

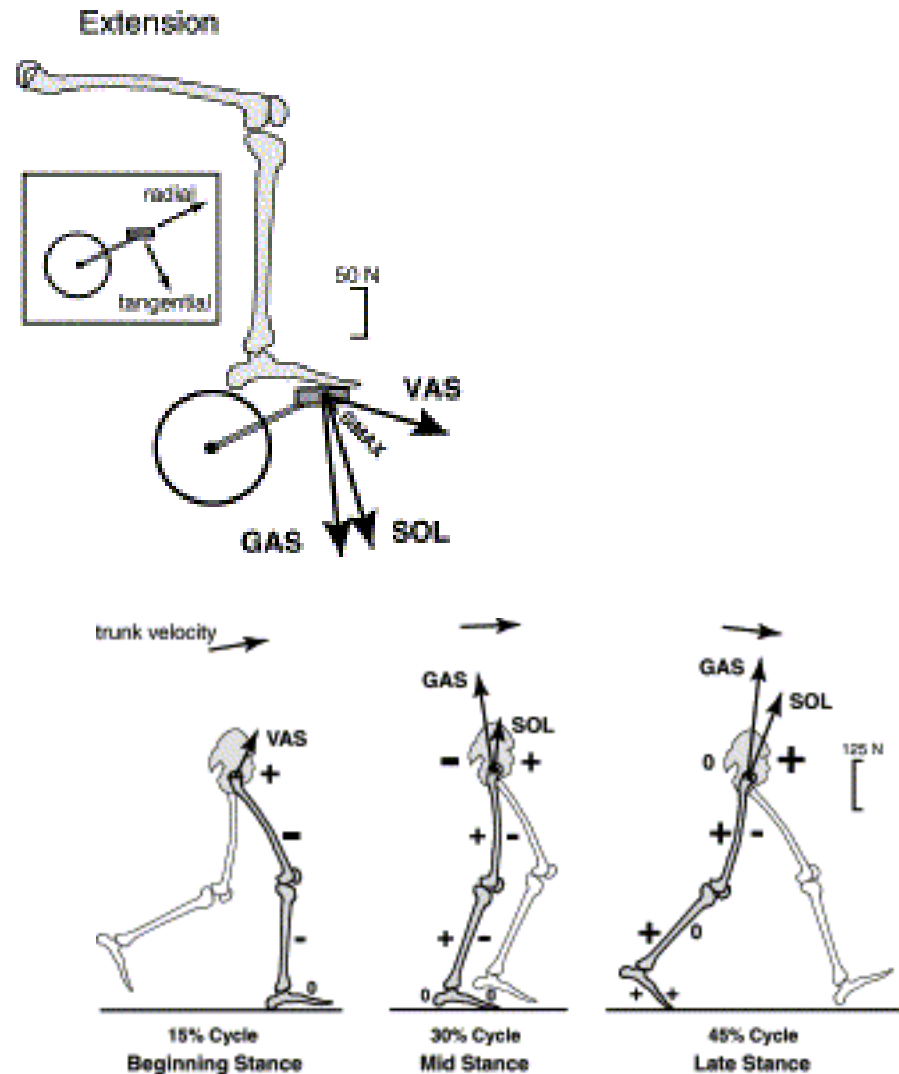
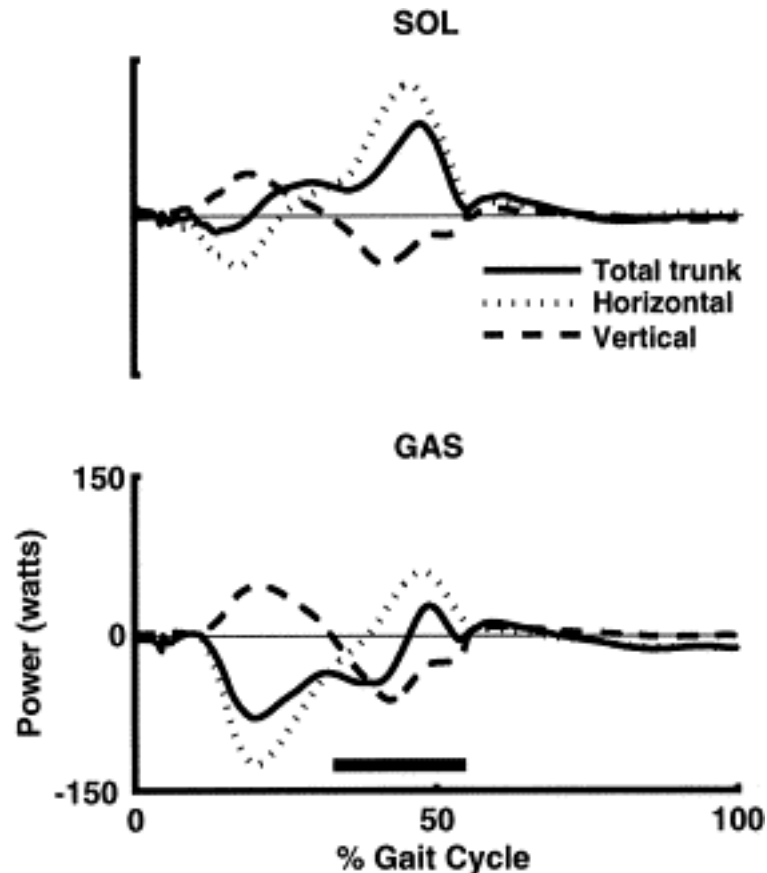
Induced Acceleration Analysis for Everyone



Art Kuo
University of Michigan, USA

Muscle induced acceleration analysis

Why does soleus produce negative vertical power?



Some history

- Zajac & Gordon (1989) Determining muscle's force and action in multi-articular movement. *Exerc Sport Sci Rev.* 17: 187 – 230.
- Kepple, T.M., Siegel, K.L., Stanhope, S.J. (1997) Relative contributions of the lower extremity joint moments to forward progression and support during stance. *Gait & Posture.* 6: 1 – 8.
- Kuo, A. D. & Zajac, F. E. (1993) A biomechanical analysis of muscle strength as a limiting factor in standing posture. *J Biomech.* 26 Suppl 1:137 – 150.
- Kuo, A. D. & Zajac F. E. (1993) Human standing posture: Multi-joint movement strategies based on biomechanical constraints. *Prog Brain Res.* 97: 349 – 358.
- Kuo, A. D. (1994) A mechanical analysis of force distribution between redundant, multiple degree-of-freedom actuators in the human: implications for central nervous system control. *Hum. Movem. Sci.* 13: 635 – 633.

Controversies in induced accelerations

Understanding muscle coordination... (Zajac, 2002 *J Biomech.*)

Biomechanics & muscle coordination...I & II (Zajac et al. 2002, 2003 *Gait Posture.*)

15 – 30% gait: Emphasize proximal muscles accelerate trunk forward, **as if to imply** importance despite actual deceleration.

0 – 15% gait: Proximal hip & knee muscles perform forward acceleration “**perplexing**” as actually trunk decelerates Letter to editor (Chen, 2004 *Gait Posture*)

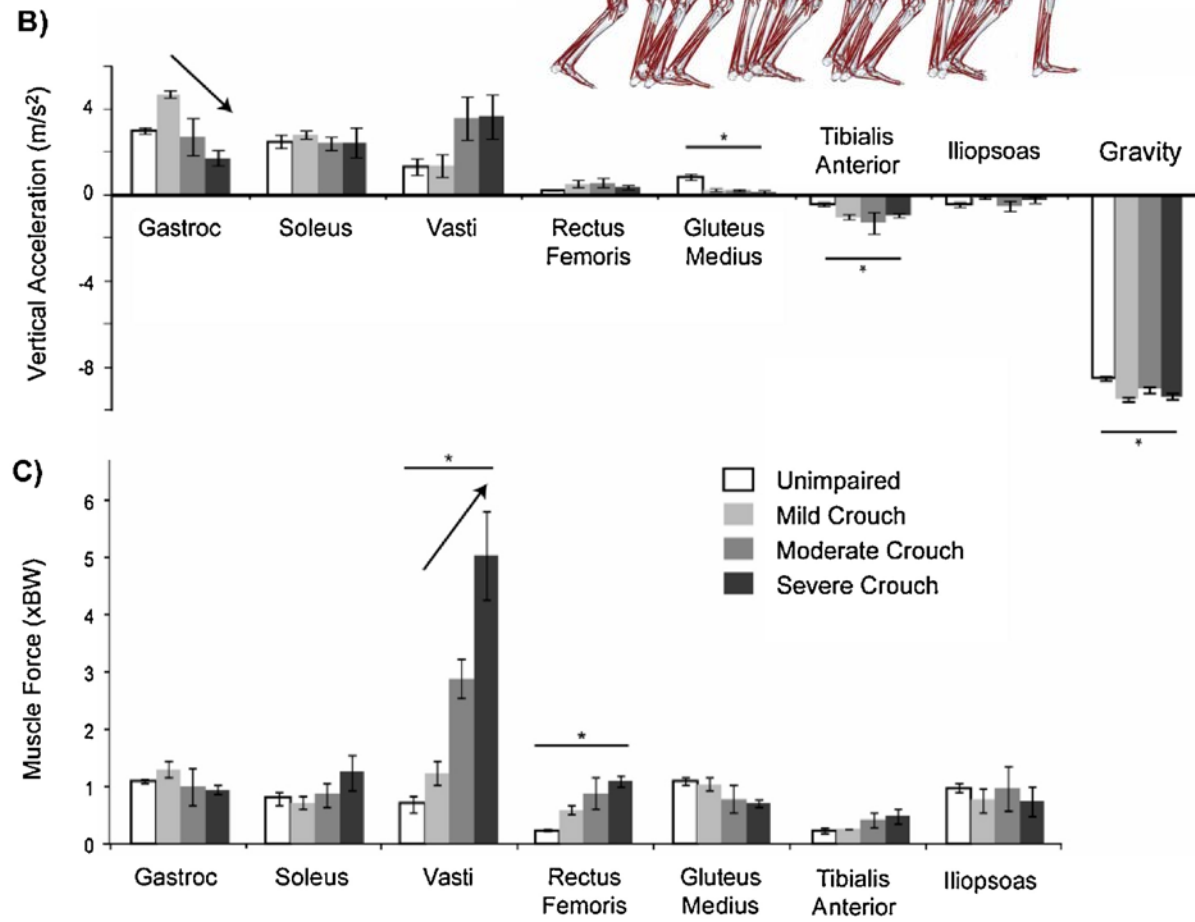
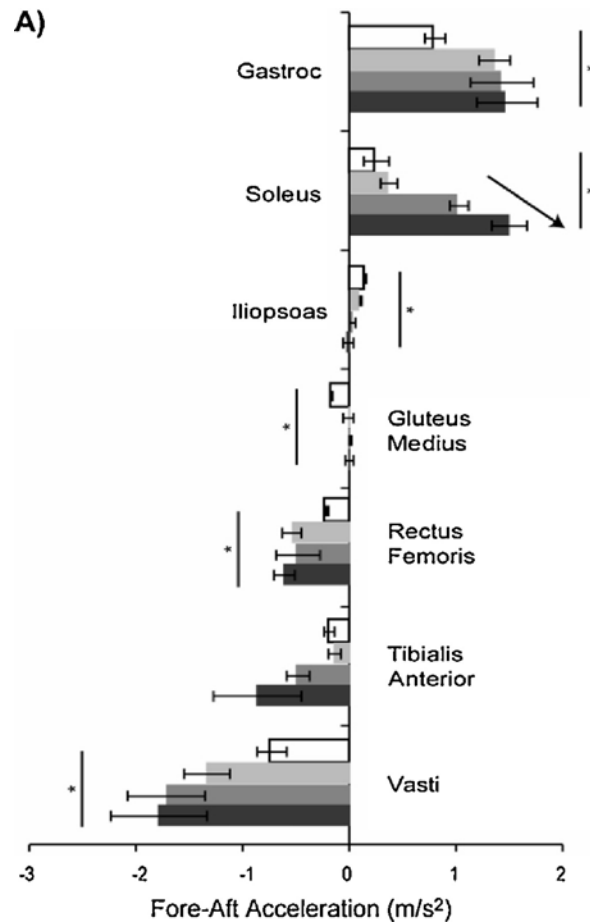
Induced acceleration contributions to locomotion dynamics are not physically well defined (Chen, 2006 *Gait Posture*)

“The muscular component of pedaling force is not well defined.” (Ruina & van Soest, 2002 *World Congr Biomech*)

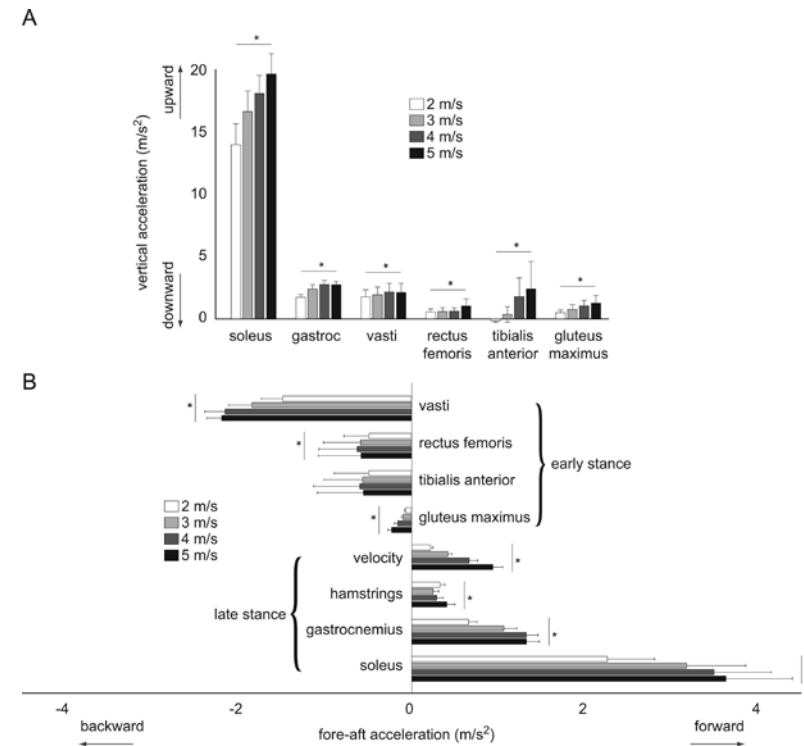
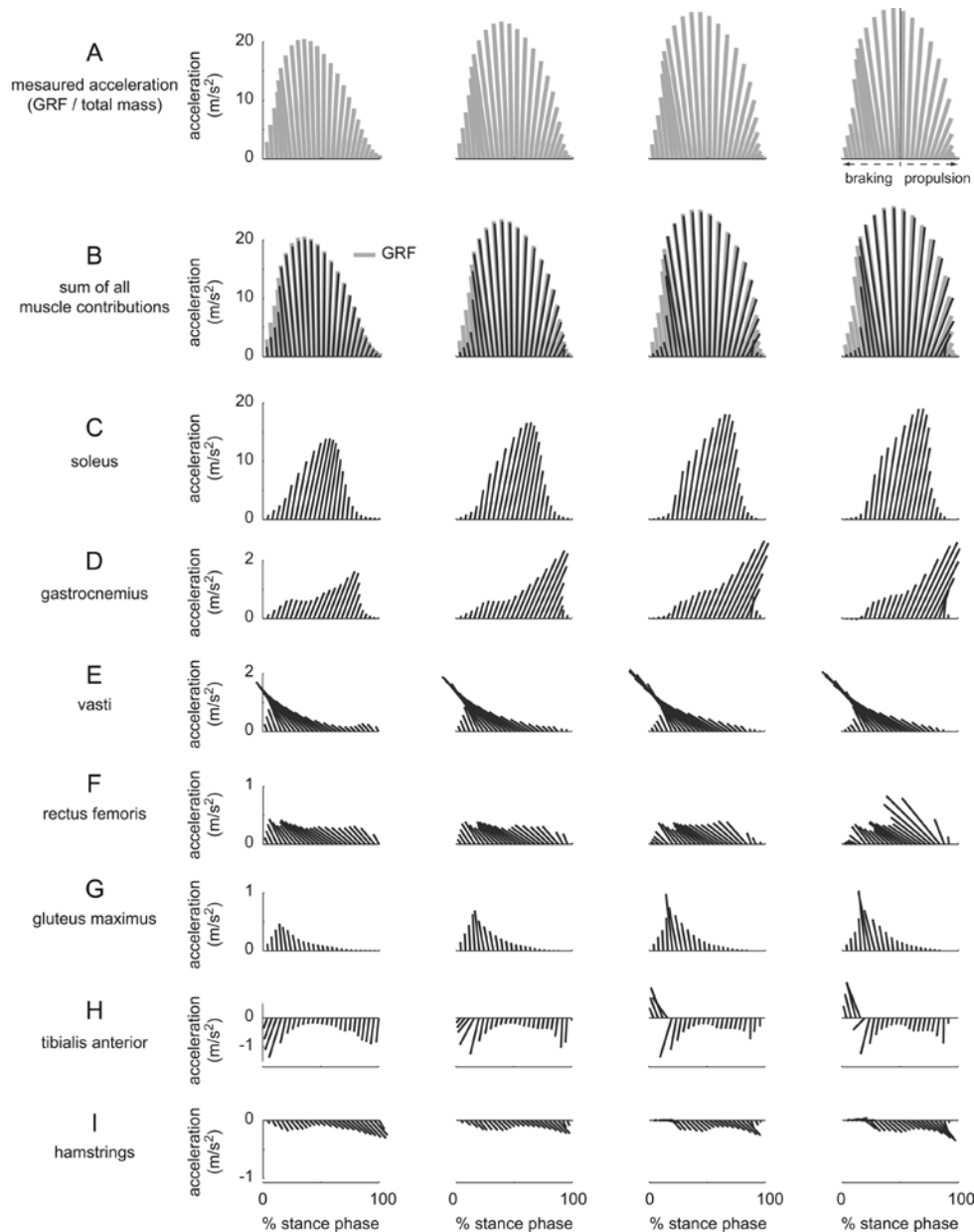
Propulsive adaptation... (Riley et al. 2001 *J Biomech*)

20 – 65% gait: Stance-leg moments impede forward propulsion because, hip flexor decelerate hip...inconsistent with well-known anterior ground reaction force (Neptune et al. 2006 *J Biomech*)

Muscle contributions to accelerations with crouch gait



Muscle contributions to accelerations in running



Goals of this tutorial

- Gain intuitive understanding of induced acceleration analysis
- Learn about controversies
- Use simple models to analyze
- Gain ability to read literature, form opinions

Intuitive induced accelerations

Case study: Pedaling

Case study: Throwing a ball

A quick study of gait

Discussion

Equations of motion simplified

$$M\ddot{\theta} + V(\theta, \dot{\theta}) + G(\theta) = R^T \cdot F_m$$

mass matrix

Coriolis & centripetal

gravitational

moment arms

muscle forces

$$\ddot{\theta} = -\cancel{M^{-1}V(\theta, \dot{\theta})} - M^{-1}G(\theta) + M^{-1}R^T \cdot F_m$$

Coriolis forces are usually small (gyroscopes, rockets, divers?)

Centripetal forces are often small (leg swing, riding in car)

Gravity is relatively easy

Muscle moments can be guessed (in sign)


Inertial coupling (from mass matrix) is main issue

Equations of motion: Accelerations

$$\ddot{\theta} = -M^{-1}G(\theta) + M^{-1}R^T \cdot F_m$$

$[A]$
 $\begin{bmatrix} a_1 & a_2 & \dots & a_n \end{bmatrix}$
 $\begin{bmatrix} f_{m1} \\ f_{m2} \\ \vdots \\ f_{mn} \end{bmatrix}$

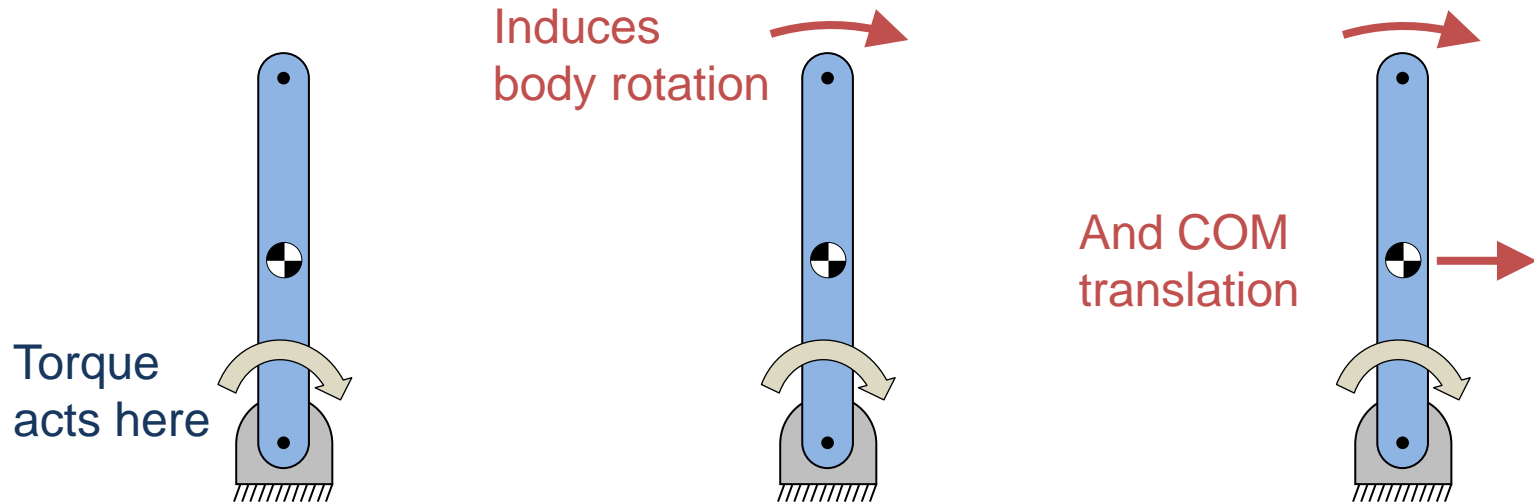
$$\ddot{\theta} = \text{gravity} + \underbrace{a_1 \cdot f_{m1} + a_2 \cdot f_{m2} + \dots + a_n \cdot f_{mn}}_{\text{individual muscle forces}}$$


 accelerations

Accelerations are linear sum of effects from individual muscles plus gravity...


So we can examine each muscle individually, right?

Forces induce rotation and translation



Key:

Torques  T

Forces  F

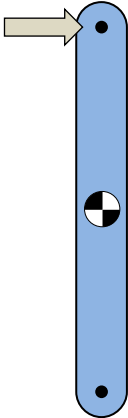
Rotations  $\ddot{\theta}$

Translations  \ddot{x}

Exercise 1: Find the accelerations

Sketch arrows here

Impulsive
force here



1. Rotation?

2. COM translation?

3. This point?

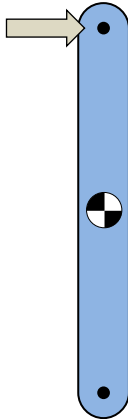


(no gravity, floating in space)

Exercise 1: Find the accelerations

SOLUTION

Impulsive
force here

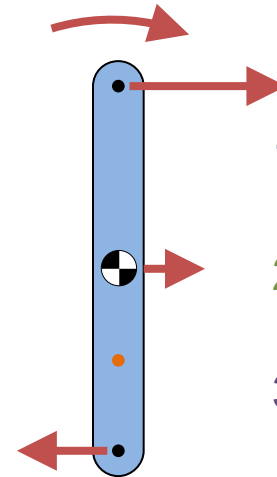


1. Rotation?

2. COM translation?

3. This point?

(no gravity, floating in space)



1. Clockwise

2. Forward

3. Backward!

(one point doesn't move)

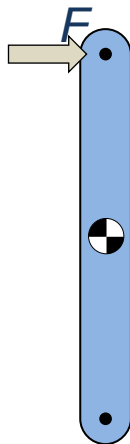
Take-aways: Forces generally induce rotation and translation

Not all points on a body accelerate in same direction

Exercise 1: Find the accelerations

DETAILS

Impulsive
force here



$M, ML^2/12$

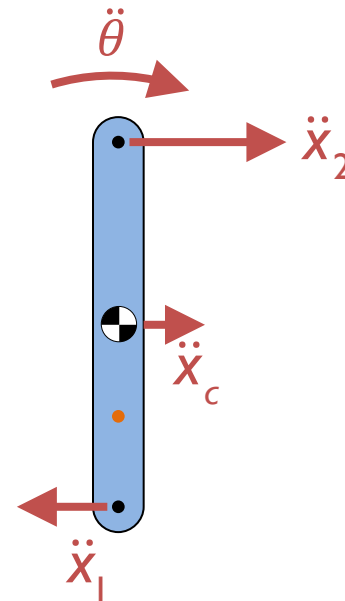
length L

Newton's law $M\ddot{x}_c = \sum \text{forces}$

Euler's eqn $I\ddot{\theta} = \sum \text{torques}$

Kinematics

$$a_l = a_c + \alpha \times r_{l/c} + \omega \times (\omega \times r_{l/c})$$



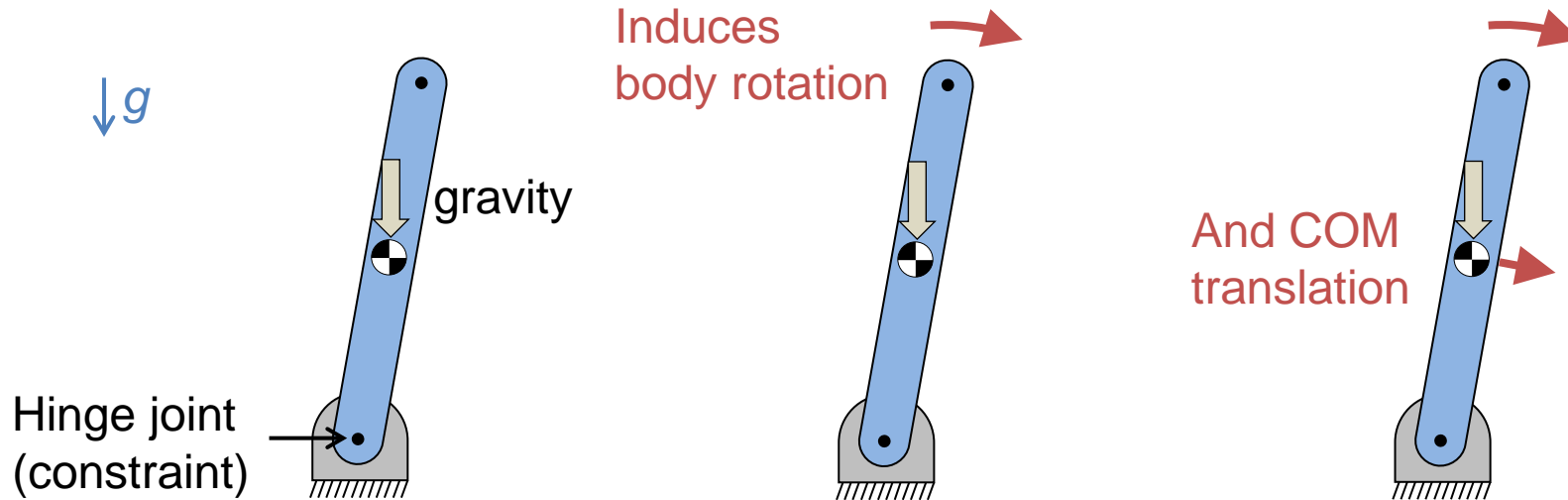
$$\ddot{x}_c = \frac{F}{M}$$

$$\ddot{\theta} = \frac{LF}{2} \cdot \frac{12}{ML^2} = \frac{6F}{ML}$$

$$\ddot{x}_1 = x_c - \frac{L}{2} \ddot{\theta} = \frac{F}{M} - \frac{L}{2} \frac{6F}{ML} = -2 \frac{F}{M}$$

$$\ddot{x}_2 = x_c + \frac{L}{2} \ddot{\theta} = \frac{F}{M} + \frac{L}{2} \frac{6F}{ML} = 4 \frac{F}{M}$$

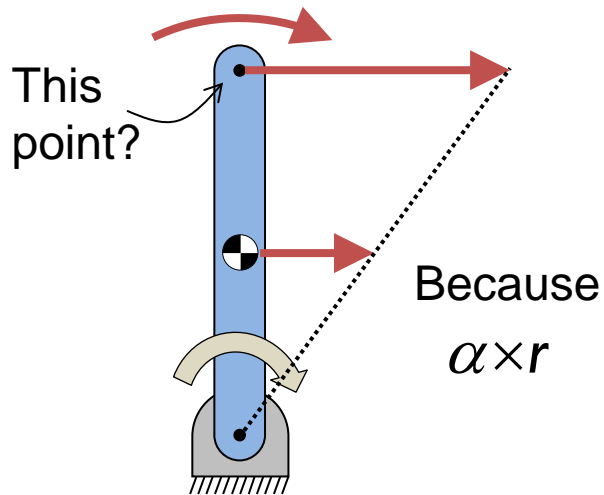
Induced accelerations respect constraints



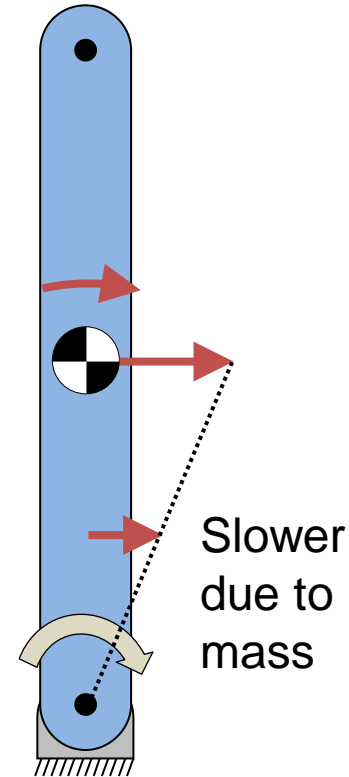
Acceleration from gravity will not be very high. Why?

Induced accelerations vs. length & mass

Translational accelerations
scale with distance from
pivot



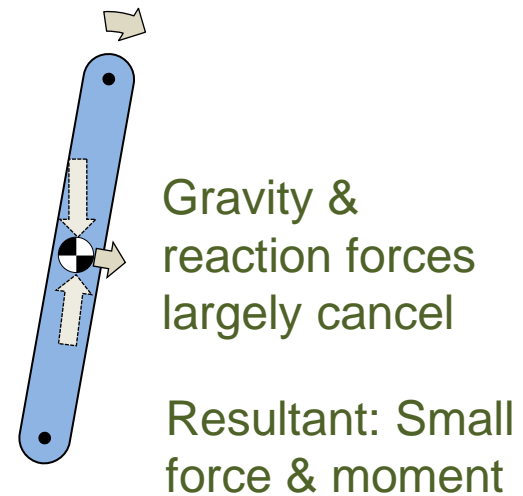
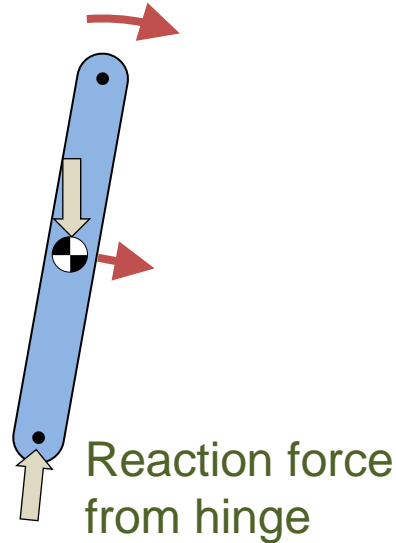
Heavier



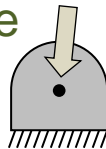
Translational & rotational
accelerations scale
inversely with mass

Active forces also induce constraint forces

Free body diagram



Reaction force on hinge



(equal and opposite)

(forces must respect hinge)

Equations of motion: Forces

$$\begin{bmatrix} M & C^T \\ C & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ -\lambda \end{bmatrix} = \begin{bmatrix} R^T F_m + \text{grav} + \dots \\ -\dot{C}\dot{q} \end{bmatrix}$$

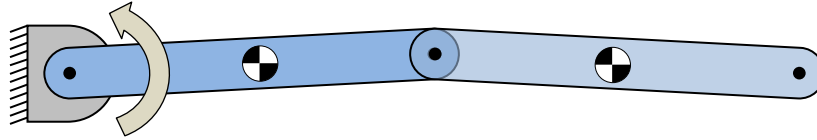
constraint matrix

constraint reaction forces

Reaction forces are also linear sum of individual muscles plus gravity...

Examine accelerations & reaction forces as linear sum
e.g., ground reactions, pedal forces, COM accelerations

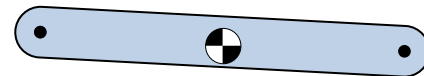
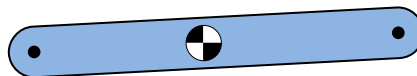
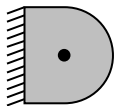
Exercise 2: Find the accelerations & reactions



1. Rotations?

2. End point?

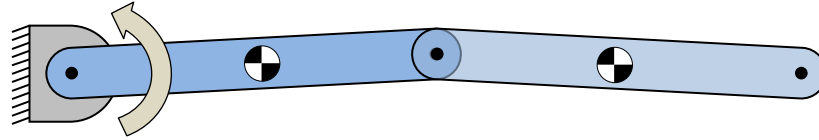
3. Reaction forces?



Take-aways: Multi-body systems induce multiple constraint forces

Not all points on a body accelerate in same direction

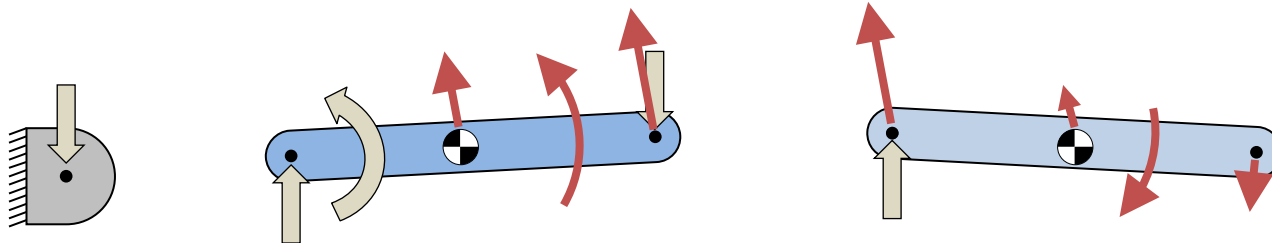
Exercise 2: Find the accelerations & re SOLUTION



1. Rotations? Counter-clockwise Clockwise

2. End point? Accelerates downward

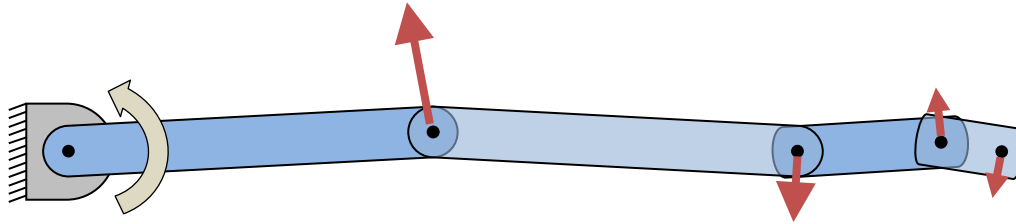
3. Reaction forces?



Take-aways: Multi-body systems induce multiple constraint forces

Not all points accelerate in same direction

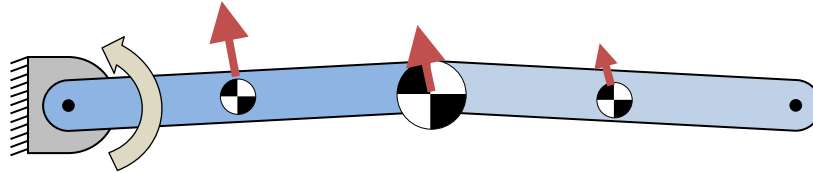
What about additional segments?



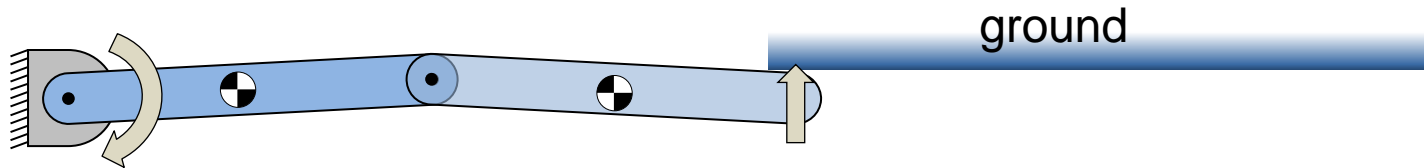
To move endpoint up, shouldn't proximal joint always rotate up?

Regardless of how many joints?

General accelerations and forces



COM acceleration is weighted sum of accelerations
(i.e., linear in accelerations)



Same with reaction forces

Any acceleration or reaction force of
interest linearly related to active forces

Intuitive induced accelerations

- Forces induce combined rotation & translation
- Induced accelerations respect constraints
- Accelerations scale with length & mass
- Forces also induce reaction forces

Intuitive induced accelerations

Case study: Pedaling

Case study: Throwing a ball

A quick study of gait

Discussion

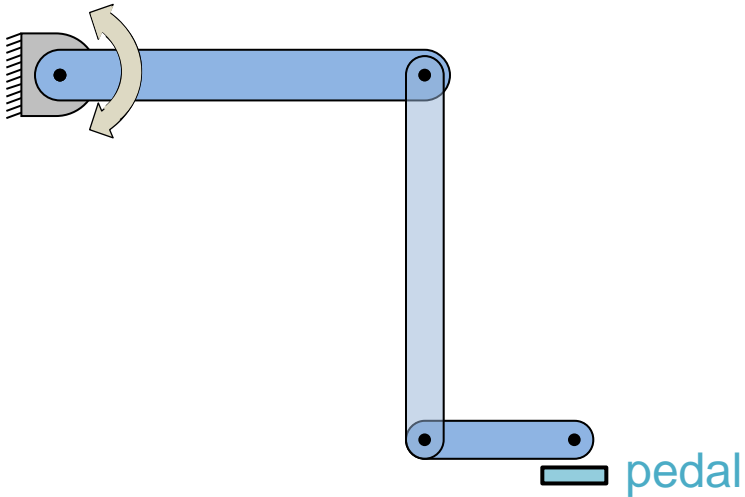
Induced Accelerations for Everyone

Case study: Pedaling

- Hip extension non-intuitive effects
- Ankle (2 ways)
- Multivariate perspective

Induced Accelerations for Everyone

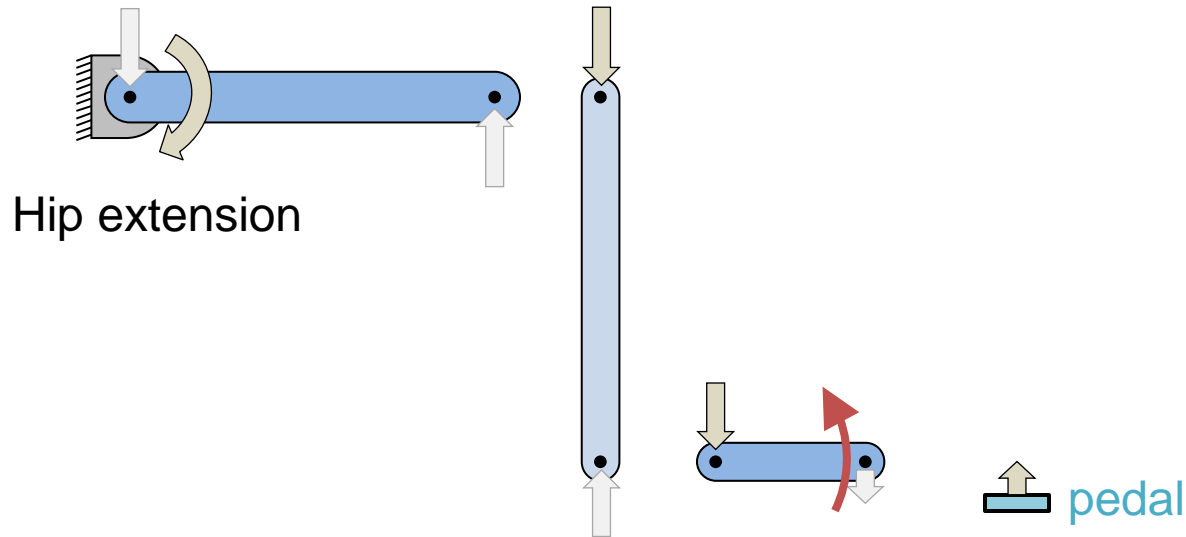
Exercise: How to pedal a bike



Goal: Perform positive work on pedal
Which direction for hip torque?

Exercise: How to pedal a bike

SOLUTION 1

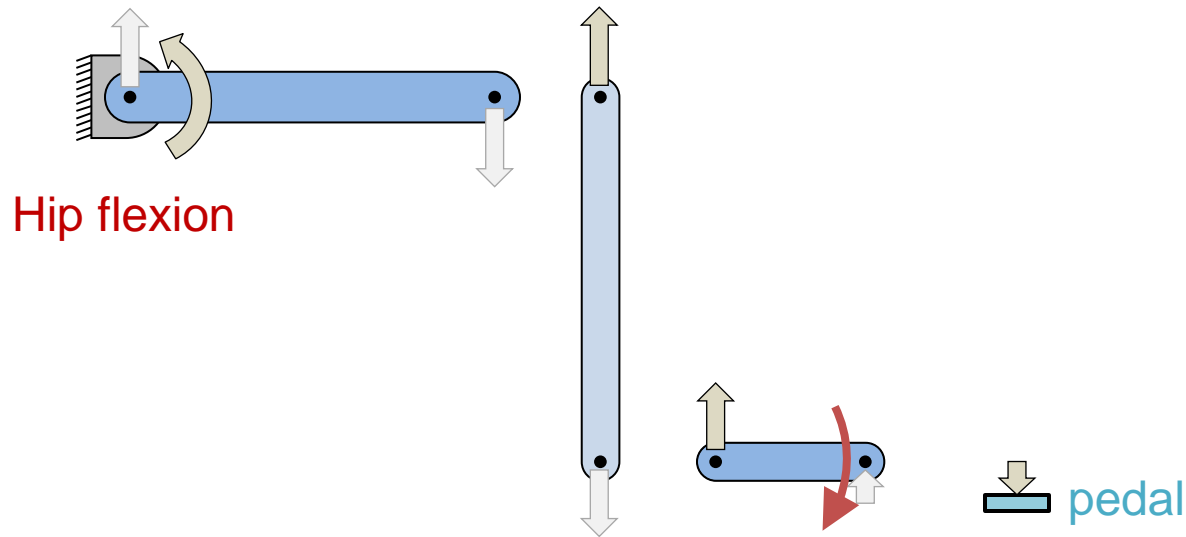


Good!
Hip extension performs positive work
but induces upward force on pedal, flexes ankle
Bad!

Paradox: Hip extension obviously desirable, but doesn't press on pedal??

Exercise: How to pedal a bike

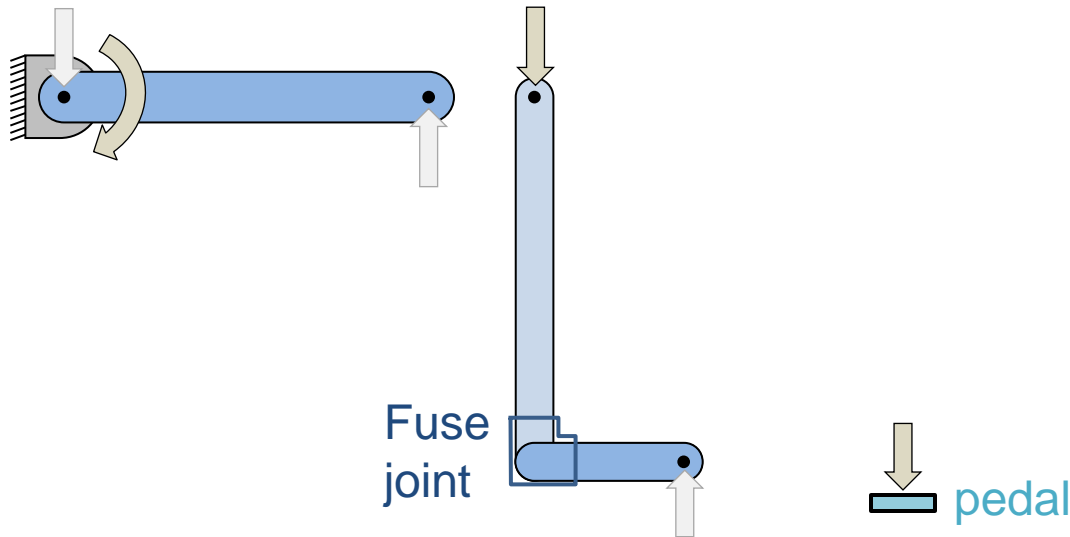
SOLUTION 2



Bad!

Hip flexion performs negative work
and induces downward force on pedal, extends ankle

Good!



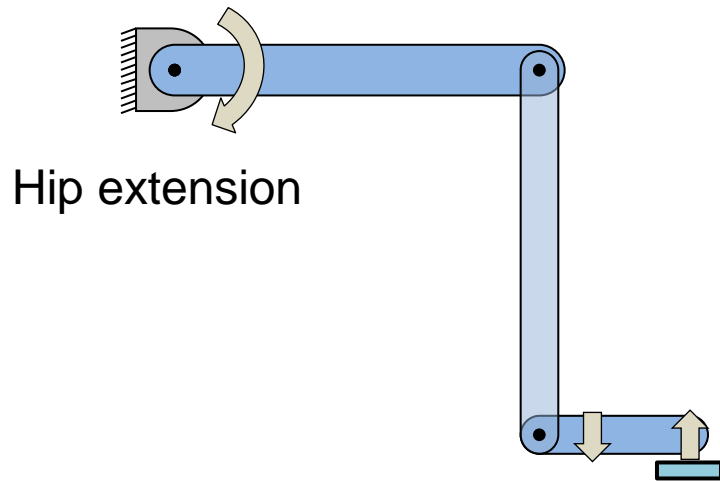
Chen (2006) *Gait Posture*

Now hip extension performs positive work and induces downward pedal force

But when is a joint a joint??

Ruina & van Soest (2002) The muscular component of pedaling force is not well defined. World Congr Biomech #5414.

Another paradox

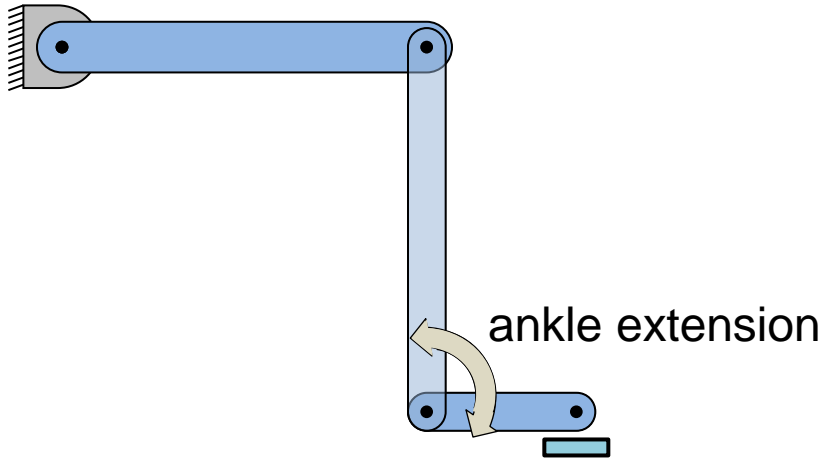


Move pedal slightly,
hip extension has reverse effect

But when is a joint a joint??

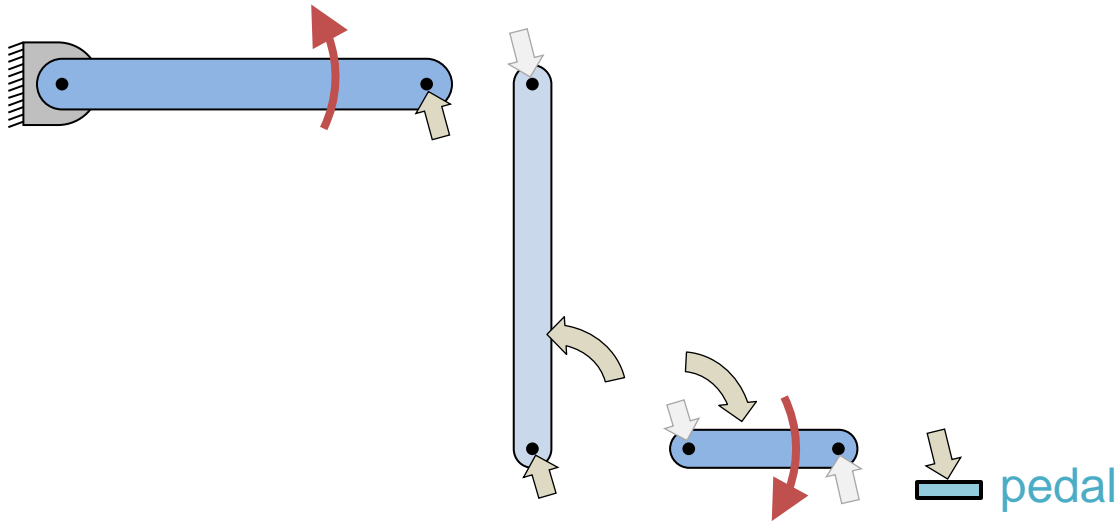
Ruina & van Soest (2002) The muscular component of pedaling force is not well defined. World Congr Biomech #5414.

Another component: Ankle extension



Find pedal force, thigh force from ankle extension

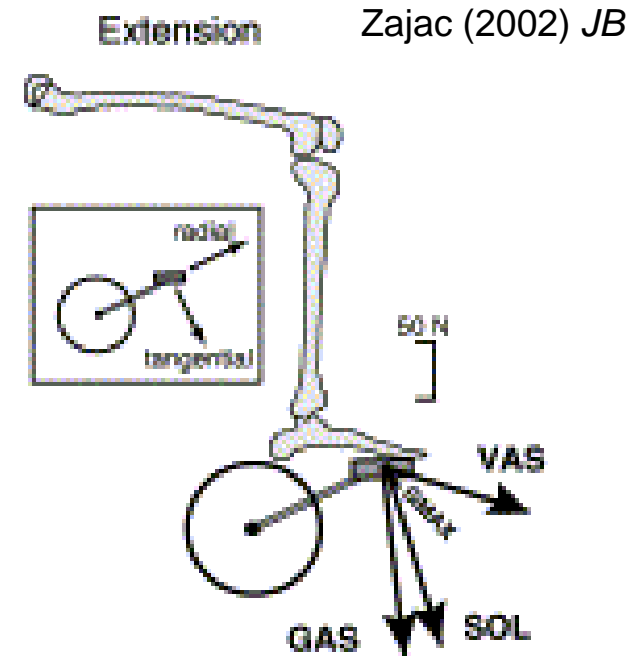
Another component: Ankle extension



Ankle extension induces downward force on pedal, flexion of thigh

Bad!

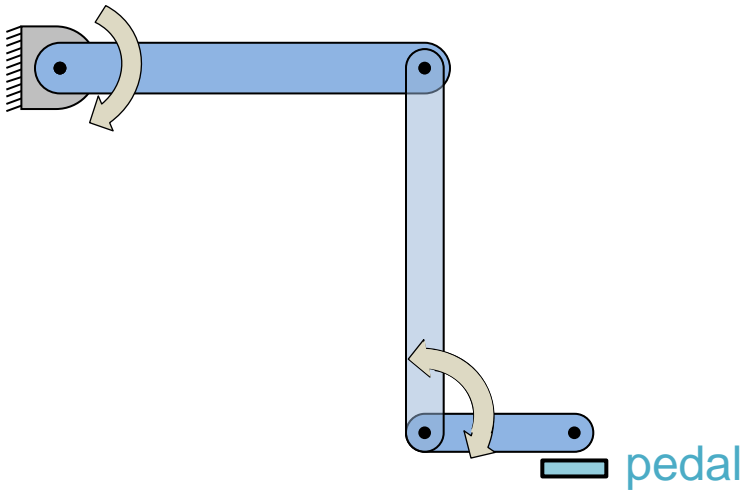
Good!



“Don’t do this”

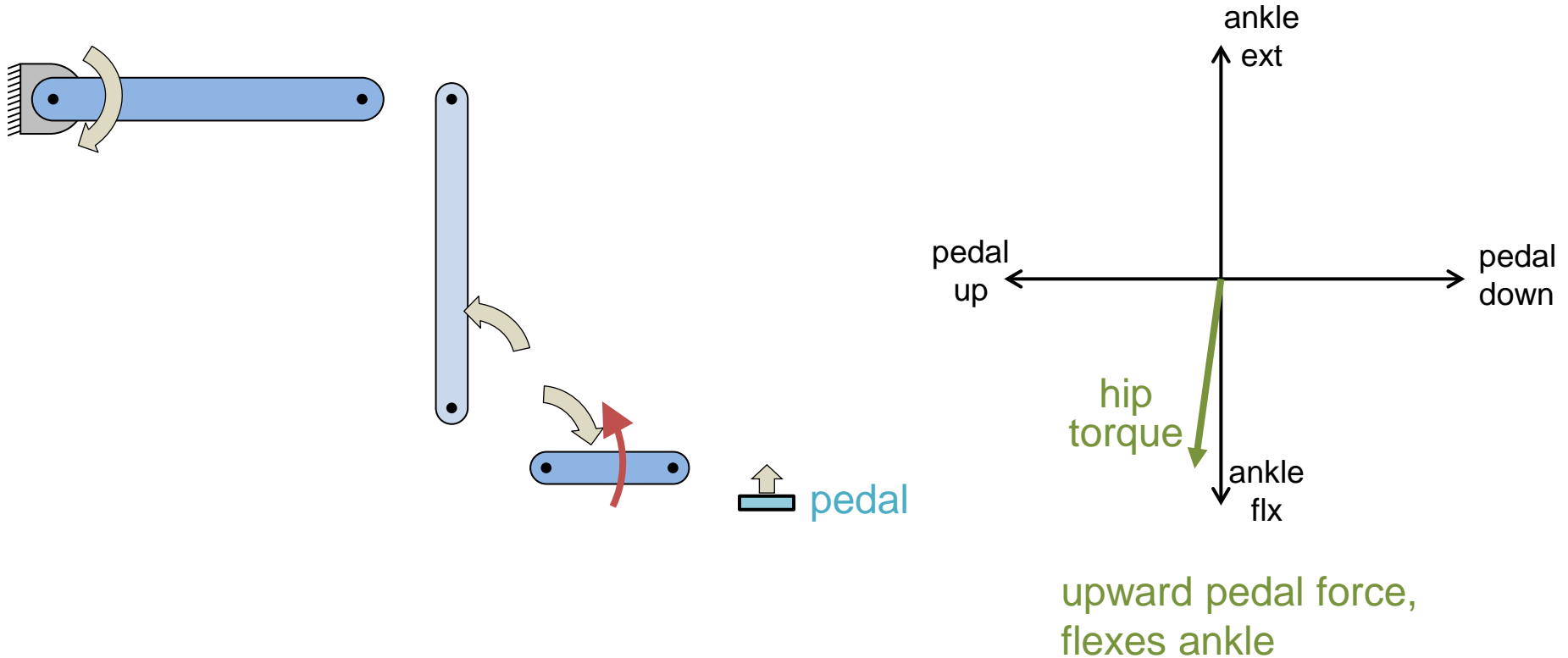
Ruina & van Soest (2002) WCB

“Extensor synergy” Zajac (2002) *J. Biomech.*

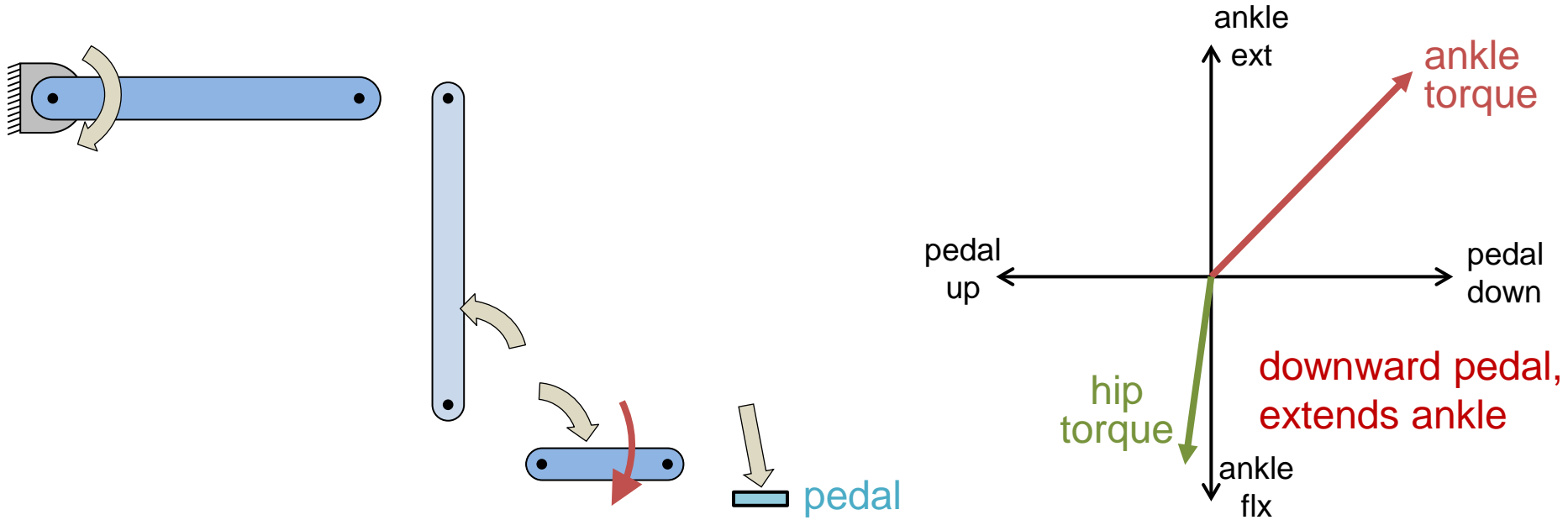


Find pedal force, thigh force from both
ankle & hip extension

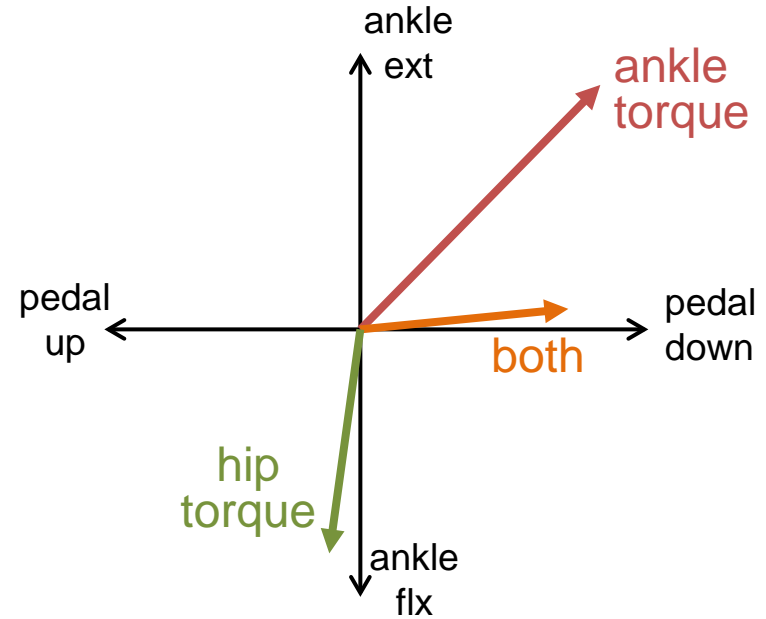
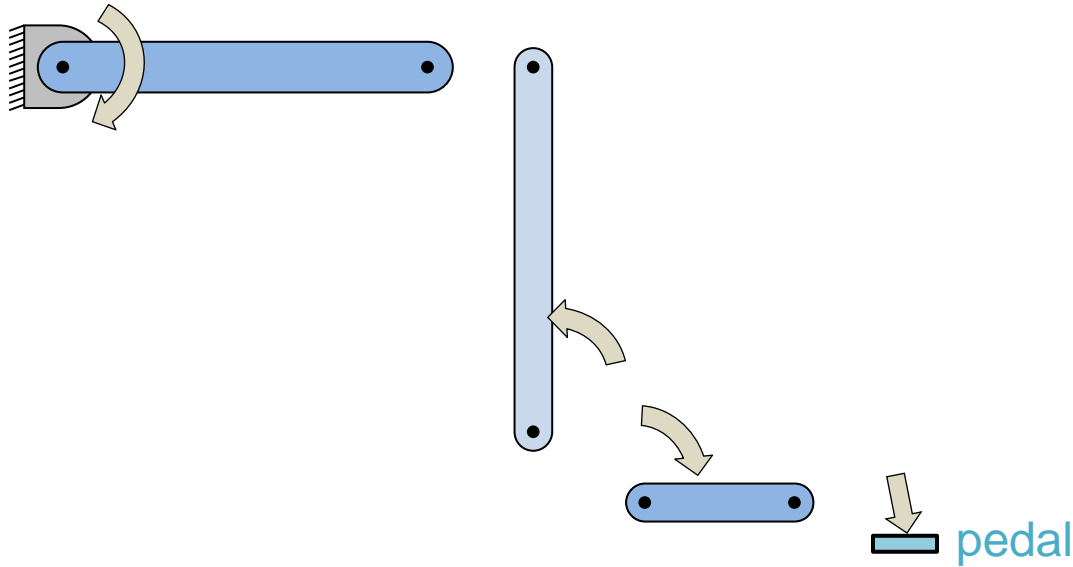
Multivariate perspective: Combine ankle and hip



Multivariate perspective: Combine ankle and hip



Multivariate perspective: Combine ankle and hip



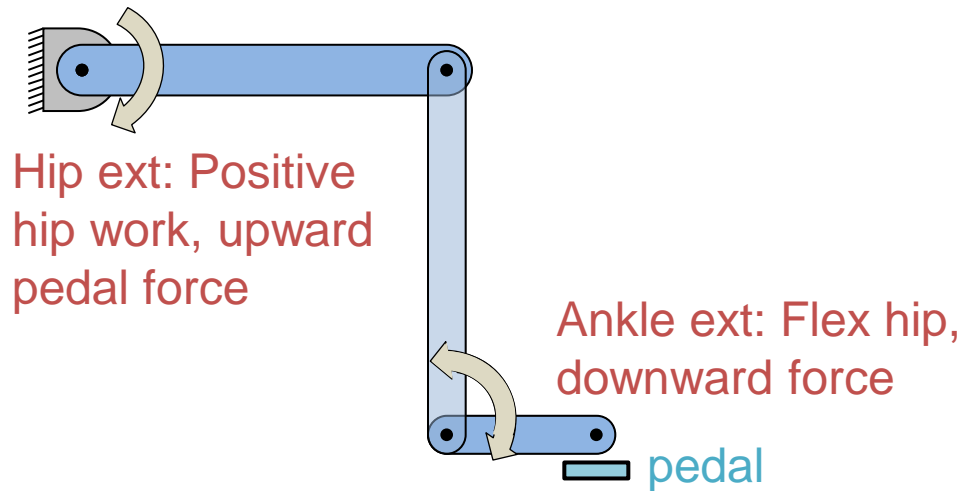
Hip & ankle extension produces positive work
(esp. at hip) and downward pedal force

Good!

Good!

Multivariate approach may help

Summary: How to pedal a bike



Pedaling (and most other tasks) are multivariate

Avoid univariate interpretations

Watch out for model dependencies

Case study: Pedaling

- Hip extension non-intuitive effects
- Ankle (2 ways)
- Multivariate perspective

Induced Accelerations for Everyone

Intuitive induced accelerations

Case study: Pedaling

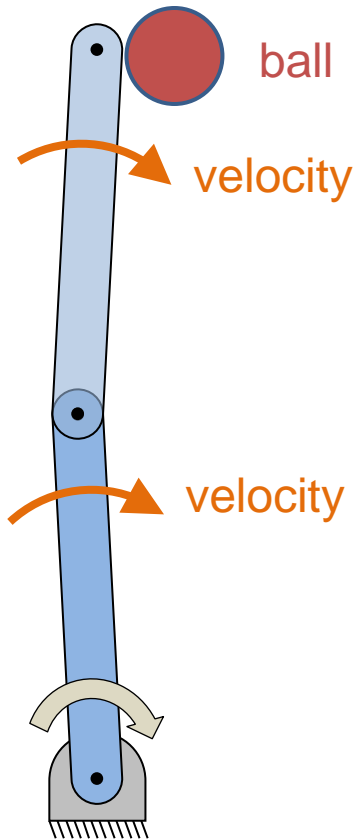
Case study: Throwing a ball

A quick study of gait

Discussion

Induced Accelerations for Everyone

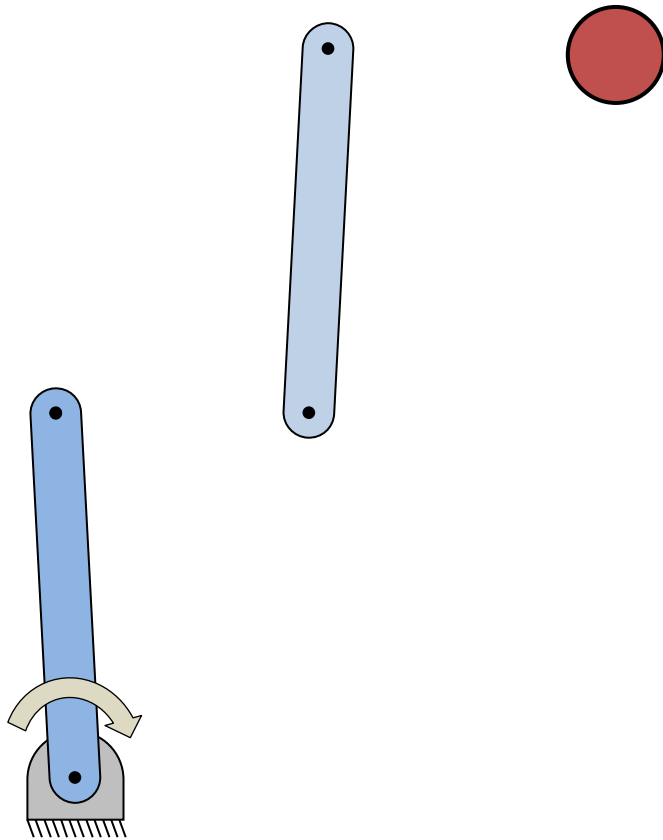
How to throw a ball



Throw this way →

Should torque be exerted to perform positive work, add energy?

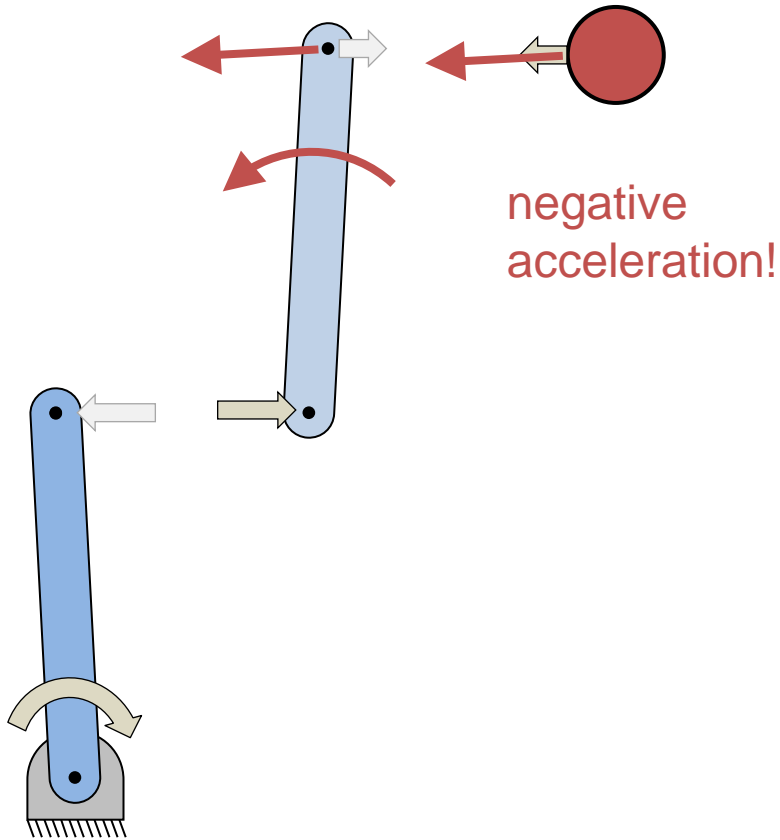
Exercise: Sketch induced accelerations



Throw this way →

Should torque be exerted to perform positive work, add energy?

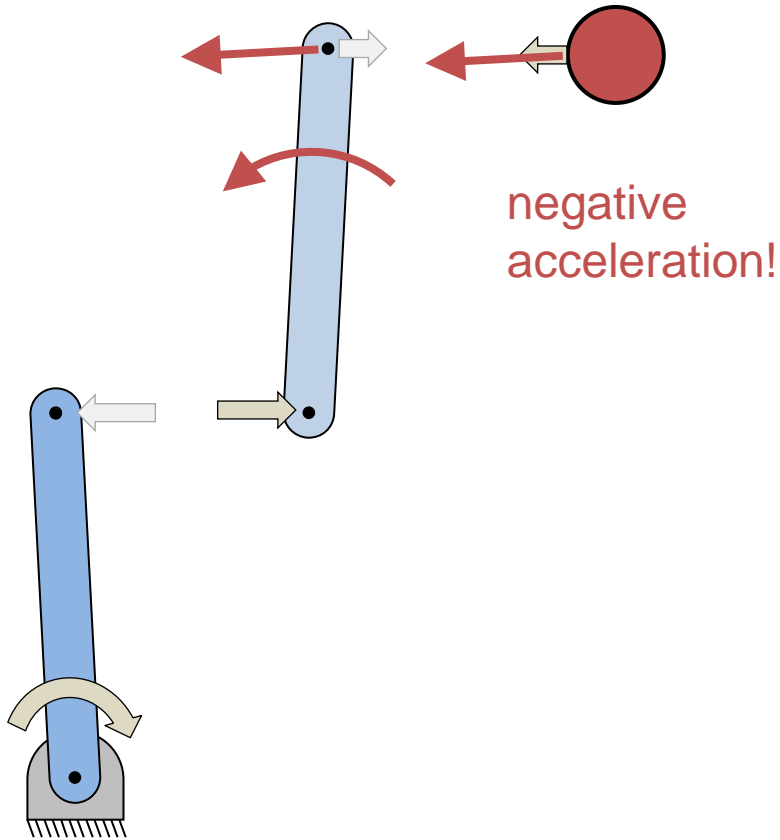
Exercise: Sketch induced acceleration SOLUTION



Throw this way →

Should torque be exerted to perform positive work, add energy?

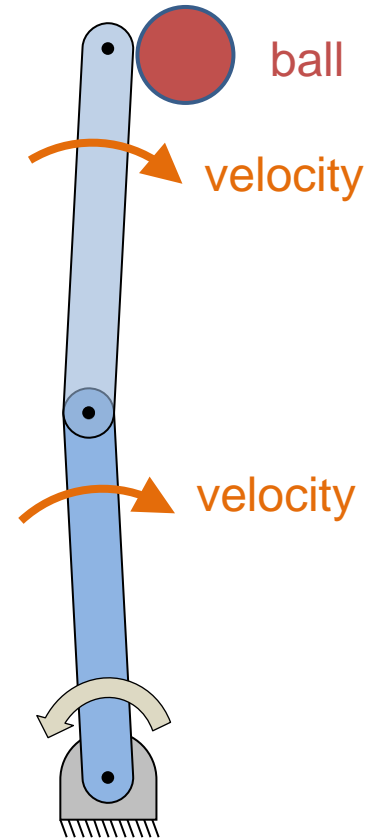
Positive work doesn't help ball



Shoulder flexion induces
wrong acceleration for
throwing

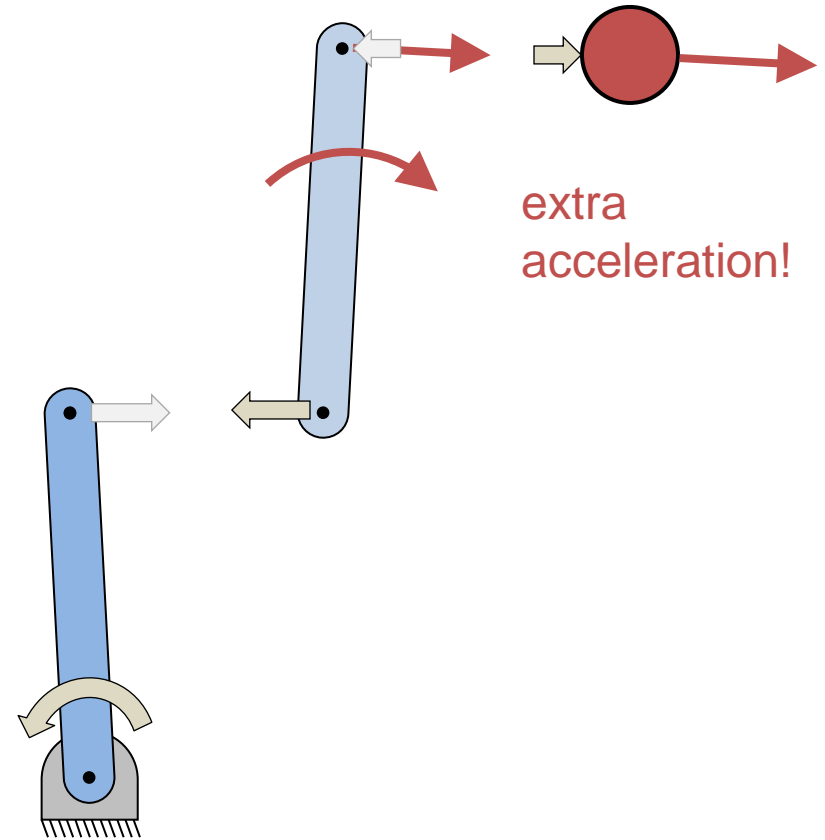
Should torque be exerted to perform
positive work, add energy?

Alternative strategy: Braking



Or opposite to accelerate ball?

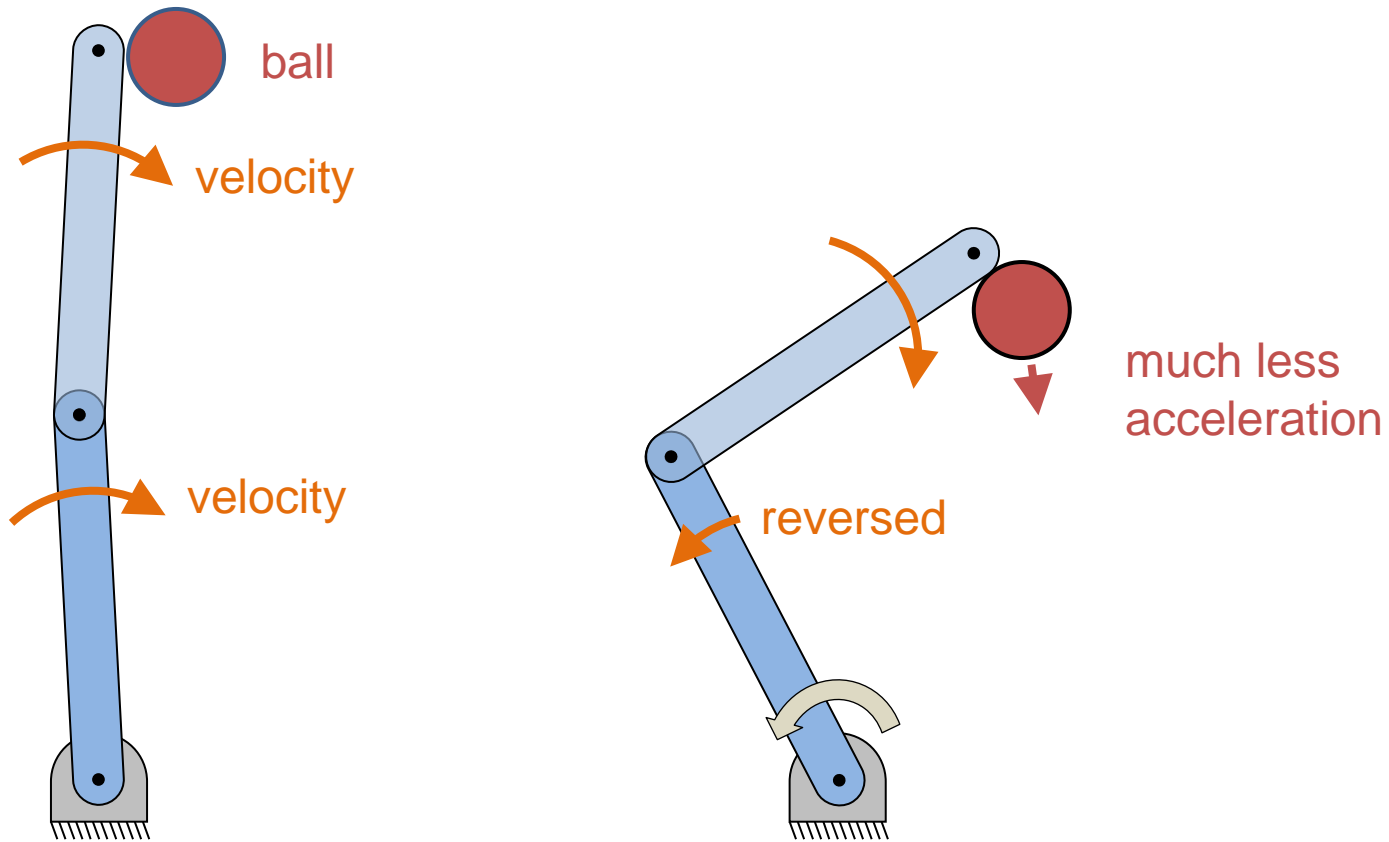
Braking strategy analysis



Or opposite to accelerate ball?

despite negative work

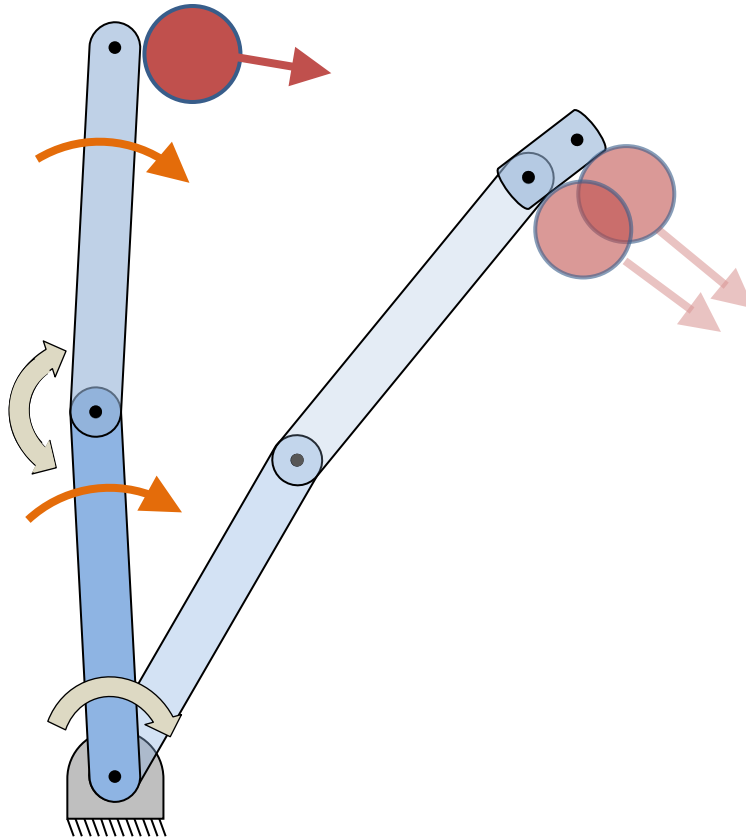
Braking strategy a moment later



Or opposite to accelerate ball?

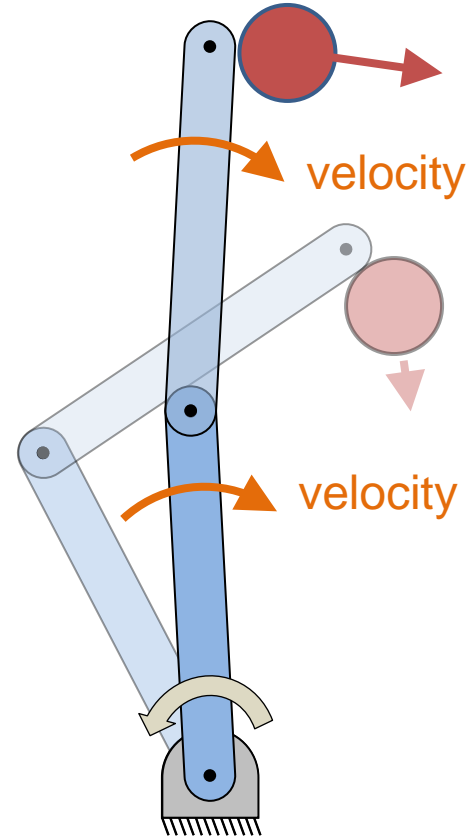
Braking works, but fleetingly

Summary: How to throw a ball



- 1 Flex shoulder: Add energy
~~but decelerate ball~~

Flex elbow: Add energy
and accelerate ball



- 2 Reduce energy but accelerate ball
for an instant

BEWARE snapshot analysis
when designing movement

Case study: Throwing a ball

- Shoulder flexion, extension both viable
- Beware snapshot analysis

Induced Accelerations for Everyone

Intuitive induced accelerations

Case study: Pedaling

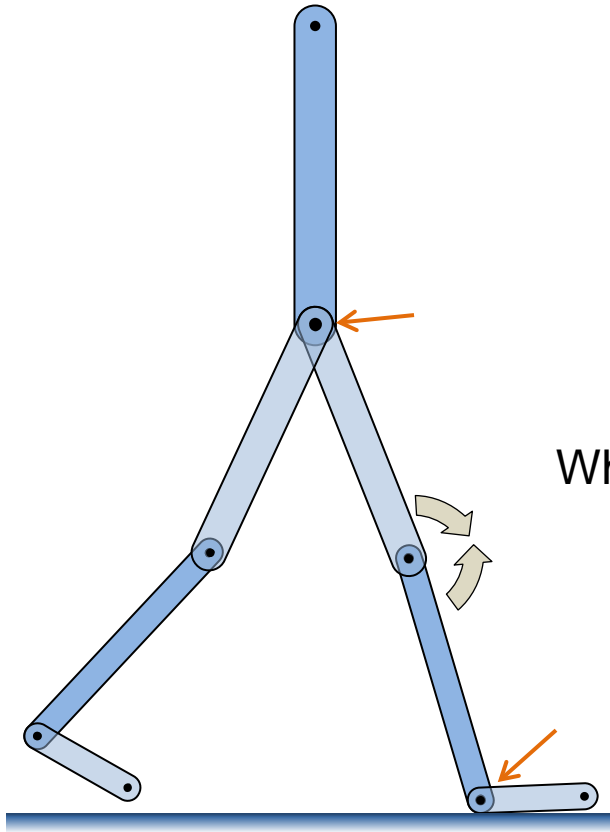
Case study: Throwing a ball

A quick study of gait

Discussion

Induced Accelerations for Everyone

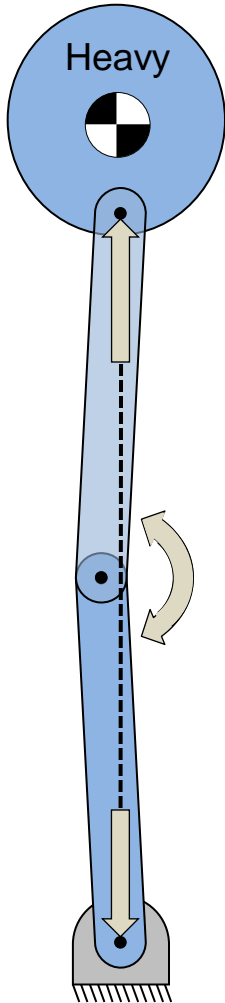
A quick study of gait



What is effect of knee extension moment?

How does joint moment cause forces on neighboring segments?

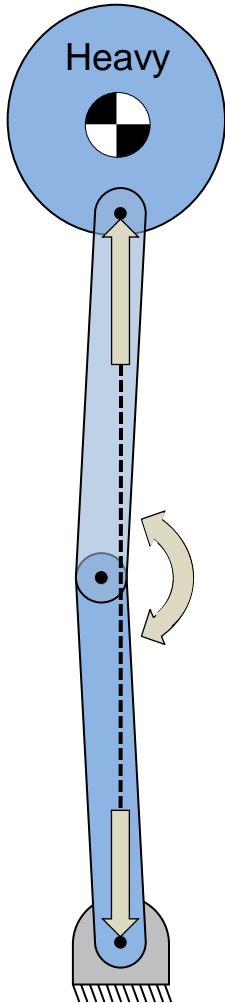
More tips on induced reaction forces



Heaviest masses
attract reaction forces

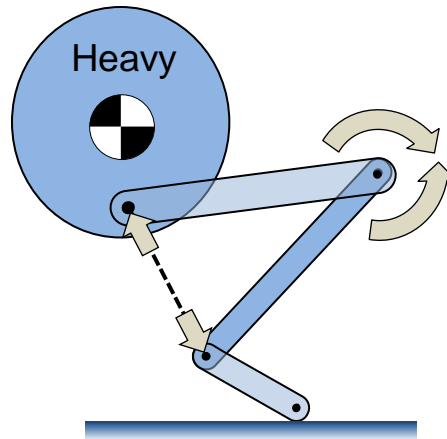
1. Draw line between joints on either side of active torque.
2. Reaction forces fall on that line.

More tips on induced reaction forces

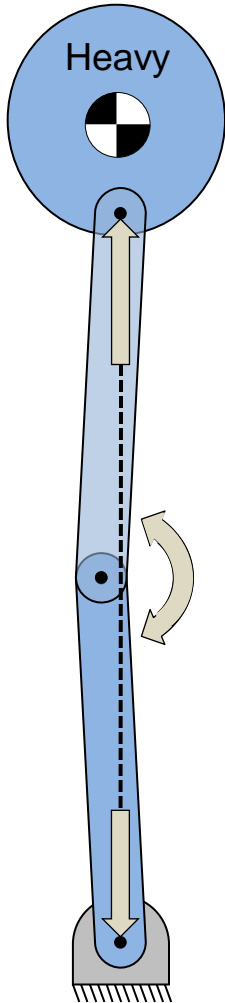


Heaviest masses
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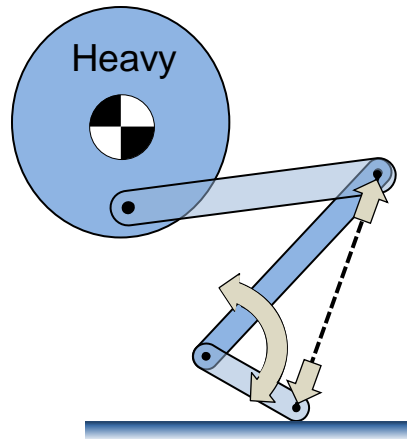


More tips on induced reaction forces

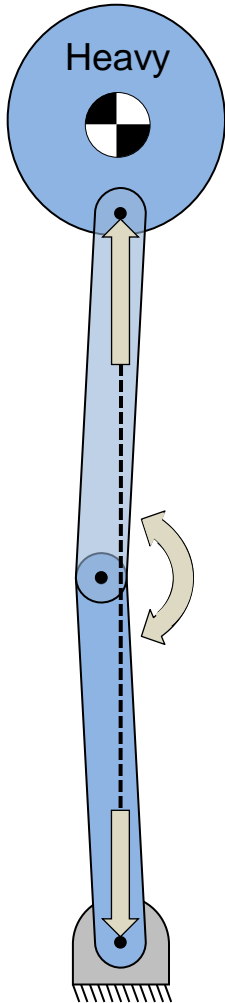


Heaviest masses
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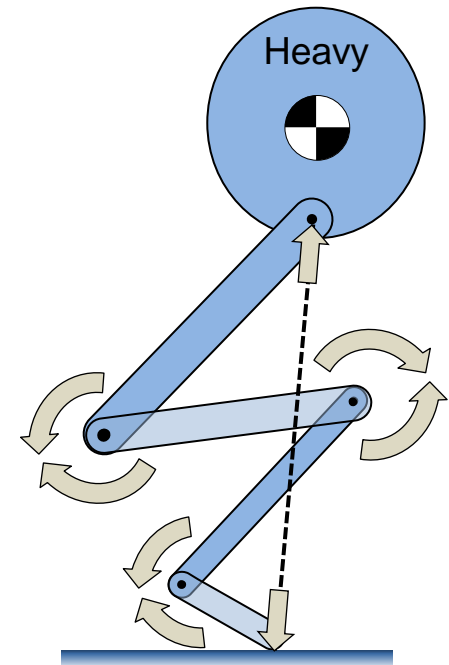
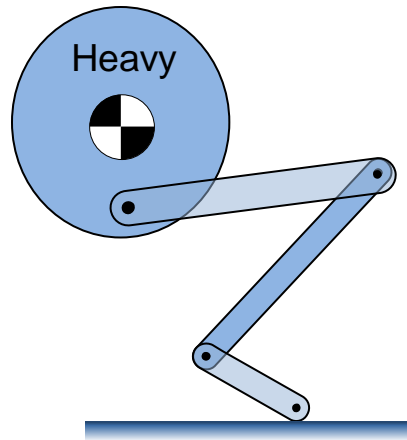


More tips on induced reaction forces

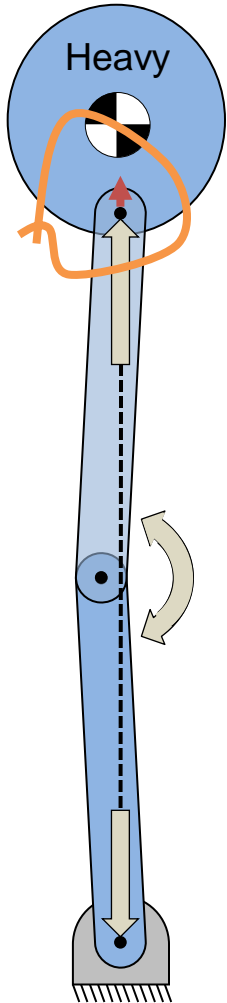


Heaviest masses attract reaction forces

1. Draw line between joints on either side of active torque.
2. Reaction forces fall on that line.

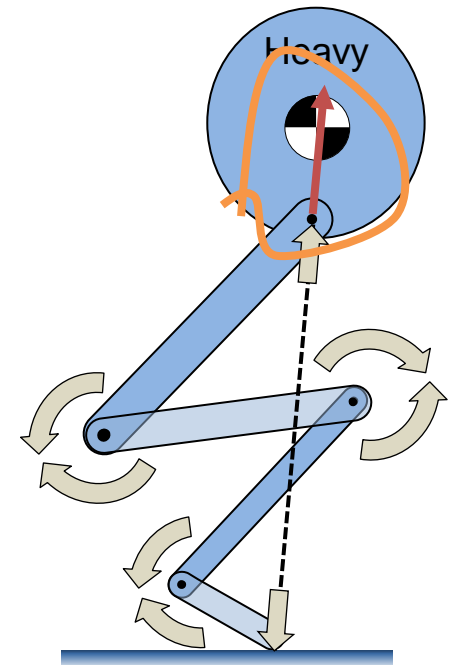
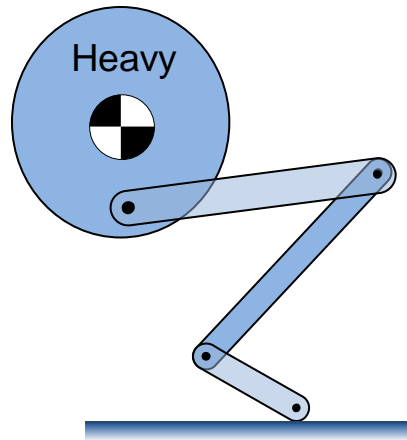


More tips on induced reaction forces



Heaviest masses attract reaction forces

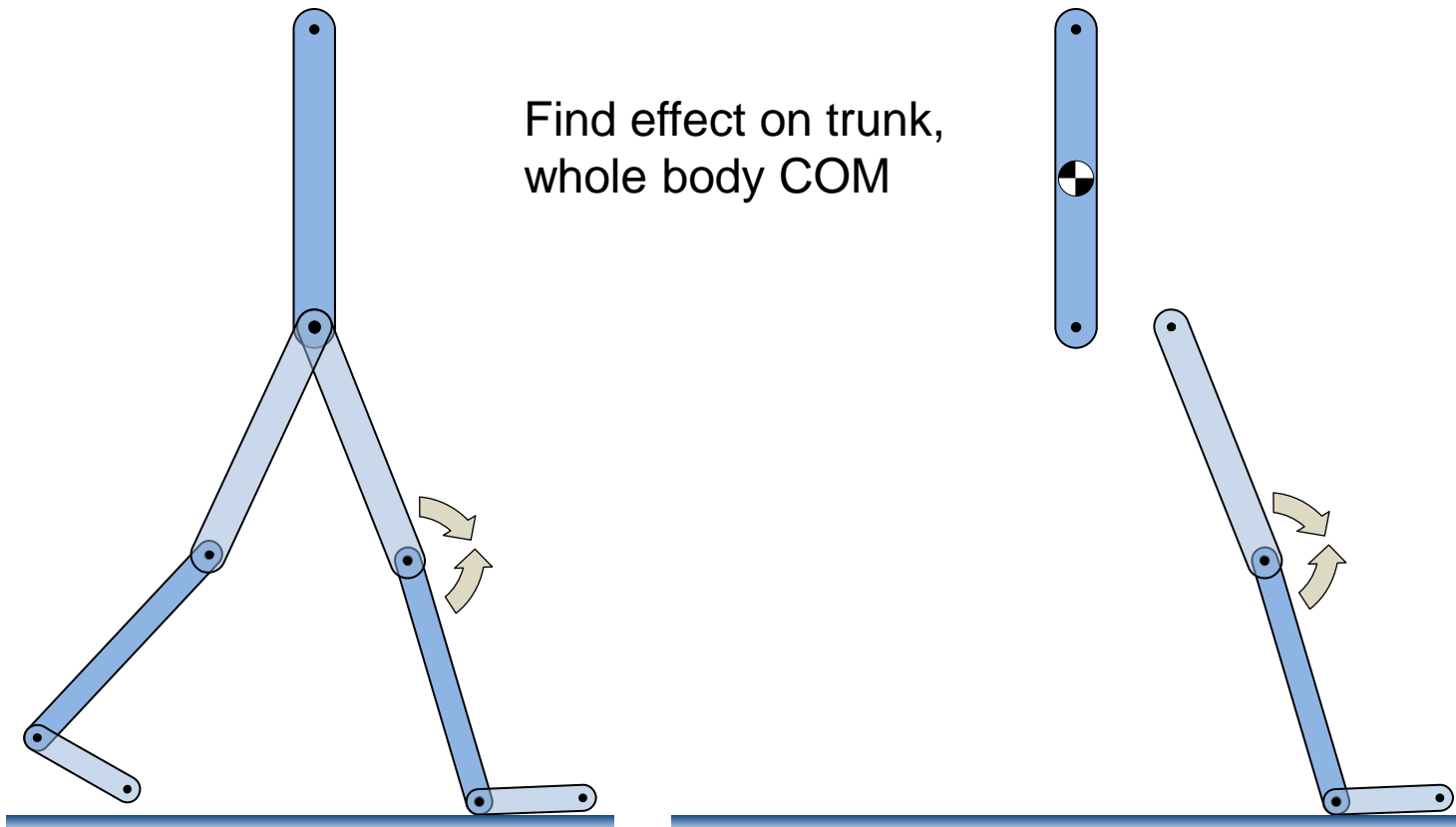
1. Draw line between joints on either side of active torque.
2. Reaction forces fall on that line.



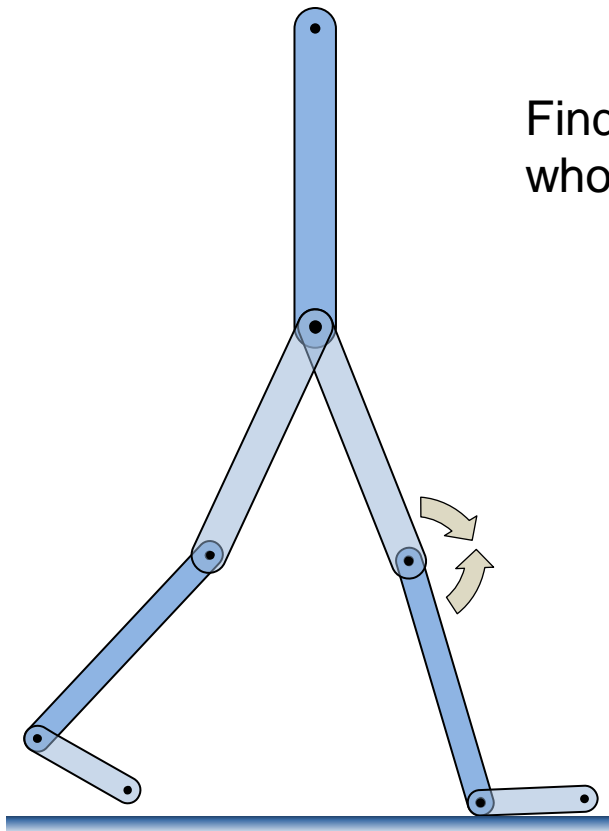
3. Aligned joints usually induce higher forces, lower in-line accelerations
4. Non-aligned usually induce lower reactions, higher in-line accelerations

Not always
(Discuss)

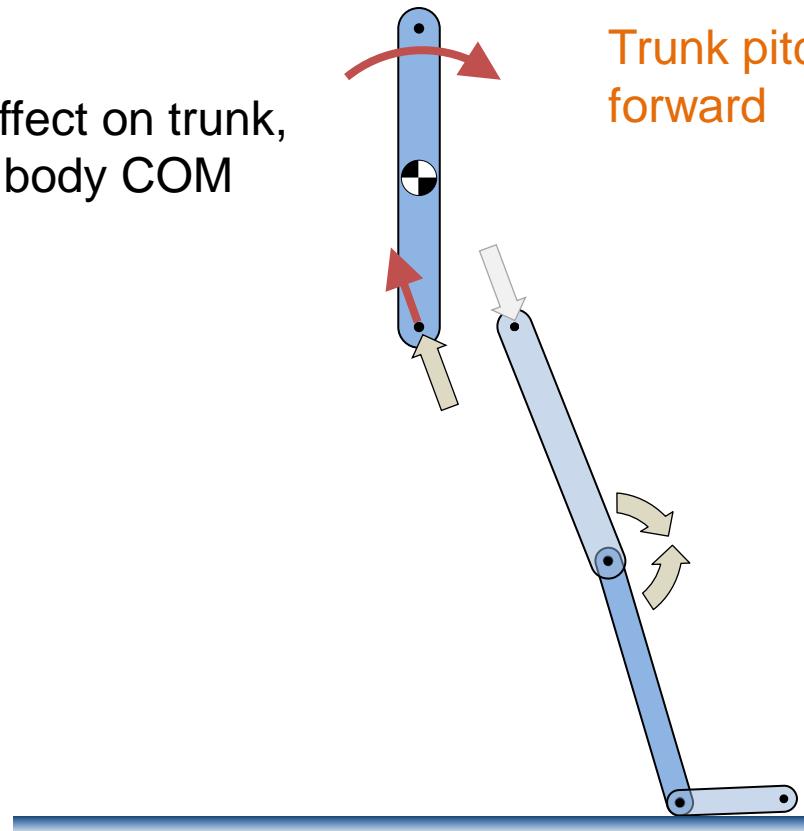
A quick study of gait: Trunk



A quick study of gait: Trunk

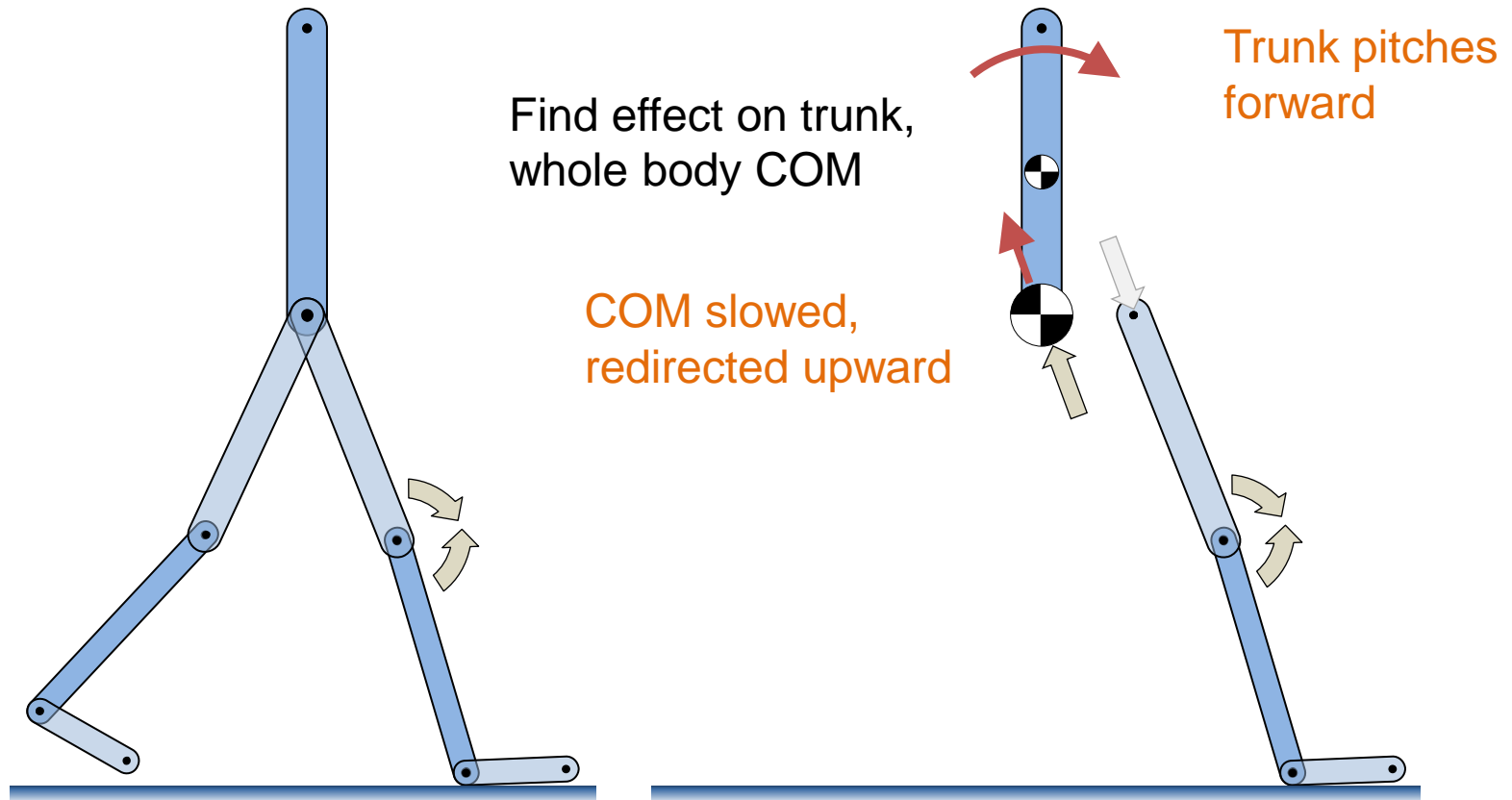


Find effect on trunk,
whole body COM



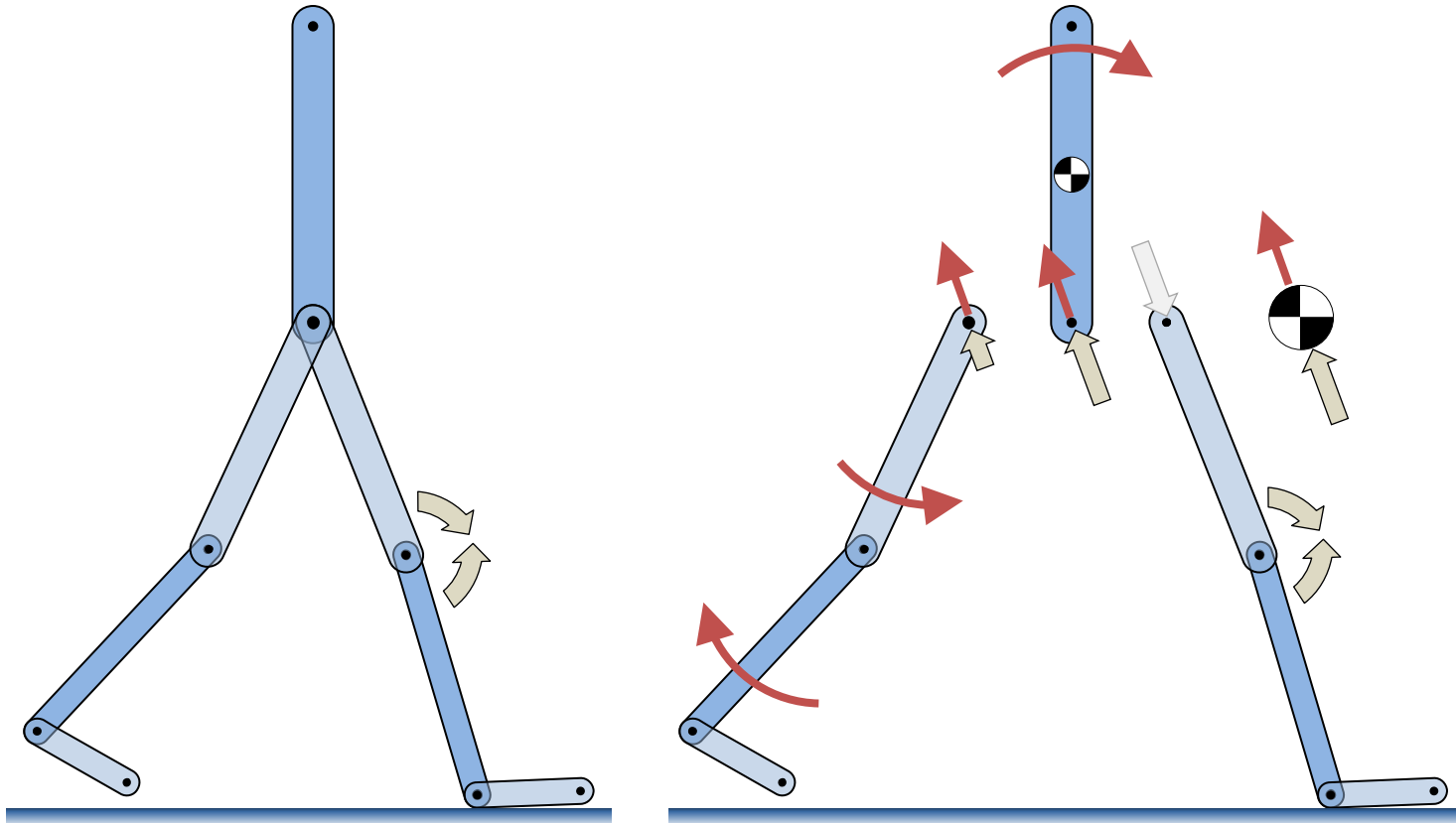
Trunk pitches
forward

A quick study of gait: Trunk



Trunk & COM need not experience
same accelerations

A quick study of gait: Swing leg



Leading leg can induce
swing phase

A quick study of gait

- Stance knee extension slows COM, speeds trunk
- Can induce swing phase

Induced Accelerations for Everyone

Intuitive induced accelerations

Case study: Pedaling

Case study: Throwing a ball

A quick study of gait

Discussion

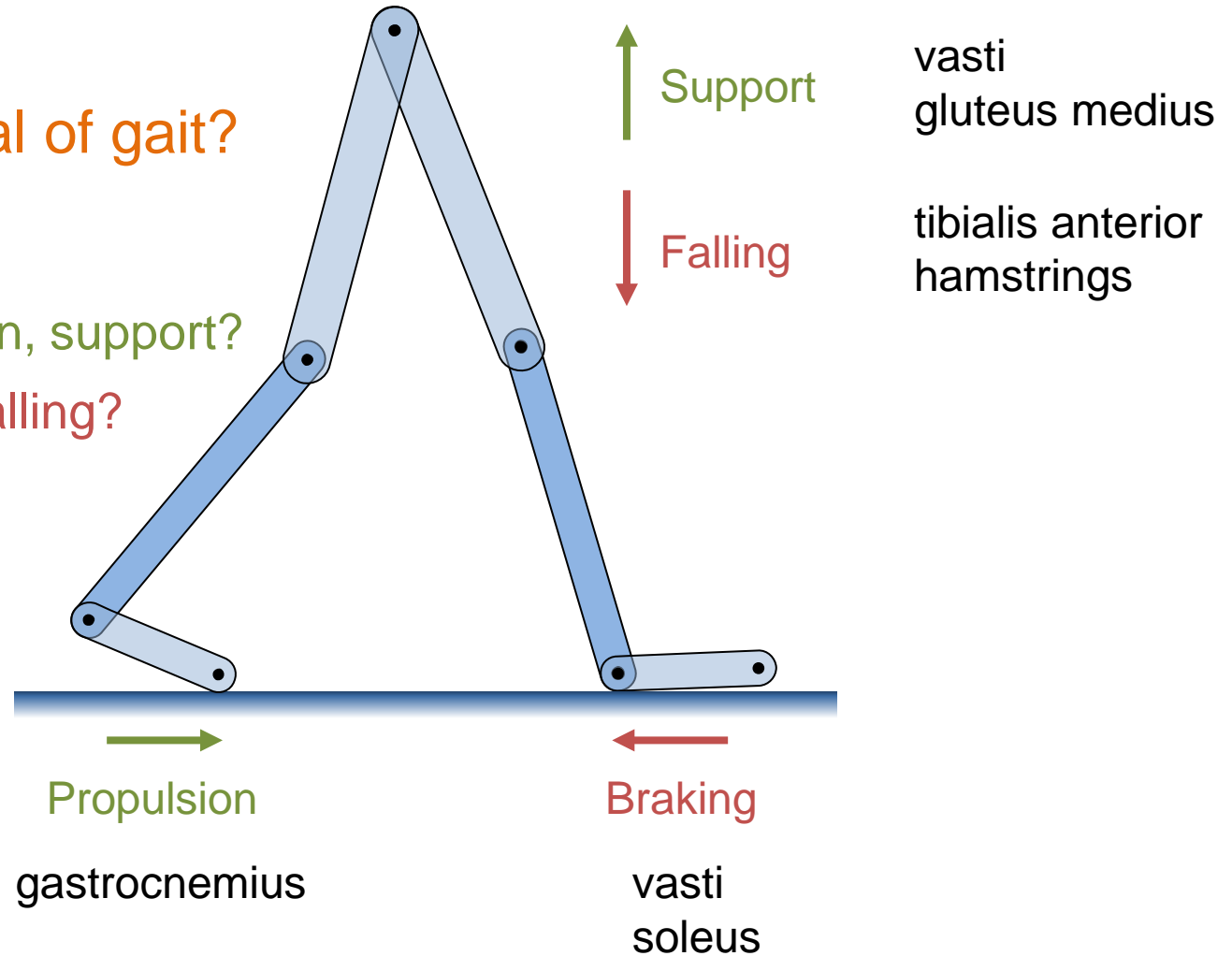
Induced Accelerations for Everyone

The connotation problem

What is the goal of gait?

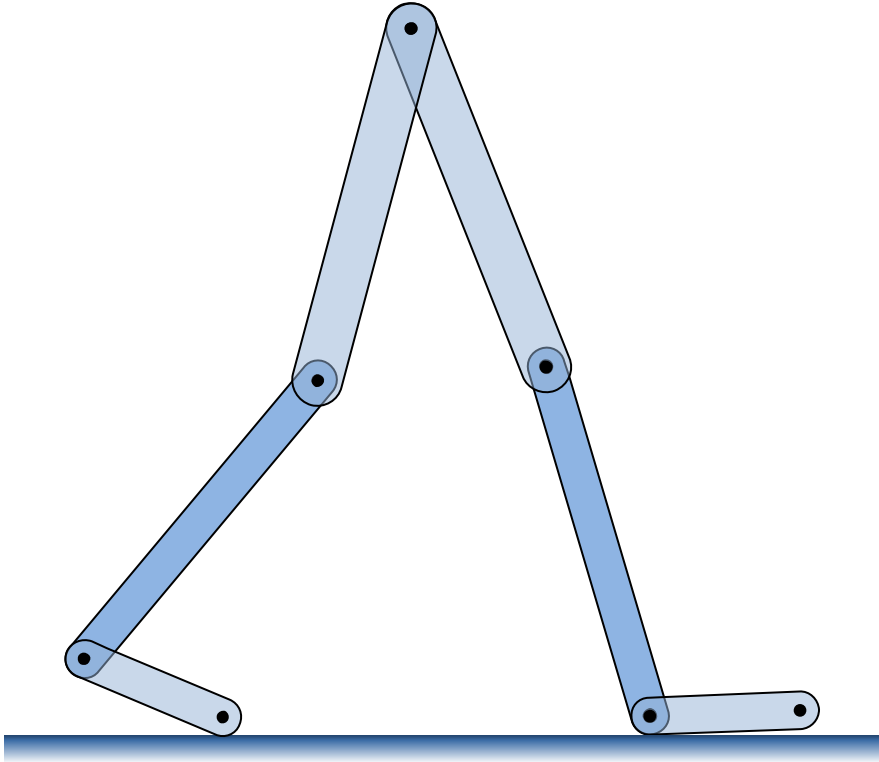
Promote propulsion, support?

Reduce braking, falling?



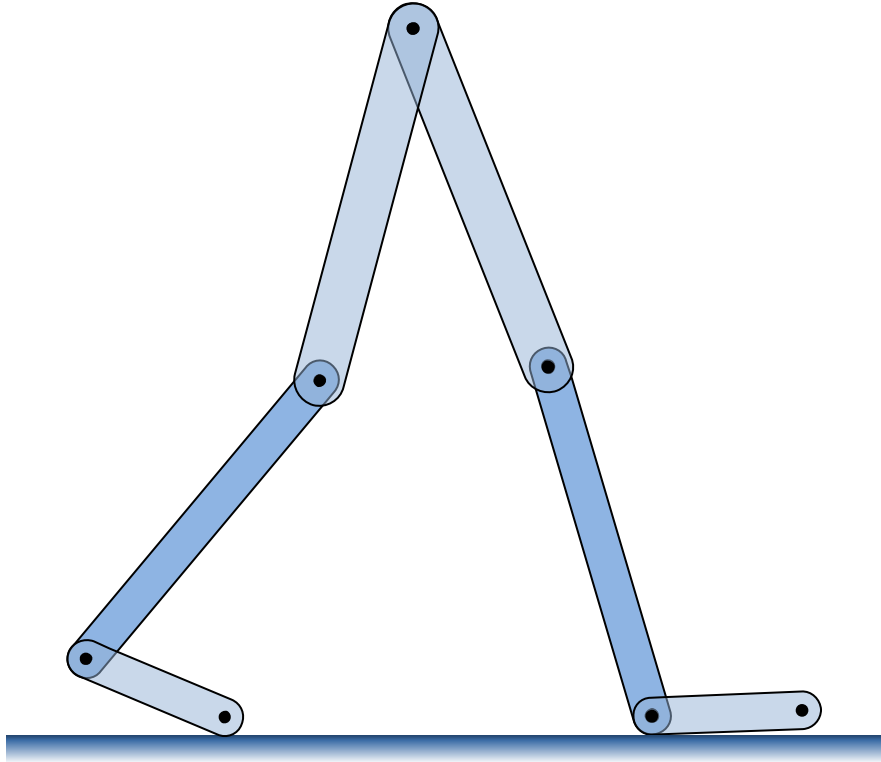
Walking requires zero net work, zero net acceleration, zero net vertical displacement

Some issues with induced accelerations



- Issues
 - Connotation problem
 - Model dependencies
 - Snapshot analysis
- Interpretations often not predictive
- Conclusions have not been well tested

The future of induced accelerations



- Recognize model dependencies
- Avoid connotations
- Study movement over time
- Perform more and better experimental tests

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