa)

Failure modes of HDPE pipes



- **High stress level :**
  - Ductile failure with localized buckling.
  - Ballooning, Hydro-bursting.
- **Intermediate stress level :**
  - Brittle fracture with SCG.
  - Continuous, discontinuous or combined SCG mode.
- **Low stress level and aggressive chemicals :**
  - Mechano-chemical brittle cracking.
  - Multiple cracking (SCC or ESC).

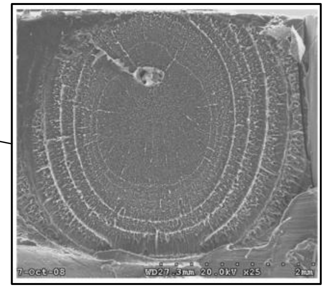
- Lifetime extrapolation must be performed in the same failure mechanisms (cannot be extrapolated across the different failure modes).
- Most relevant fracture mechanisms in field conditions (design stress) : Brittle fracture.
- Lifetime for brittle fracture = Crack initiation + Slow crack growth + Dynamic fracture.
- Dynamic fracture occurs very fast, thus this stage is often neglected in the total lifetime.

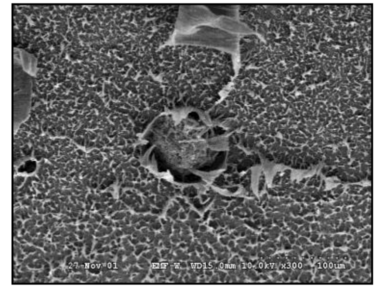
Brittle fracture of HDPE pipes – Crack initiation

Crack initiation

Slow crack growth, SCG

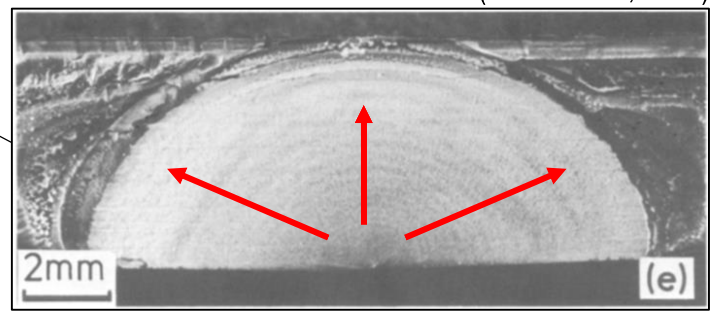
Rapid crack growth, RCP



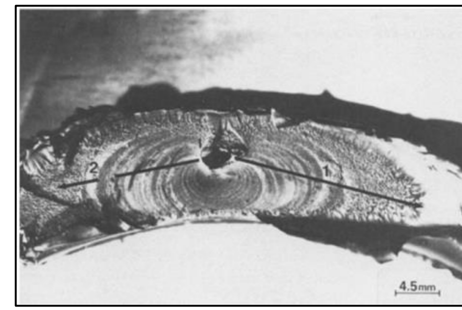


(Chudnovsky et al., 2012)

(Barker et al., 1983)



(K. Sehanobish et al., 1985)

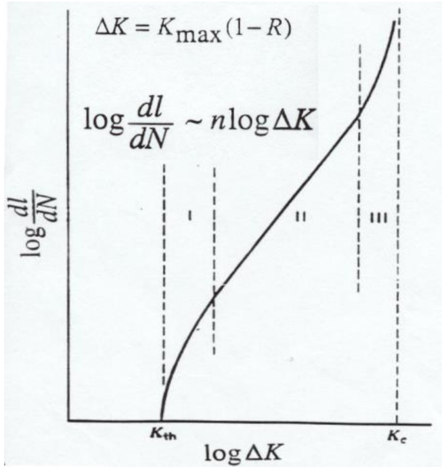


Crack initiation

- The pre-existing defect such as cavities, agglomerate gel, dusts, or the catalyst residues.
- The internal initial crack gives much longer lifetime than the initial crack at pipe surface.
- Because the location of such defects is random, the lifetime distribution of thermoplastic pipes generally reveals the large scatter even in the same test condition; therefore, the accurate lifetime prediction of thermoplastic pipes requires the proper characterization and stochastic study of the initial defects for the reliability analysis.

Slow crack growth, SCG

- **Empirical SCG model based on Paris law**



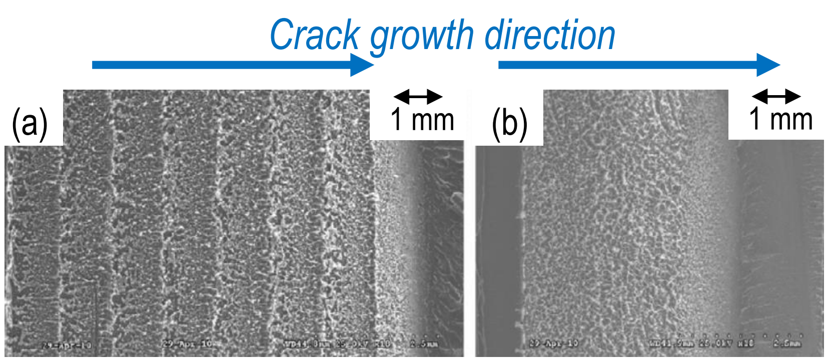
$$\frac{da}{dN} = A(\Delta K)^m$$ Paris-Erdogan ; 1963

$$\frac{da}{dN} = A\frac{(\Delta K)^m}{(1-R)^p}$$ Walker 1970 ;
- Capturing the R-ratio effect.

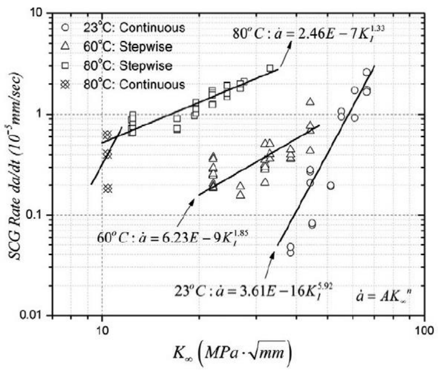
$$\frac{da}{dN} = A(\Delta K_{eff})^m$$ Elber 1971;
- Consider the crack closure effect.

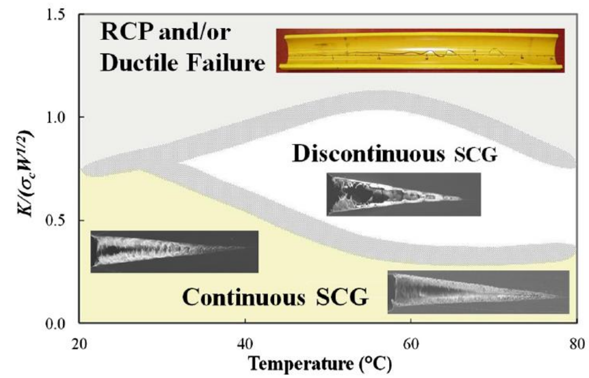
$$\frac{da}{dN} = \frac{A(1+\beta)^m(\Delta K - \Delta K_{th})^n}{(1-R)\Delta K_c - (1+\beta)\Delta K}$$ Erdogan and Ratwani 1970;
- Covers all three stages.

- **SCG behavior of HDPE**



Discontinuous growth   Continuous growth





Brittle fracture of HDPE pipes – Slow crack growth

Slow crack growth, SCG

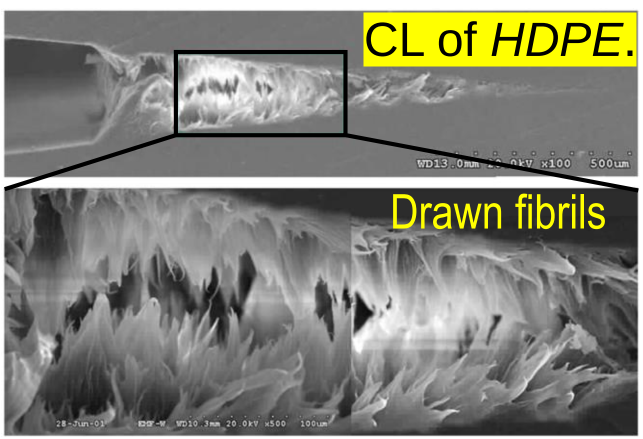
- Slow crack growth (quasi-equilibrium process) under unstable crack propagation (load-controlled)

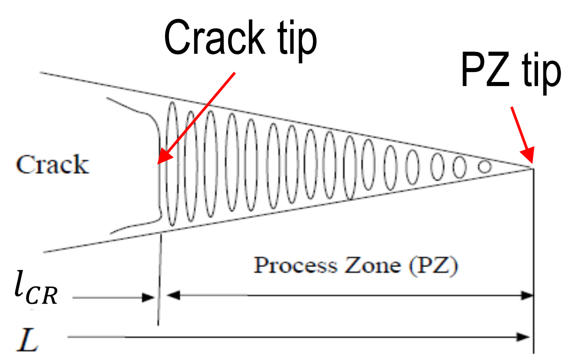
  → *Physical interaction between main crack and surrounding PZ.*

- Many empirical relationships employ only crack driving term, *i.e.*, SIF at crack tip, without consideration of the interplay with surrounding PZ.

- The unique SCG kinetics in HDPE cannot be modelled by empirical SCG laws.

- General model considering the phase transformation based on the irreversible thermodynamics is required. → ***Crack layer (CL) theory***





Thermodynamic approach: crack layer theory

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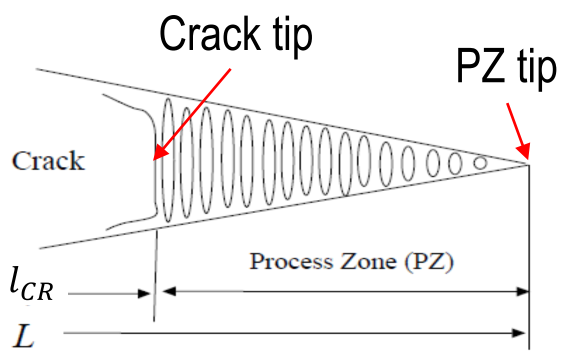
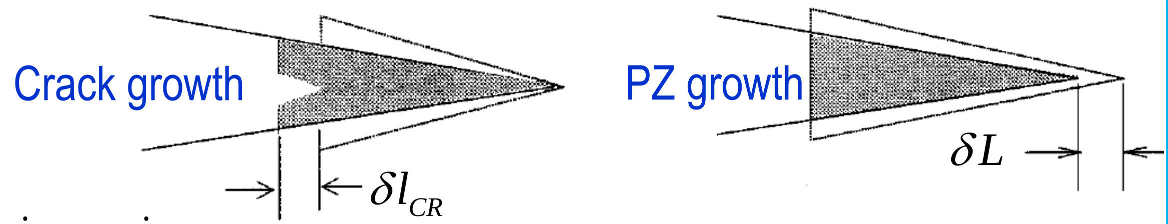
SCG model for HDPE – Crack layer theory

- **Crack layer = Main crack + Surrounding damage zone (Process zone, PZ)**



CL system of HDPE



$$T\dot{S}_{irr} = -\dot{G}$$
$$= \dot{l}_{CR}\left(J_1^{CR} - 2\gamma\right) + \dot{L}\left(J_1^{PZ} - \gamma^{tr} R_1\right) + \dot{w}_{PZ}\left(J_2^{PZ} - \gamma^{tr} R_2\right) + Diss\left(PZ\right).$$

- Thermodynamic force for crack growth;

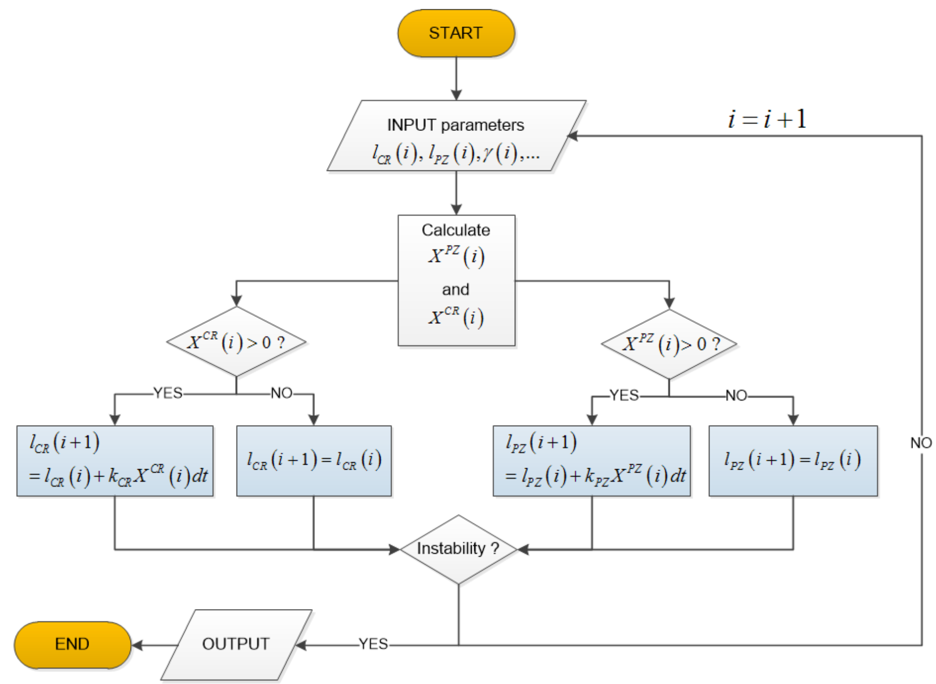
$$X^{CR} = -\left.\frac{\partial G}{\partial l_{CR}}\right|_{L=const.} = J_1^{CR} - 2\gamma.$$

- Thermodynamic force for PZ growth;

$$X^{L} = -\left.\frac{\partial G}{\partial L}\right|_{l_{CR}=const.} = J_I^{PZ} - \gamma^{tr} R_I$$

- Rate of CL growths;

$$\frac{d\left(l_{CR}\right)}{dt} = k_{CR} X^{CR} \text{ and } \frac{d\left(L\right)}{dt} = k_L X^{L}.$$



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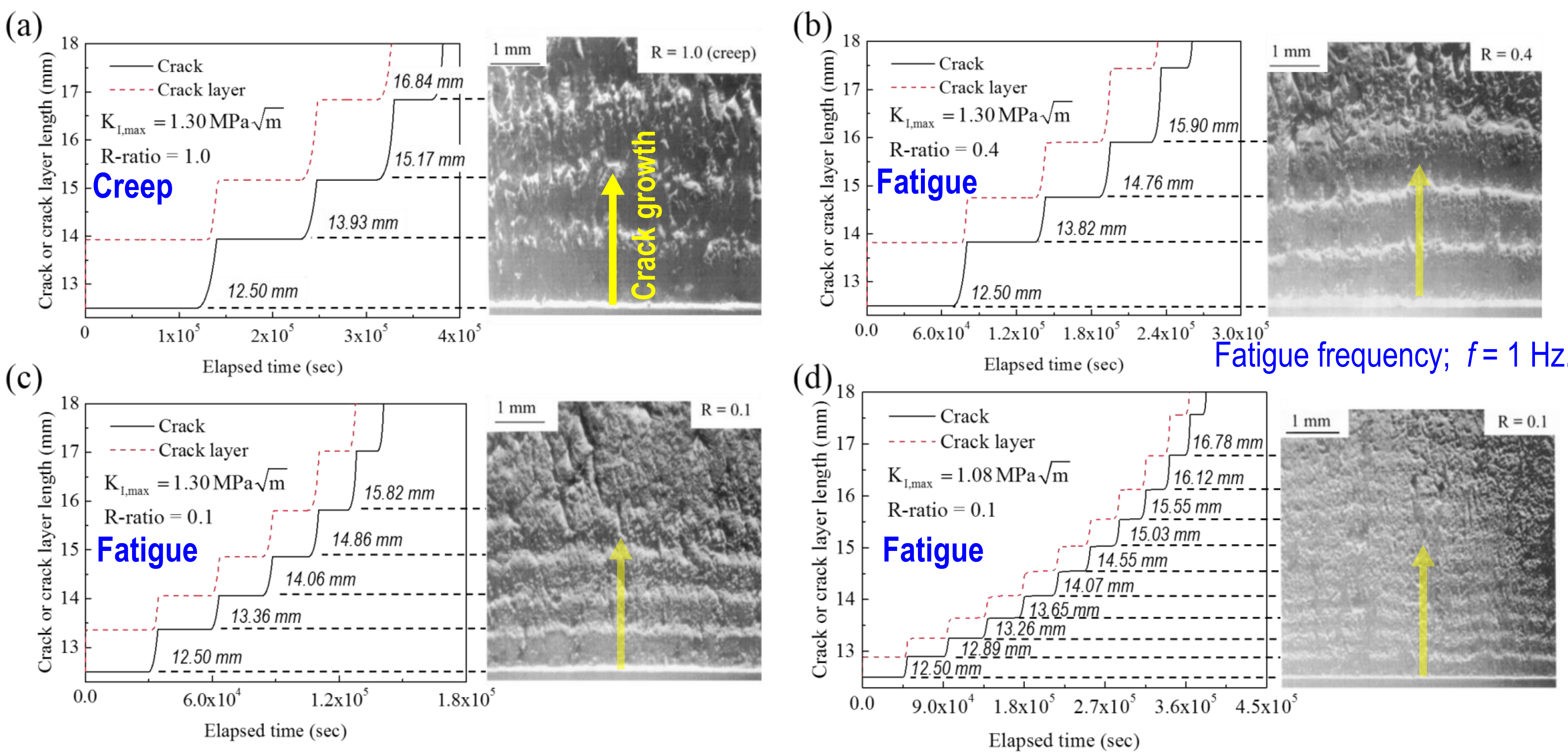
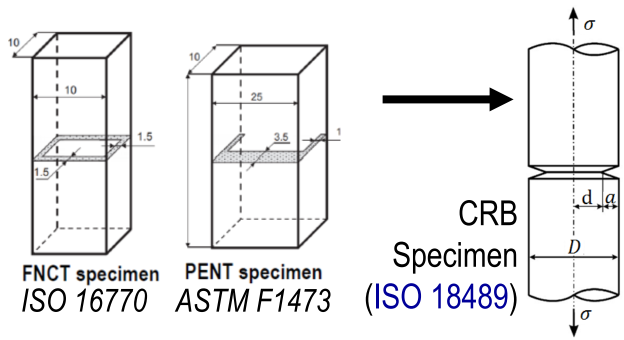
Compact tension specimen (Fatigue and creep crack growth)

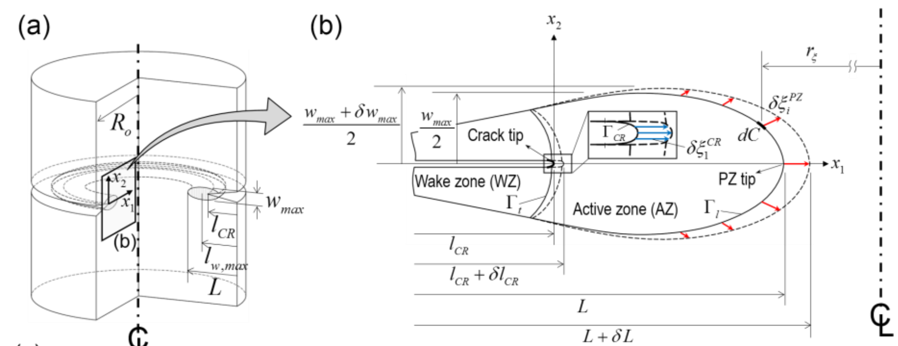


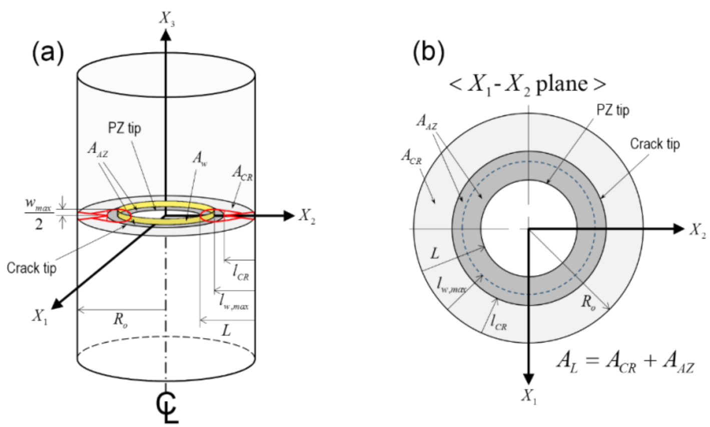
Fatigue frequency; $f = 1$ Hz.

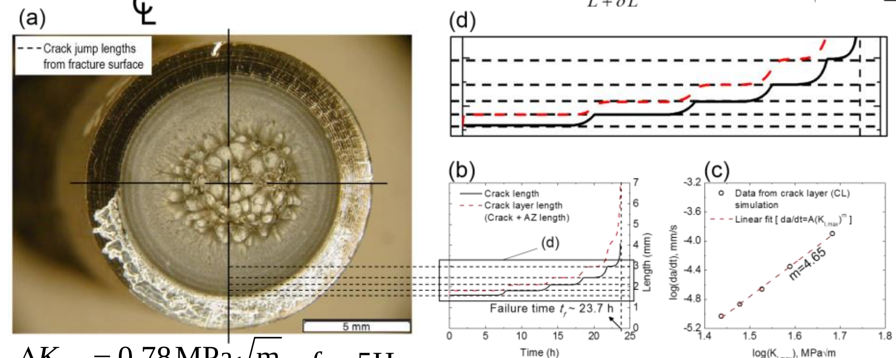
*Wee, Jung-Wook, and Byoung-Ho Choi. *International Journal of Fatigue, 2016*.

CRB specimen









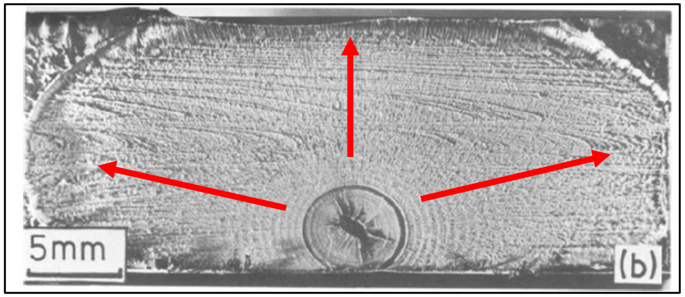
$\Delta K_{I,0} = 0.78\,\mathrm{MPa}\sqrt{\mathrm{m}}$ $f = 5\mathrm{Hz}$

R-ratio $= 0.1$ 23°C

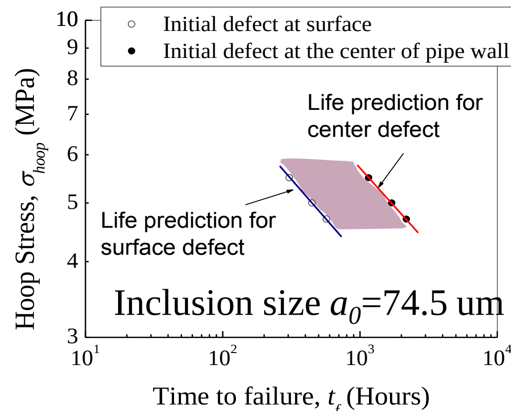
*Wee, Jung-Wook, and Byoung-Ho Choi. *International Journal of Solids and Structures,* 2016.

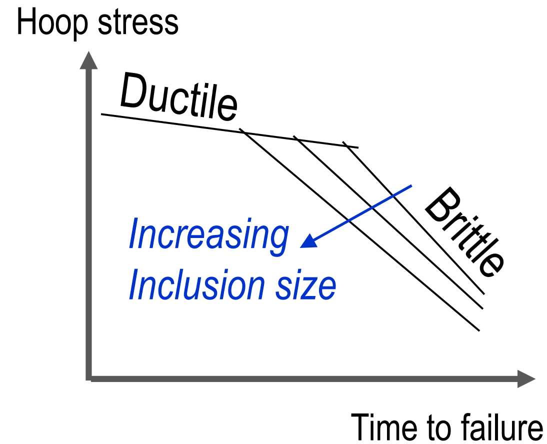
Research objectives

- **Lifetime dependency on the initial defect *location and size***



(Barker et al., 1983)





In order to expand the existing CL model to the internal initial crack and further application to the reliability analysis, the present study aims at

(1) Developing the CL model for the DSCG behavior from the internal initial defect, and

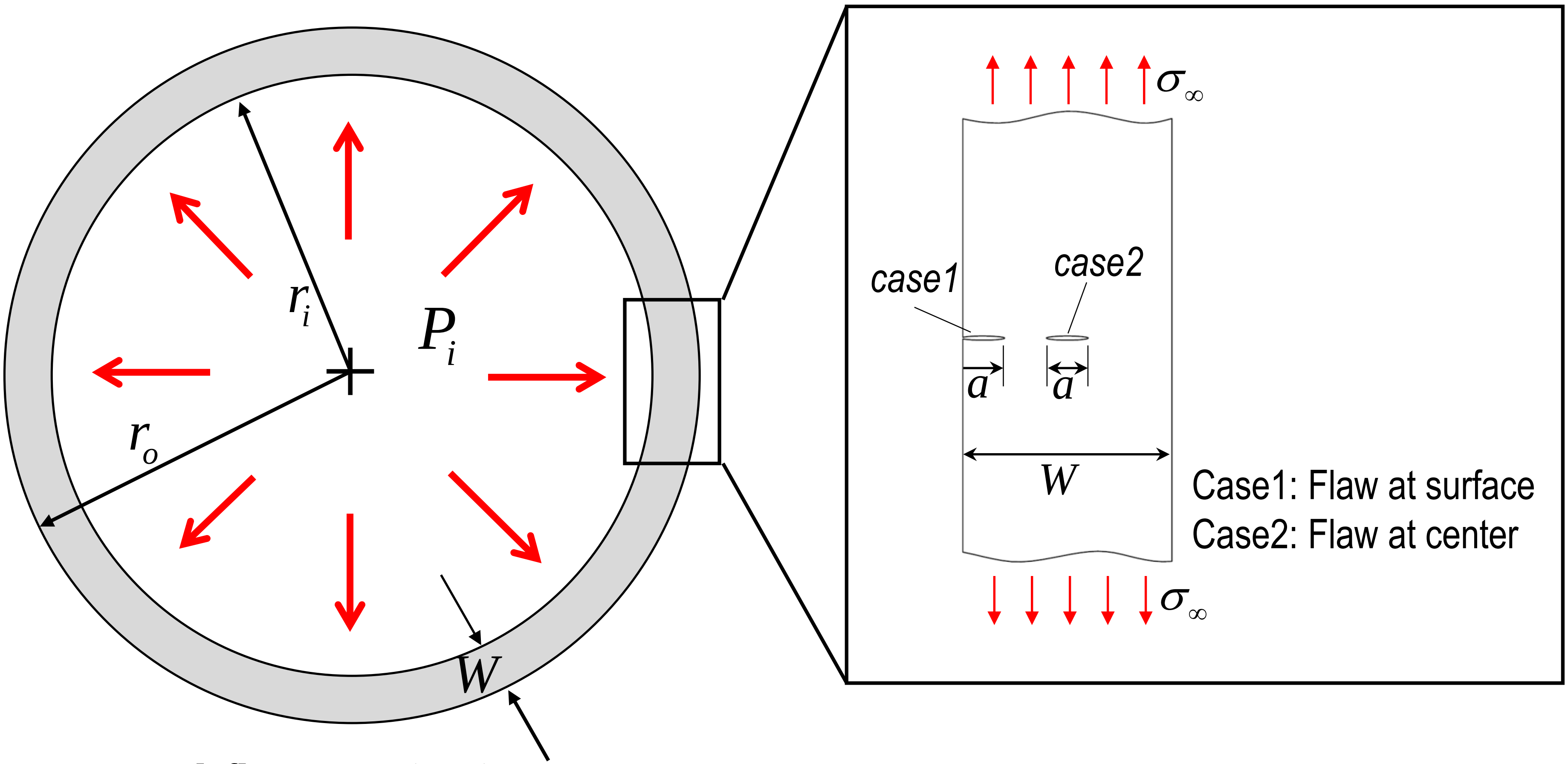
(2) Obtaining the lifetime distribution according to the applied stress level, initial crack size and location (i.e., surface or internal initial defect),

(3) Fitting of the lifetime distributions through the two kinds of probabilistic distribution functions.

Probabilistic approach on the internal defect

2019.2.25. Jung-Wook Wee, E-mail: yjw0914@korea.ac.kr

Problem definition



a : Initial flaw size (IFS)

W : Specimen width, mm

$\bar{a}$: Mean value of normalized IFS

S_a : Stdev of normalized IFS

Probabilistic distribution of IFS

⇩

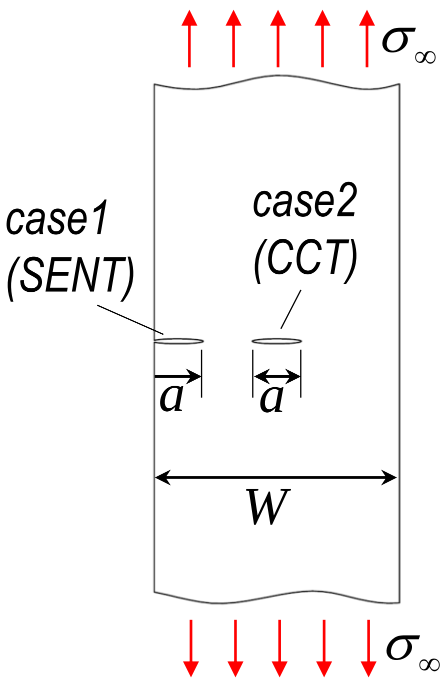
Lifetime distribution

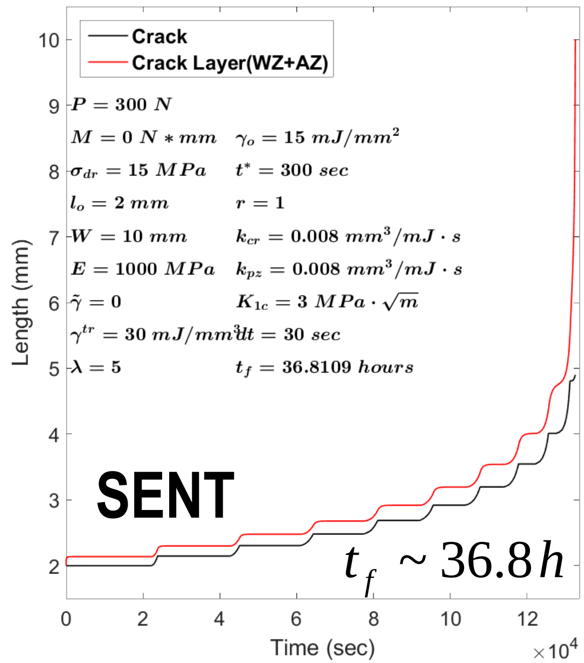
Deterministic crack layer model for center and edge crack

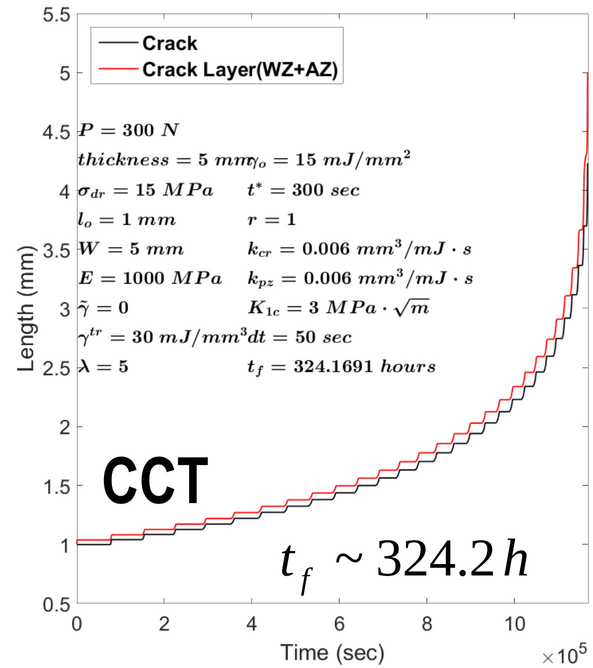
$$\sigma_\infty = 6\,\text{MPa}$$

$$a = 2\,\text{mm}$$

$$RT$$







Much longer lifetime for center cracked tension (CCT) specimen.

Probabilistic approach

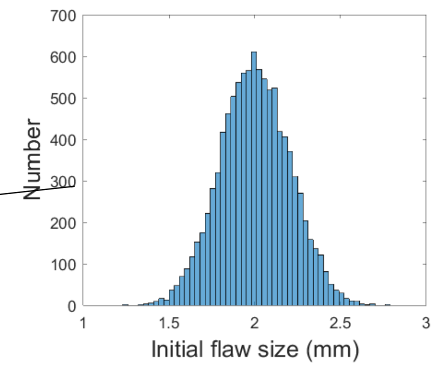
Random IFS generation following the normal distribution (generating 500 IFS)

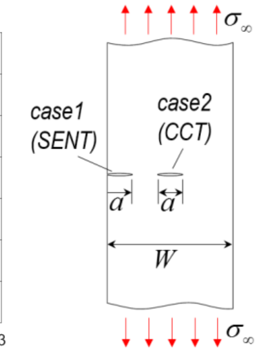
Independent CL growth simulation for each IFS from the surface or center location (500 simulations)

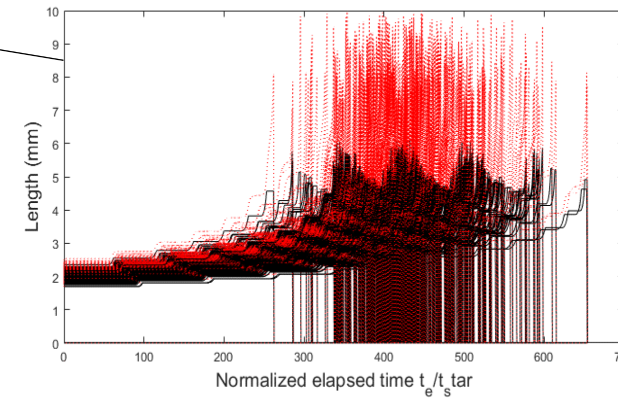
Obtaining the lifetime distribution

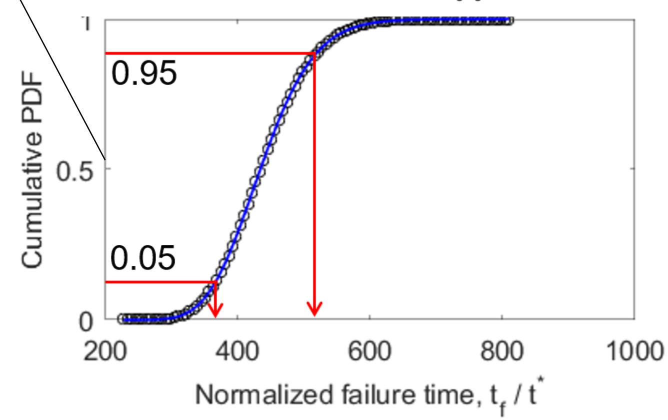
Lifetime distribution fit using the probabilistic distribution functions

Establishing the parameters for the appropriate probabilistic function

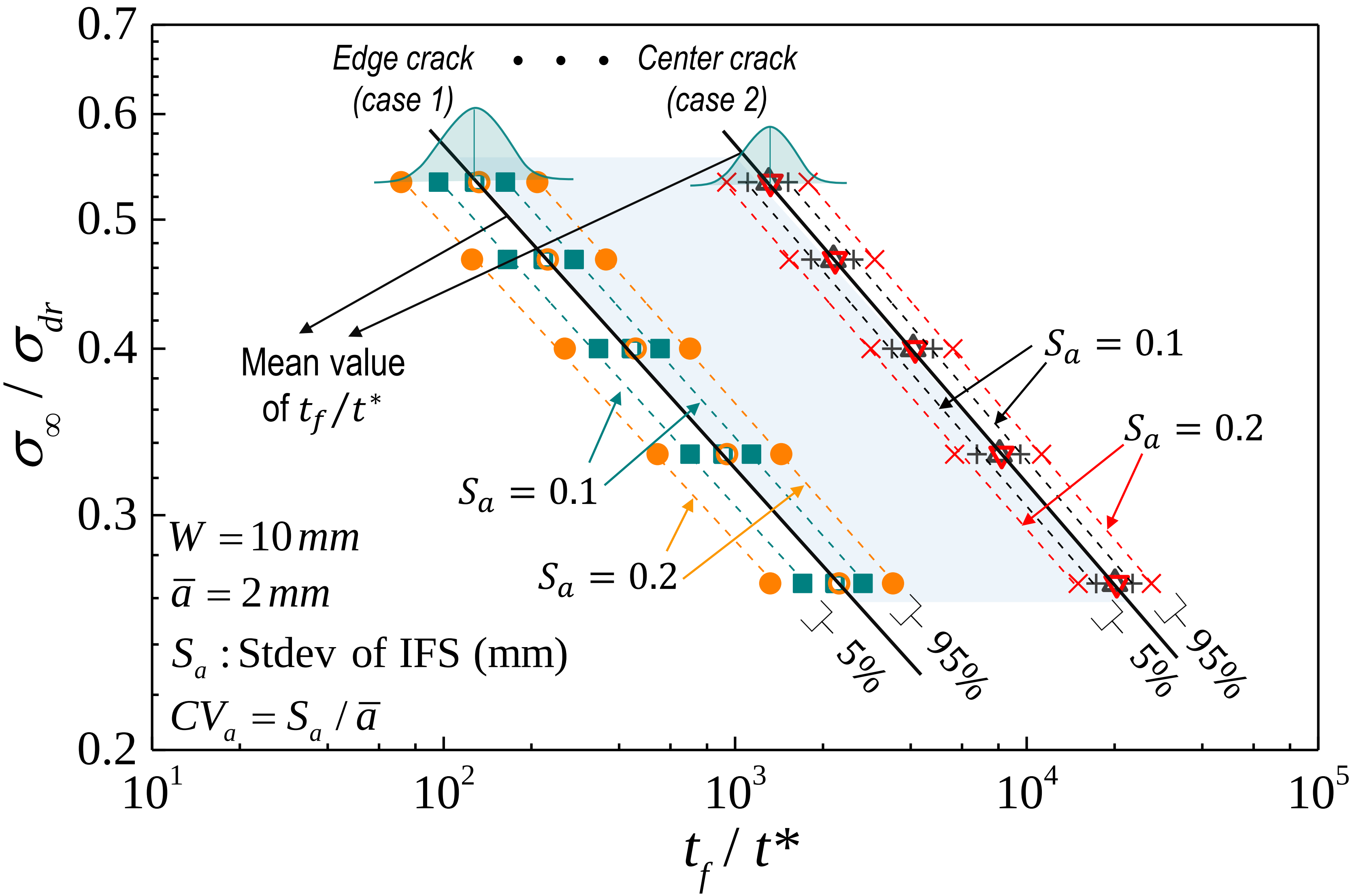




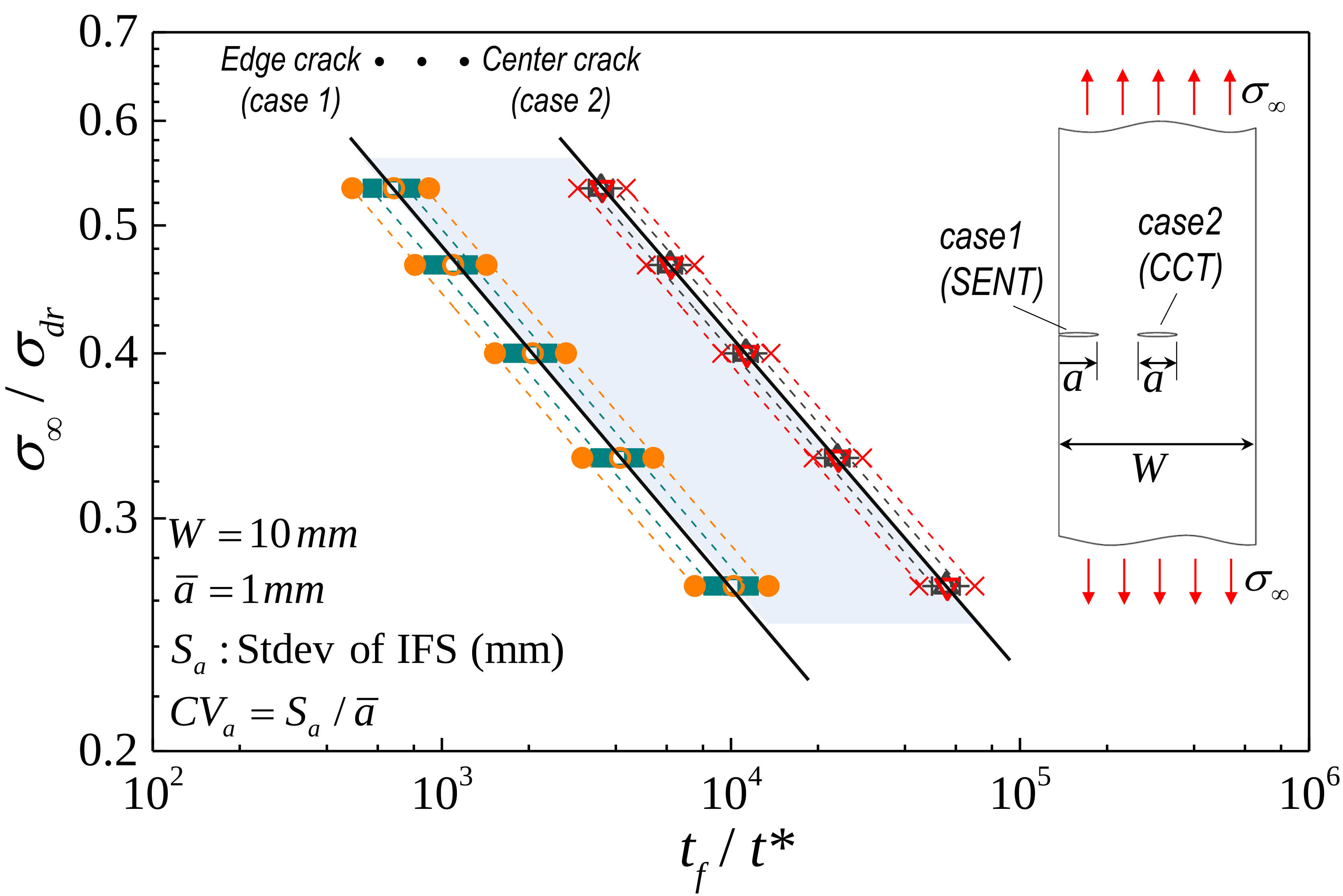




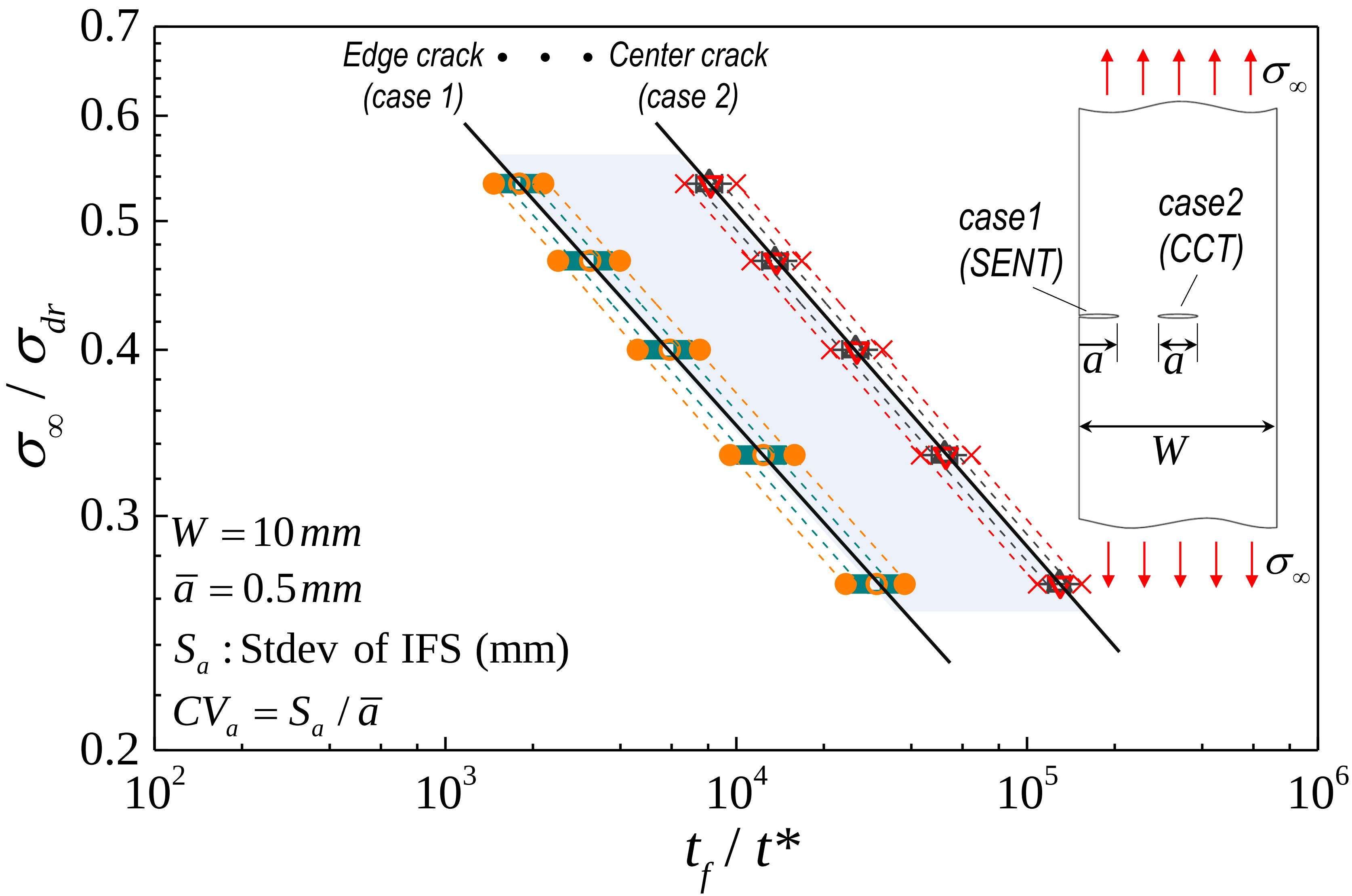
Lifetime distribution (Stress-lifetime curve)



Lifetime distribution (Stress-lifetime curve)

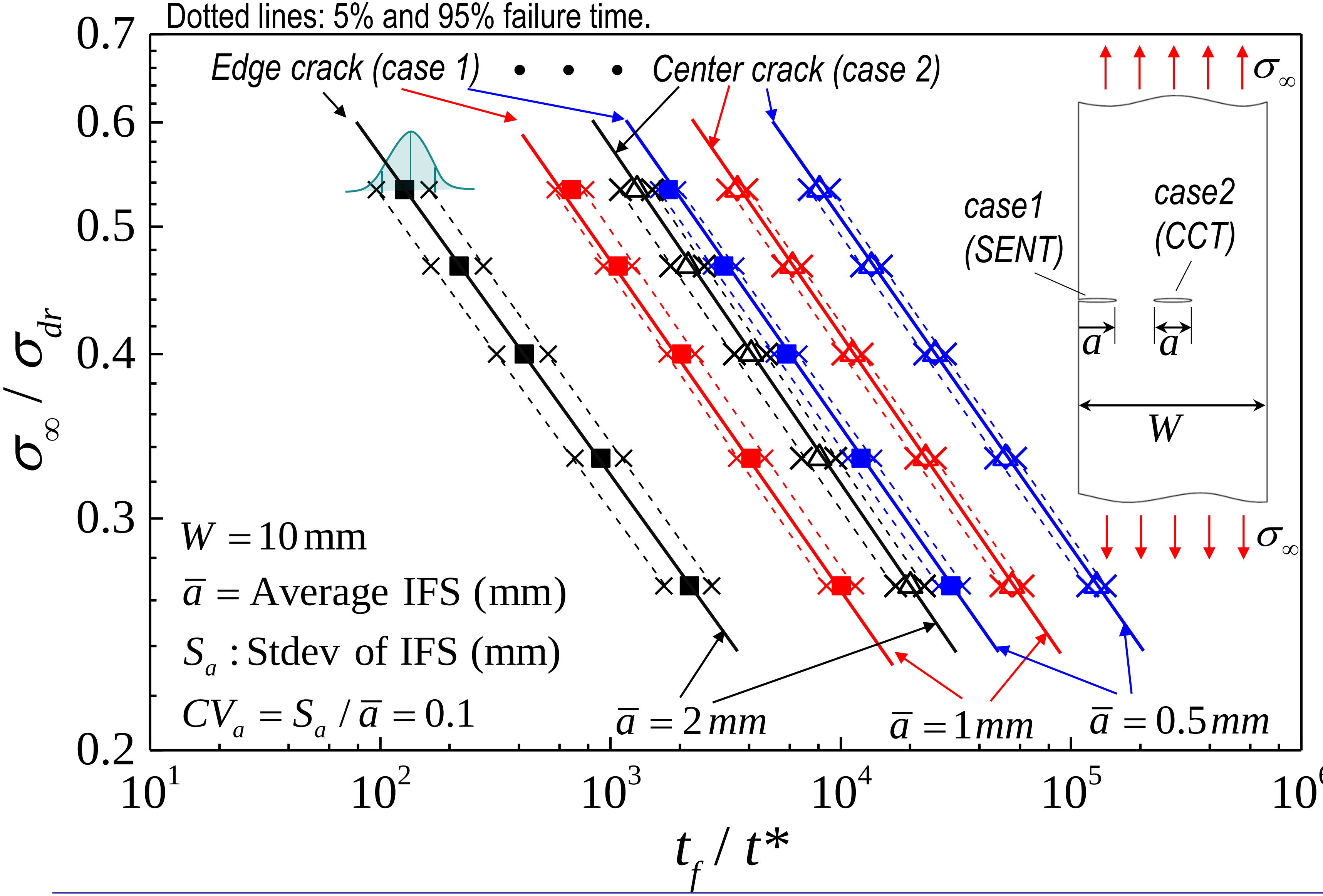


Lifetime distribution (Stress-lifetime curve)



Lifetime distribution (Stress-lifetime curve)

*Solid lines: Mean value of failure time.
Dotted lines: 5% and 95% failure time.



Lifetime distribution fit

Weibull distribution function:

- Flexibility to describe the decreasing, increasing, and constant hazard function.
- Most popular, extensive applications to the reliability analysis of wide range of engineering materials.

CDF: $$F(x;\lambda,k,\gamma)=1-e^{-\left(\frac{x-\gamma}{\lambda}\right)^k} \quad \text{for} \quad x\geq\gamma.$$

λ: scale parameter
k: shape parameter
γ: location parameter (threshold)

Birnbaum-Saunders (B-S) distribution function:

- Based on the damage accumulation, manifested in fatigue crack growth.
- If the crack increment (fatigue striations) under the fatigue loading is normally distributed, the distribution of failure time follows the B-S distribution function.

CDF: $$F(x;\alpha,\beta)=\Phi\left[\frac{1}{\alpha}\left(\sqrt{\frac{x}{\beta}}-\sqrt{\frac{\beta}{x}}\right)\right]$$

α: shape parameter
β: scale parameter
$\Phi(z)$: CDF of standard nornal distribution

- ❑ Although Weibull function has successfully demonstrated the appropriate fit of lifetime distributions of various engineering materials, the detailed investigation on the suitability of the Weibull function to the lifetime of HDPE (discontinuous SCG) is required.

- ❑ It is also meaningful to determine if there is other probability distribution function which describes the lifetime better.

- ❑ Thus, the resulting lifetime distributions are fitted by Weibull function and Birnbaum-Saunders (B-S) function.

Lifetime distribution fit

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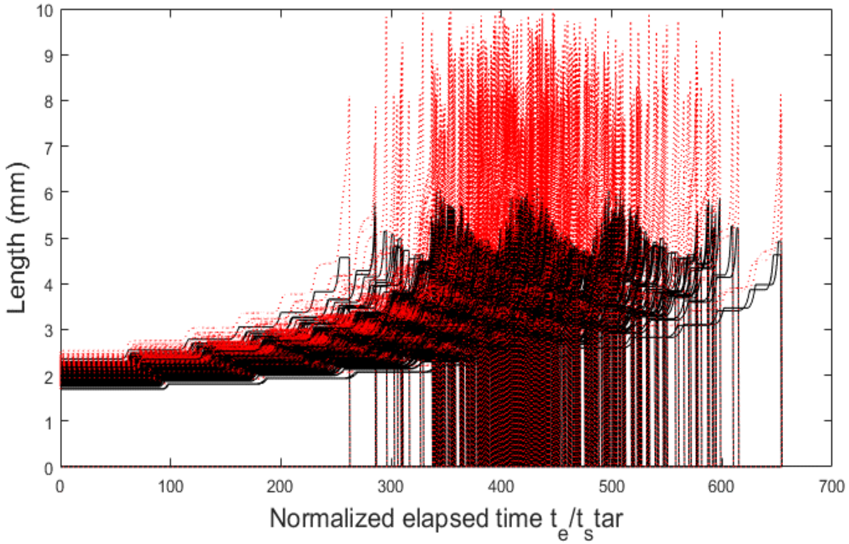
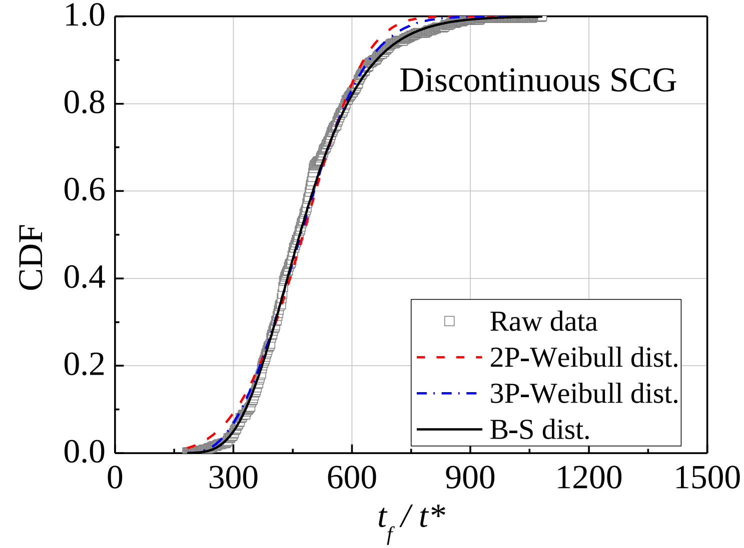
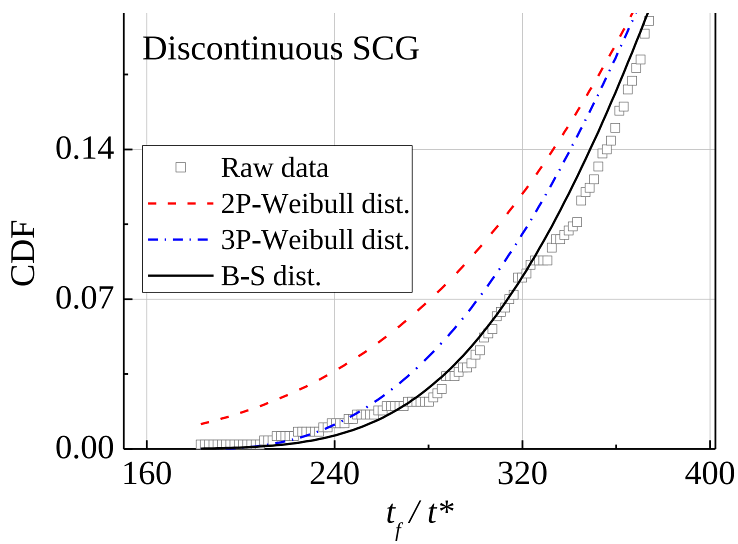
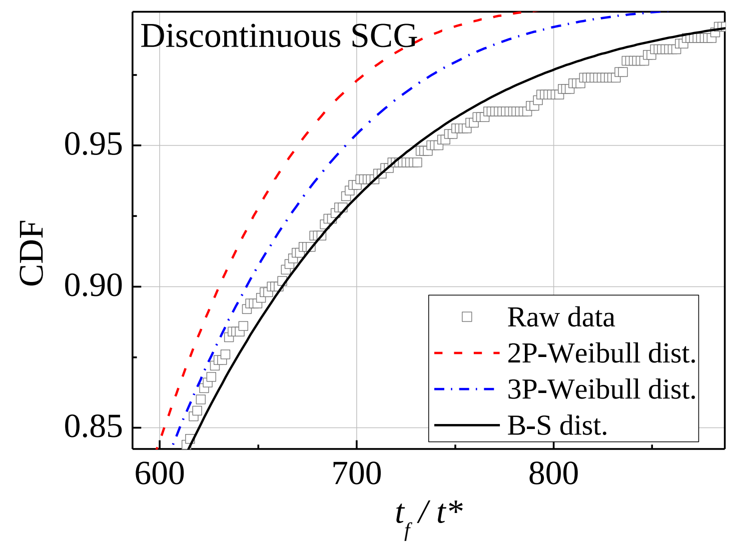
- **Weibull vs. B-S distribution function (Discontinuous SCG)**



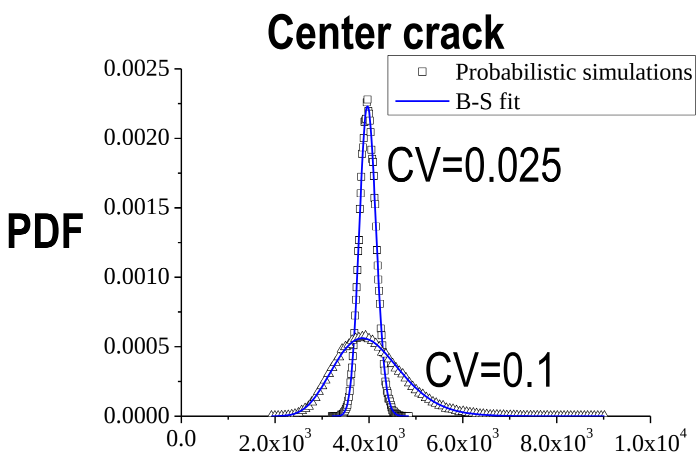


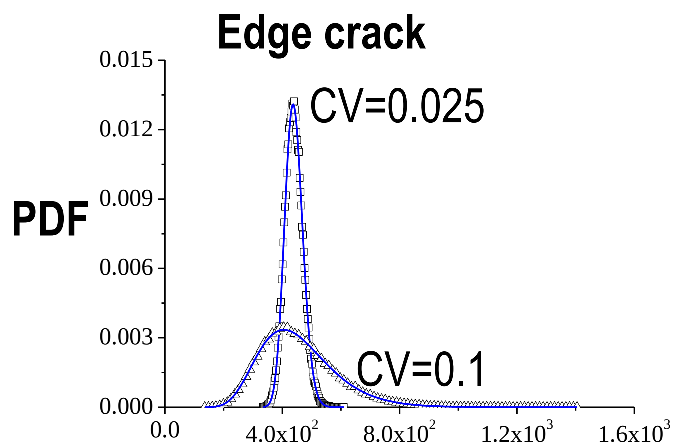


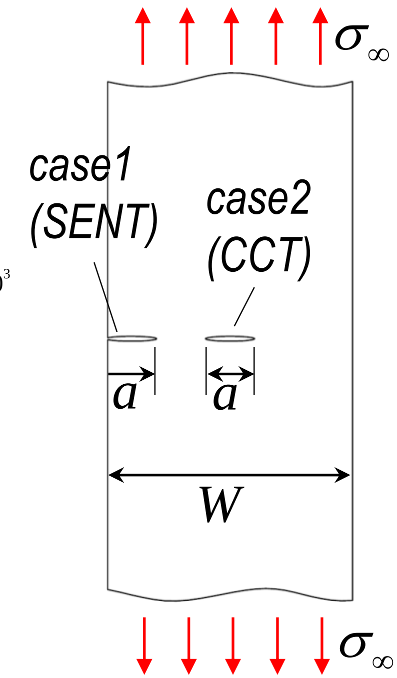


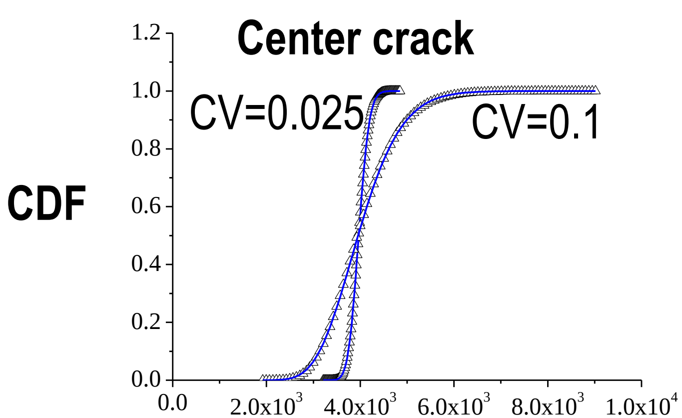
Lifetime distribution fit (B-S function)

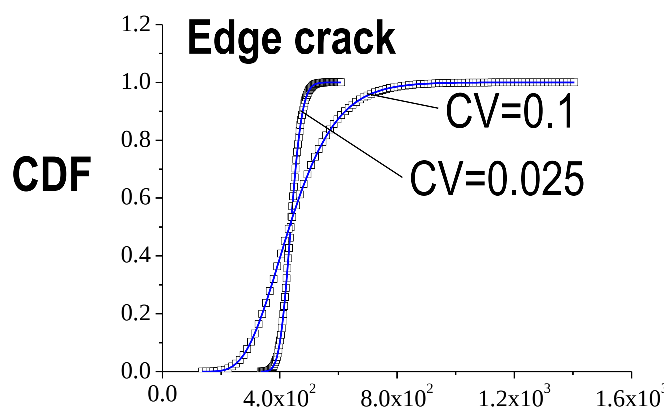
- **B-S distribution function**











Lifetime distribution fit

- **Scale and shape parameter of B-S function**

$$\frac{t_f}{t^*} = f_{B-S\ fit}\left(\beta,\gamma\right). \quad \text{Variables:} \quad \frac{\bar{a}}{W},\frac{\sigma_\infty}{\sigma_{dr}},CV_a$$

- Birnbaum-Saunders distribution

Scale parameter $\beta = \beta\left(\frac{\bar{a}}{W},\frac{\sigma_\infty}{\sigma_{dr}},CV_a\right)$

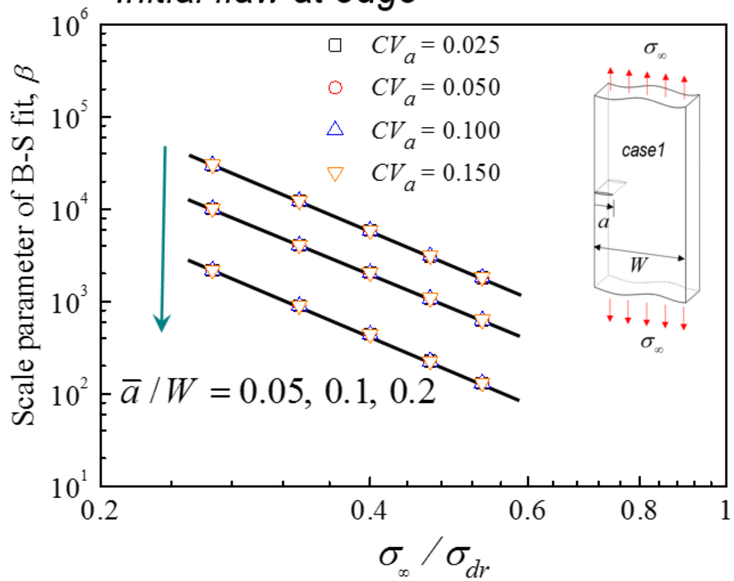
Shape parameter $\gamma = \gamma\left(\frac{\bar{a}}{W},\frac{\sigma_\infty}{\sigma_{dr}},CV_a\right)$

$$CV_a = \frac{\text{standrad deviation of initial crack length (mm)}}{\text{mean value of initial crack length (mm)}} = \frac{S_a}{\bar{a}}$$

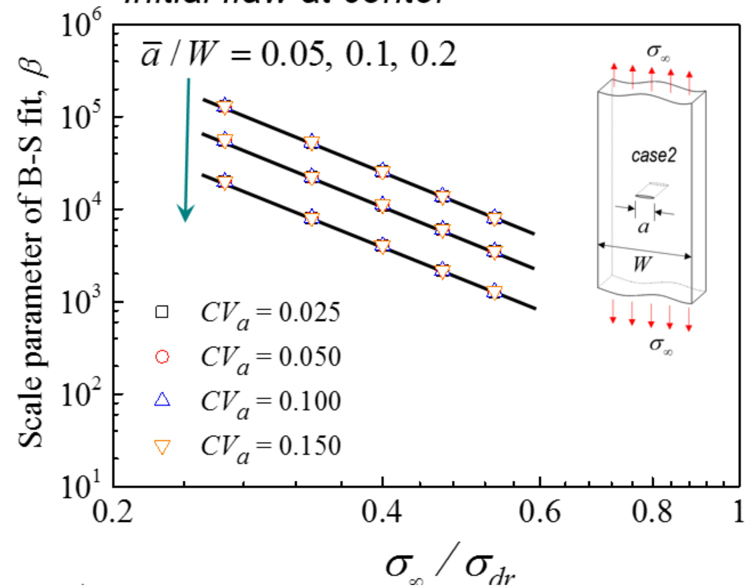
Lifetime distribution fit

- **Scale parameter, β**

Initial flaw at edge



Initial flaw at center



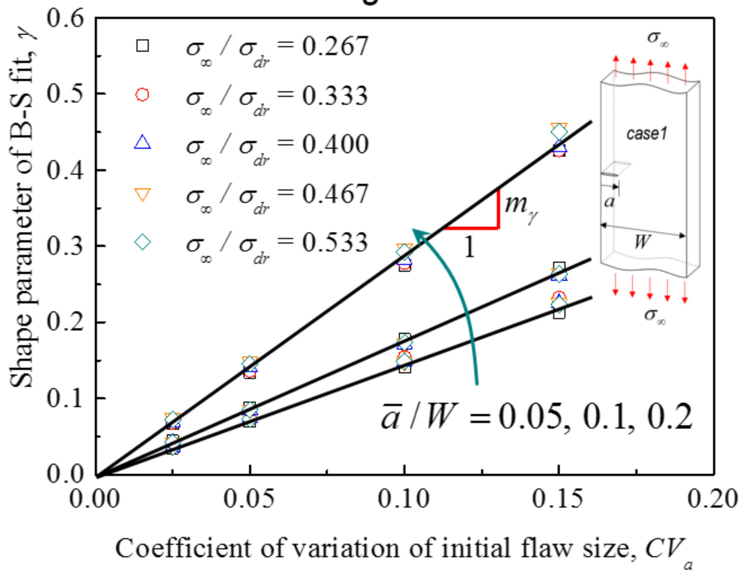
$$\beta = \beta\left(\frac{\bar{a}}{W}, \frac{\sigma_\infty}{\sigma_{dr}}, \cancel{CV_a}\right) \Rightarrow \log(\beta) = A_\beta \log\left(\frac{\sigma_\infty}{\sigma_{dr}}\right) + B_\beta,$$

- Scale parameter β is independent on the CV of initial crack size, and dependent on the mean value of initial crack size and stress level.
- β is decreased with the stress level and mean value of initial crack size.

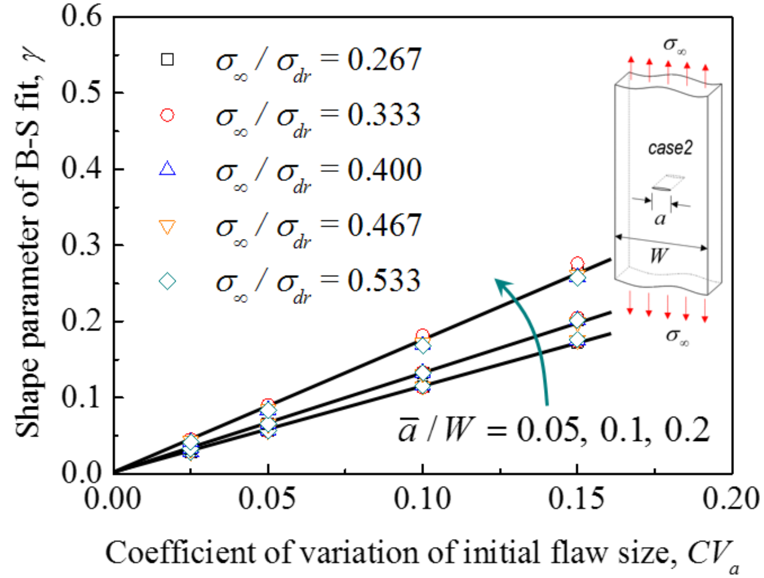
Lifetime distribution fit

- **Shape parameter, γ**

Initial flaw at edge



Initial flaw at center



$$\gamma = \gamma\left(\frac{\bar{a}}{W}, \cancel{\frac{\sigma_\infty}{\sigma_{dr}}}, CV_a\right) \Rightarrow \gamma = \gamma\left(\frac{\bar{a}}{W}, CV_a\right) = m_\gamma \cdot CV_a$$

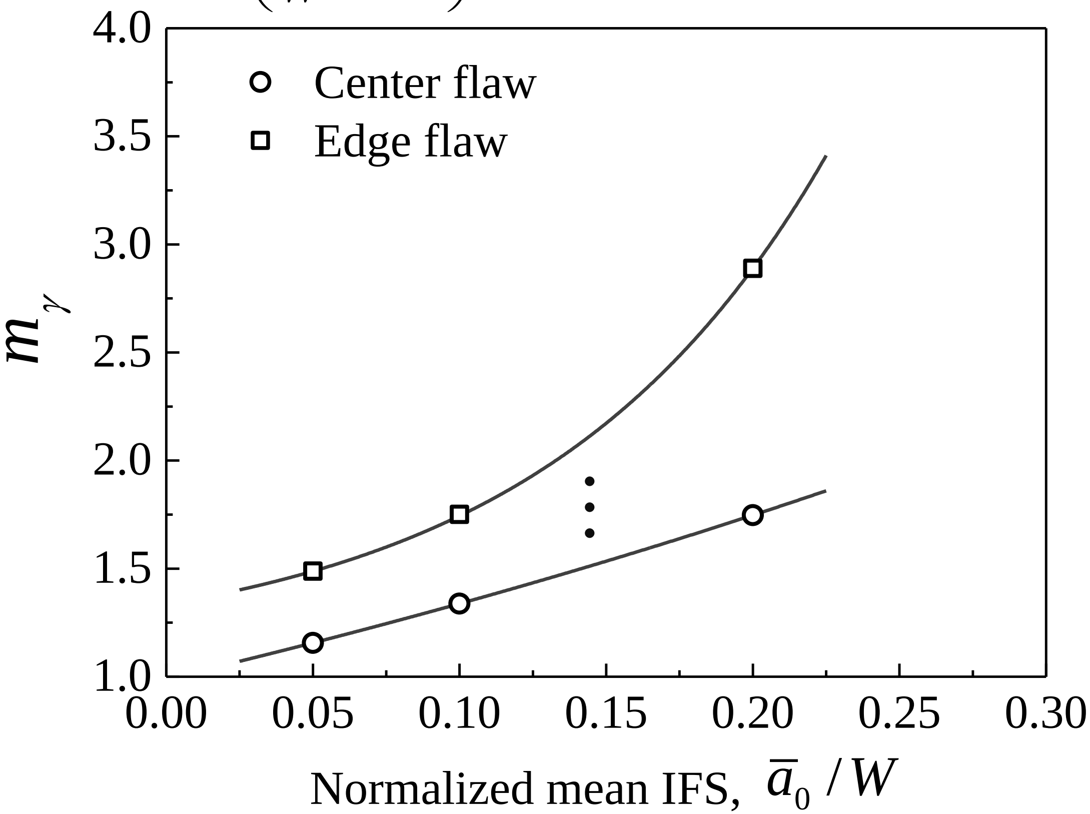
- Shape parameter γ is independent on the stress level, and dependent on the mean and CV of initial crack size.
- γ is linearly proportional to the *CV* of initial crack size.
- γ is increased with the mean value of initial crack size.

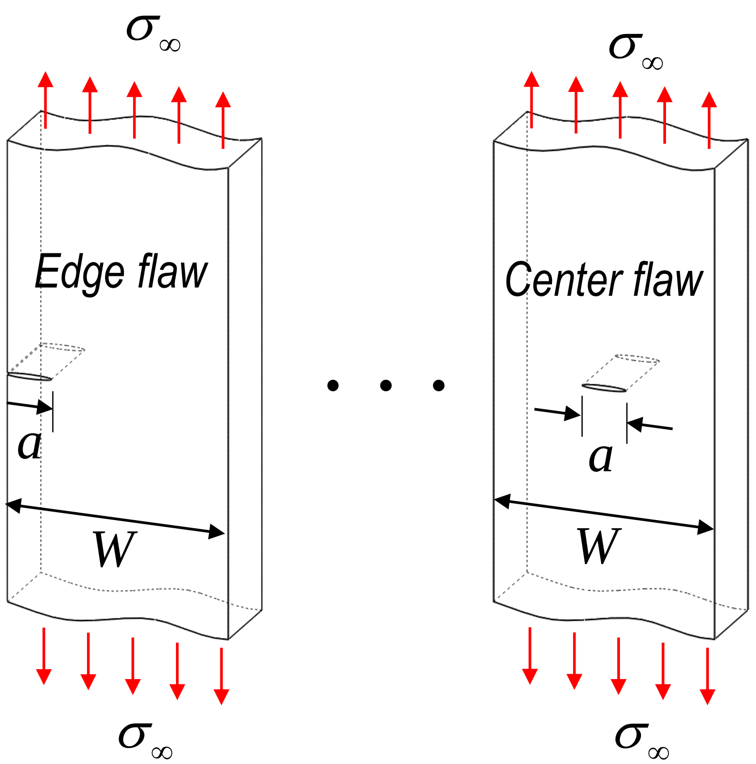
Lifetime distribution fit

- **Shape parameter, γ**

$$\gamma = \gamma\left(\frac{\bar{a}}{W}, CV_a\right) = m_\gamma \cdot CV_a$$

$$CV_a = \frac{\text{standrad deviation of IFS (mm)}}{\text{mean value of IFS (mm)}} = \frac{S_a}{\bar{a}}$$





Lifetime distribution fit

- **Parameter estimation for B-S fit**

1) Scale parameter β

$$\log(\beta) = A_\beta \log\left(\frac{\sigma_\infty}{\sigma_{dr}}\right) + B_\beta,$$

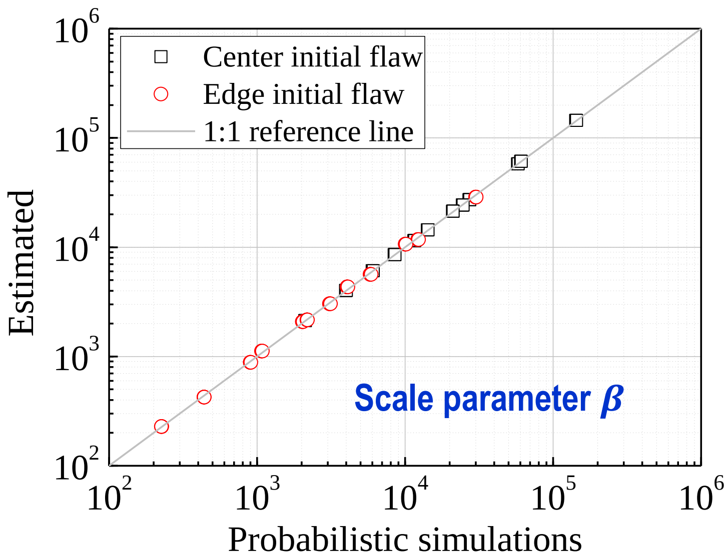
$$A_\beta = -4.07 \qquad \left(\frac{\bar{a}_0}{W} = \alpha\right)$$

$$B_\beta = 3.38 - 14.3\alpha + 58.2\alpha^2 - 110\alpha^3$$

For an center initial flaw,

$$B_\beta = -32.6\alpha^3 + 23.0\alpha^2 - 11.5\alpha + 2.68$$

For an edge initial flaw,



2) Shape parameter γ

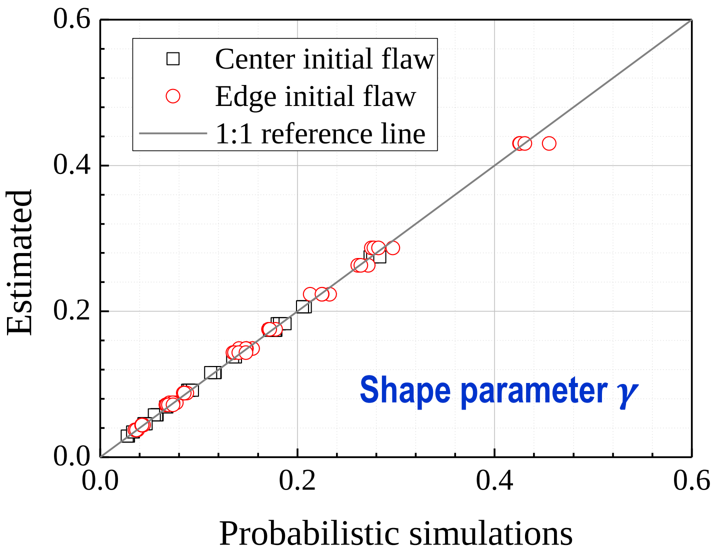
$$\gamma = m_\gamma \cdot CV_a$$

- **For an center initial flaw,**

$$m_\gamma = 0.949 + 4.10\alpha + 1.51\alpha^2 \qquad \left(\frac{\bar{a}_0}{W} = \alpha\right)$$

- **For an edge initial flaw,**

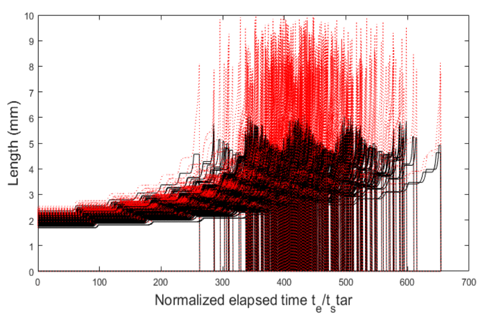
$$m_\gamma = -317\alpha^3 + 150\alpha^2 - 11.7\alpha + 1.74$$

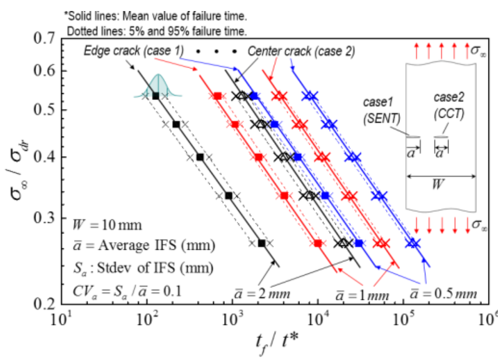


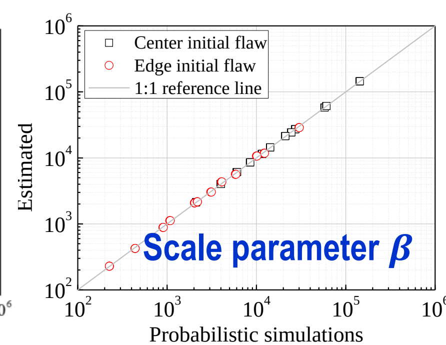
Conclusions

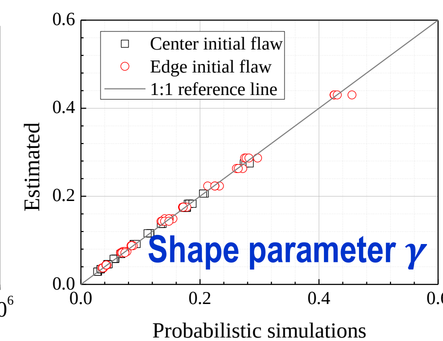
Conclusion

- Discontinuous SCG can be fundamentally modeled by the CL theory.
- To assess the lifetime distribution regarding the location and size of initial defect, the probabilistic CL simulations were performed.
- The lifetime distribution followed by the discontinuous SCG can be accurately explained by B-S function.
- The parameters of B-S function with various initial crack sizes were estimated for the center and surface initial crack cases.









Thank you very much for your attention!