Selecting The Best Idler Size

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ABSTRACT

When thinking about idler diameter, we usually think smaller is better. But smaller is not always best. An idler's bearing drag and rotating inertia combine during acceleration and deceleration to upset the web's tension. Even the best tension control system can't correct for idler induced tension upsets during transients. In lines with large accumulators and many idlers, this can result in missed splices, and long times to steady state production. The right diameter can minimize the upset caused by idlers. This paper shows how to "right size" idler diameter and discusses factors that must be considered when selecting idler diameter.

NOMENCLATURE

A	Linear acceleration of web, m/s^2
E	Young's modulus of shell, N/m^2
J	Rotational moment of inertia, kg-m^2
h_a	Air film thickness, m
h_w	Web thickness, m
k	SI natural frequency constant
l	Idler shell length, m
m	Rotating mass of idler, kg
P	Resultant tension force on idler, N
ΔT, *Drag*	Tension difference across an idler, N
ΔT_b	Tension difference due to bearings, N
ΔT_i	Tension difference due to inertia, N
ΔT_{peak}	Total tension difference near the end of acceleration, N
T_n	Tension on web in span "*n*", force, N
V	Velocity of web, m/s
w	Idler shell wall thickness, m
α	Angular acceleration, rad/s^2
β	Total wrap angle on roller, rad
γ	Idler dynamic torque, N-m/rad/s
η	Dynamic viscosity of air, 18.3x10^{-6} N-sec/m^2
θ	Angular position, rad
μ	Coefficient of friction
ρ	Idler shell material density, kg/m^3
τ	Idler static-starting torque, N-m
ω	Angular velocity, rad/s
Subscripts	
b	Bearing related
i	Inertia related
n	Span number

INTRODUCTION

In low tension webs, idler bearing drag and idler rotating inertia are important. They work together to create tension transients during acceleration and deceleration. Idler drag increases with web speed and inertial drag increases with acceleration rate. Together, these limit how fast lines can accelerate and how long it takes to achieve steady state production. They are one of the factors that determine both high and low tension targets and therefore what range of material gauges can be run. For a given bearing size, a larger diameter idler has less steady state drag due to its longer torque arm, but the larger diameter often means greater rotating mass and inertia and bigger upsets during acceleration and deceleration. Idler diameter can be optimized to minimize their combined effect on the web.

Figure 1 shows a simple unwind zone with parent roll, two plain idlers, a tension measuring idler, and lastly, a driven roller.

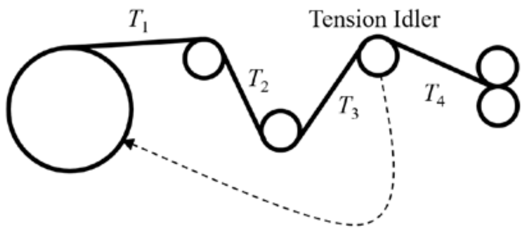

Figure 1 - An unwind zone with a parent roll, three idlers, and a driven roller

Figure 2 shows the tension in each span as the web accelerates, runs at steady speed, and then decelerates. At the start of acceleration, we see a sudden difference in tension develop across each idler due to the idler's static bearing drag and the imposition of the idler's inertial drag. As the line accelerates, the inertial drag remains the same, but the bearing drag and tension difference increase across the idlers as the velocity increases. The tension control system will try to keep the tension constant at the tension idler, but tensions upstream of the tension idler will be lower than target and tensions downstream will be higher. The result is none of the tensions are at target.

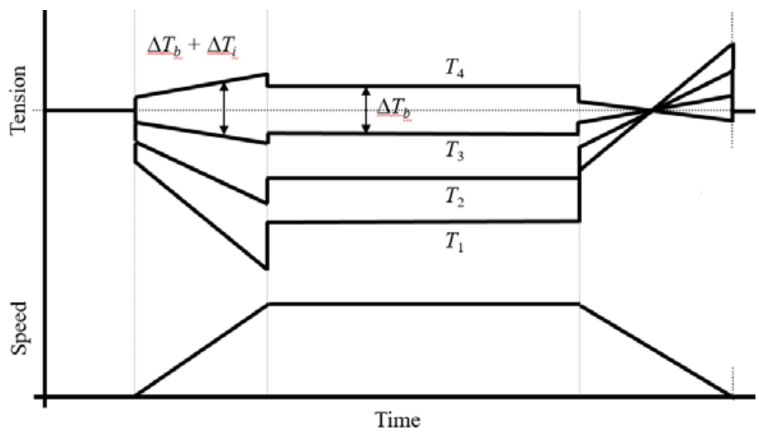

Figure 2 - Tension distribution across the four spans shown in Figure 1

The lowest tension occurs near the end of acceleration where line speed and bearing drag are greatest and the web's tension is still being used to accelerate the idlers. Target tension must be set high enough at the tension idler so that the start-up survives this low tension period in span 1. During deceleration, the web's tension is used to decelerate the idlers and there may be a reverse of the tension profile as the line decelerates. The tension target must be low enough that the peak tension during deceleration does not damage the web. The extremes of tension are seen during acceleration and deceleration creating the greatest risk to the process

Similar bearing and inertial drag transients occur in accumulators, dancers, draw control zones, and tension control zones containing one or more idlers. The magnitude of the changes depends on the design and number of idlers, the acceleration rate, and the final velocity.

If the driven roller following span 4 is the pacer roller for the path, the upset in tension will be seen in all downstream spans. If the driven roller is the primary process roller, it will see the transients directly.

Figures 3 and 4 show a more dramatic transient at the pacer or process roller created by moving the tension idler to the first idler in the zone.

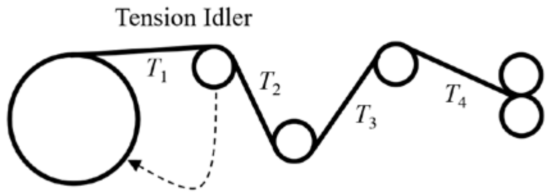

Figure 3 - Tension idler upstream.

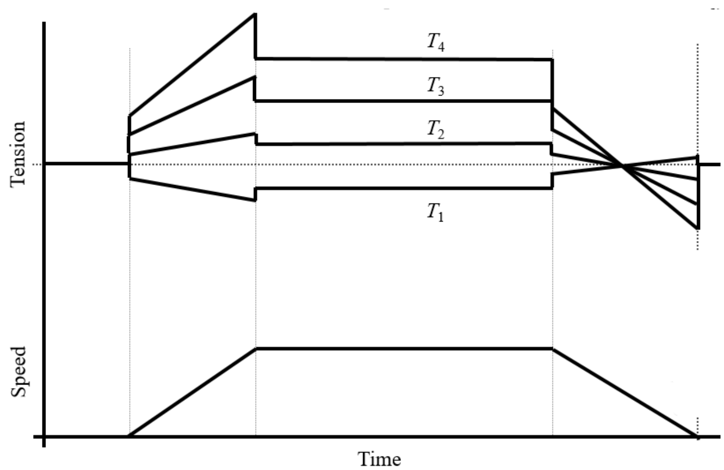

Figure 4 - Tension distribution resulting from Figure 3's arrangement.

In both tension cell configurations, the highest and lowest tensions seen by the web, and the off-target errors, are the result of bearing and inertial drag. Minimizing their combined effect can result in a more stable web path, fewer tension related problems, and can allow running at lower tension.

OPTIMIZING BEARING AND INERTIA DRAG

In its simplest form, idler bearing drag torque can be described as a linear relationship with angular velocity, and modeled as

$$Torque_{bearing} = \tau + \gamma\omega \qquad \{1\}$$

where τ is the static or starting torque and γ is the torque increase with rotational velocity, ω. The torque to turn the bearing is generated by the frictional force transmitted from the web to the idler's surface. The force transmitted to the idler results in a web tension difference across the idler. Bearing drag will always want to slow the idler, so web tension must always be higher downstream of the idler to overcome bearing drag.

Formula 1 can be restated in terms of linear velocity, V, and the tension difference, ΔT_b, across the idler as

$$\Delta T_b = \tau/r_o + \gamma V/r_o^2 \qquad \{2\}$$

From this, we can see that increasing the idler outer radius, r_o, will reduce the tension difference across the idler caused by bearing drag torque.

The inertial drag torque during acceleration is

$$Torque_{ineria} = J\alpha \qquad \{3\}$$

The idler's mass moment of inertia, J, depends on its design, but in its simplest form for a hollow shell idler it can be reduced to

$$J = mr_o^2 \qquad \{4\}$$

where the rotating mass, m, is concentrated near the outer radius.

Formulas 3 and 4 can be combined and can also be restated in terms of the tension difference across the idler due to inertia, ΔT_i, and the linear acceleration, A, to become

$$\Delta T_i = mA \qquad \{5\}$$

Assuming most of the idler's mass is in the outer rotating shell of a hollow idler, mass can be restated in the formula in terms of outer radius, wall thickness, w, shell length, l, and density, ρ, as

$$\Delta T_i = \pi (r_o^2 - (r_o - w)^2)\ l\rho A \qquad \{6\}$$

For idlers, we normally think of "low inertia" as better and this can be achieved by decreasing the radius, wall thickness, or density, i.e., decreasing the rotating mass.

The formulas 2 and 6 can be added to show the total tension difference, ΔT_{peak}, seen across the idler as a function of its properties, linear velocity and linear acceleration.

$$\Delta T_{peak} = \tau/r_o + \gamma V/r_o^2 + \pi (r_o^2 - (r_o - w)^2)\ l\rho A \qquad \{7\}$$

The highest total drag occurs near the end of acceleration when velocity and bearing drag are at their highest and the inertial drag is still present. This value is the difference in tension between adjacent spans shown in figures 2 and 4. Figure 5 is a plot of formulas 2, 6 and 7 for the "typical" idler properties shown in Table 1. It shows the bearing, inertial and total drag near the end of acceleration as a function of radius.

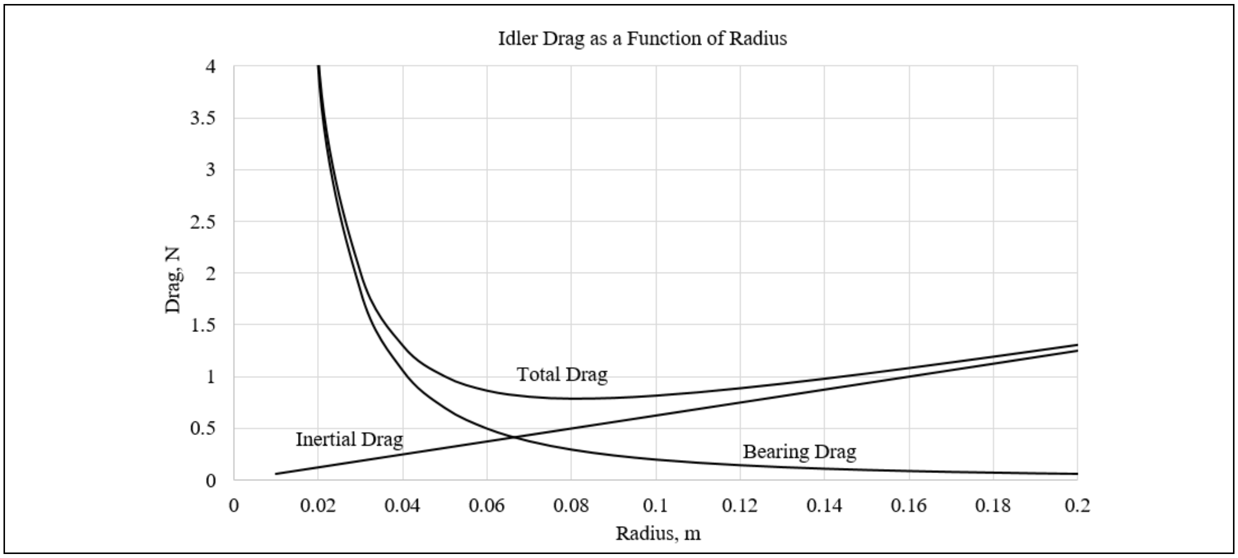

Figure 5 - Idler drag as a function of radius

At small radii, the inertial drag is low, but the rotational velocity is high creating high bearing drag. At large radii, there is low rotational velocity and bearing drag, but higher inertial drag. For the example values in Table 1, the lowest total drag of 0.79 N occurs at a radius of approximately 0.082 m. Higher acceleration or higher density will raise the right side of the plot and shift the optimum diameter to the left. Increased bearing drag or higher velocity will shift it to the right.

l =	1	meters	idler shell length
w =	0.0025	meters	shell wall thickness
ρ =	1600	kg/m^3	material density
A =	0.25	m/sec^2	linear acceleration
V =	5	m/sec	linear velocity
α =	0.005	N-m	bearing static drag
γ =	0.0003	(N-m)/sec	bearing viscous drag
P =	50	N/m	resultant web load
k =	9.87		SI natural freq const
E =	7.00E+10	N/m^2	Young's modulus of shell

Table 1 - Typical idler values for a generic carbon composite idler

DEFLECTION

An important factor in selecting roller diameter is deflection under load. Deflection appears as either a misalignment or steering affect and is a common cause for web wrinkling and mistracking. For a simply supported, centered, dead shaft idler, the mode of deflection is sag of the shell between its support bearings.

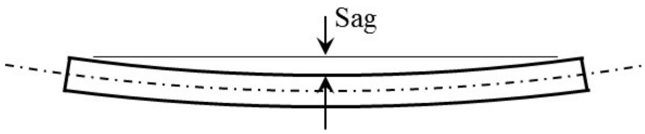

Figure 6 - Roller sag

The rule of thumb for the maximum acceptable sag at the midpoint of a roller for general applications is 0.00015 m of sag per meter of length[1]. For a simply supported hollow tube idler, the area moment of inertia, I, and maximum sag are calculated using

$$I = \pi(r_o^4 - r_i^4)/4 \quad \text{and} \quad Sag = 5Pl^4/(384EI) \qquad \{8\}$$

Using the values in Table 1, the deflections for several radii of the 1 m long idler are shown in Table 2.

Radius, m	0.04	0.045	0.05	0.055	0.06	0.065	0.07	0.075	0.08	0.085
Sag, m	0.000201	0.000139	0.000101	0.000075	0.000058	0.000045	0.000036	0.000029	0.000024	0.000020

Table 2 - Deflection as a function of radius

For the roller properties described above, any roller greater than 0.045 m radius will satisfy the sag requirement. A 0.045 m radius idler designed as in Table 1 will have 1.13 N of peak drag; 40% more peak drag than the 0.79 N of a similarly designed idler with 0.082 m radius. Its steady state drag will be 0.85 N vs. 0.28 N for a 0.082 m idler; 300% higher. The minimum diameter for sag is not the best choice to minimize the peak or steady state tension difference across the idler.

[1] The origin of this rule of thumb is uncertain. The earliest reference I find is Dave Roisum's 1996 *Mechanics of Rollers* citing "unpublished standards" on page 30.

OTHER CONSIDERATIONS

WEB BENDING RADIUS AND DIFFERENTIAL STRAIN

A web bending over the radius of a roller experiences a strain profile through its thickness and has a neutral bending axis somewhere between its surfaces. See figure 7.

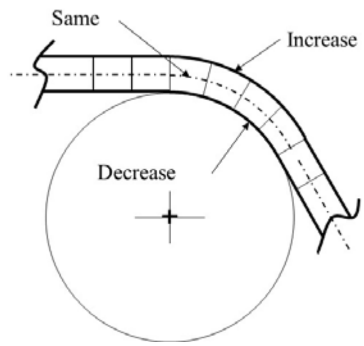

Figure 7 - Web bending strain over a roller

The strain difference, $\Delta\varepsilon$, between the inside surface and outside surface of the web is approximately equal to the web's thickness, h_w, divided by the roller radius.

$$\Delta\varepsilon \approx h_w/r_o \qquad \{8\}$$

A 1 mm thick web on a 100 mm radius roller will have approximately 1% strain difference between its inside and outside surfaces. If the pitch line is in the center of the web and is nominally at 0.25% strain, the neutral axis will continue to experience 0.25% strain, the outer surface will experience 0.75% strain and the inner surface will experience 0.25% compression. Especially with thick webs, and considering times of peak tension, care must be taken to select a roller radius that does not cause damage to either of the web's surfaces due to over extension or compression.

PITCH RADIUS AND VELOCITY ERRORS

The web's neutral bending axis is also its pitch line as it traverses a roller. The velocity of the web approaching a roller is determined by its pitch line velocity. Assuming the pitch line is in the center of the web, the pitch line velocity is

$$V = \omega(r_o + h_w/2) \qquad \{9\}$$

A web with a thickness profile, either natural or due to product design, will have a varying pitch line radius across its width and therefore a cross direction velocity profile. This is a potential cause of wrinkles and tracking error. The size of the velocity profile that can be tolerated depends on the nature of the web and cannot be easily determined. A larger roller radius will minimize the cross direction velocity profile that develops and minimizes the potential for wrinkles and mistracking.

AIR ENTRAINMENT

As a web traverses a roller, a film of entrained air is trapped between the web and roller. The air film increases with velocity and at some point can separate enough of the web from the roller surface that slipping occurs. The initial air film thickness, h_a, can be calculated by the Knox-Sweeney equation

$$h_a = 0.643 r_o \left[\frac{6\eta\left(V_{web} + V_{roller}\right)}{T/l} \right]^{2/3} \qquad \{10\}$$

Increasing the roller's radius increases the initial air film thickness and lowers the velocity at which slipping occurs. The usual means of solving air entrainment problems is to add texture or grooves to the roller surface to provide a path for the air to escape.

CRITICAL SPEED AND RPM

Critical speed and running RPM must be considered when sizing a roller. The critical speed depends on many factors in addition to the radius of the shell. However, considering just the shell alone, the first critical speed, ω_c, for a simply supported hollow shell is

$$\omega_c = \frac{k}{2\pi l^2}\sqrt{\frac{lEI}{m}} \qquad \{11\}$$

For the optimum 0.082 m shell in this example, the critical speed is almost 600 rev/s while its rotational speed is only 9.7 rev/s.

IDLER COST AND MACHINE SIZE

Major considerations in idler design are cost and size. Radius is a factor in both. Larger idlers generally will cost more and can be a factor in determining the overall size of the machine that uses them, which may be a bigger cost than the idlers themselves.

Wall thickness of the idler tube should be as thin as possible to minimize the rotating mass with the limit being dent resistance and mechanical integrity.

Size of standard materials is a factor. The optimum 0.082 m radius in this example is not a standard size. Figure 5 shows that radii from 0.075 m to 0.1 m would have similarly small ΔT_{peak} values.

CONCLUSION

Typically idlers are sized based on deflection standards alone. Some industries consider the stress profiles through the web's thickness, but for thin, flexible webs, this is not usually a factor. Designing idlers to minimize their effect on tension becomes more important when equipment is designed for high acceleration, high speed, and low tension. Of course, there is more to idler design than just radius, but selecting the right radius is the first step in designing the right idler.

APPENDICES

Bearing + Inertial drag plotted as a function of radius and thickness

	Outside Radius =	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12
Wall Thickness =	0.0005	15.512	4.0248	1.8707	1.1125	0.7625	0.5751	0.4652	0.3971	0.3535	0.3253	0.3073	0.2963
	0.001	15.524	4.049	1.9075	1.1618	0.8244	0.6495	0.5522	0.4967	0.4657	0.4501	0.4446	0.4462
	0.0015	15.535	4.0726	1.9436	1.2105	0.8857	0.7234	0.6386	0.5956	0.5772	0.5742	0.5813	0.5954
	0.002	15.545	4.0955	1.9791	1.2585	0.9463	0.7966	0.7244	0.694	0.6881	0.6976	0.7173	0.744
	0.0025	15.555	4.1178	2.014	1.306	1.0063	0.8691	0.8095	0.7917	0.7984	0.8205	0.8527	0.892
	0.003	15.564	4.1395	2.0482	1.3528	1.0657	0.9411	0.894	0.8888	0.908	0.9427	0.9875	1.0393
	0.0035	15.573	4.1605	2.0818	1.399	1.1244	1.0124	0.9779	0.9852	1.017	1.0643	1.1216	1.186
	0.004	15.58	4.181	2.1148	1.4445	1.1825	1.0831	1.0612	1.081	1.1254	1.1852	1.2552	1.3321
	0.0045	15.588	4.2007	2.1472	1.4894	1.24	1.1531	1.1438	1.1762	1.2332	1.3055	1.388	1.4776
	0.005	15.594	4.2199	2.1789	1.5337	1.2969	1.2226	1.2258	1.2708	1.3403	1.4252	1.5203	1.6224
	0.0055	15.6	4.2384	2.21	1.5774	1.3531	1.2914	1.3071	1.3647	1.4468	1.5443	1.6519	1.7666
	0.006	15.606	4.2564	2.2405	1.6204	1.4087	1.3595	1.3879	1.458	1.5527	1.6627	1.7829	1.9102
	0.0065	15.61	4.2736	2.2703	1.6629	1.4637	1.4271	1.468	1.5507	1.6579	1.7805	1.9133	2.0531
	0.007	15.614	4.2903	2.2995	1.7046	1.5181	1.494	1.5475	1.6427	1.7625	1.8977	2.0431	2.1954
	0.0075	15.618	4.3063	2.3281	1.7458	1.5718	1.5603	1.6263	1.7342	1.8665	2.0143	2.1722	2.3371
	0.008	15.621	4.3217	2.3561	1.7863	1.6249	1.6259	1.7046	1.8249	1.9699	2.1302	2.3007	2.4782
	0.0085	15.623	4.3365	2.3834	1.8262	1.6773	1.691	1.7822	1.9151	2.0726	2.2455	2.4285	2.6186
	0.009	15.624	4.3506	2.4101	1.8655	1.7292	1.7554	1.8591	2.0046	2.1747	2.3602	2.5558	2.7584
	0.0095	15.625	4.3641	2.4362	1.9041	1.7804	1.8192	1.9355	2.0936	2.2762	2.4742	2.6824	2.8976
	0.01	15.626	4.377	2.4617	1.9421	1.831	1.8823	2.0112	2.1818	2.377	2.5876	2.8084	3.0361

Table 3 - Values of total drag vs. radius and wall thickness.